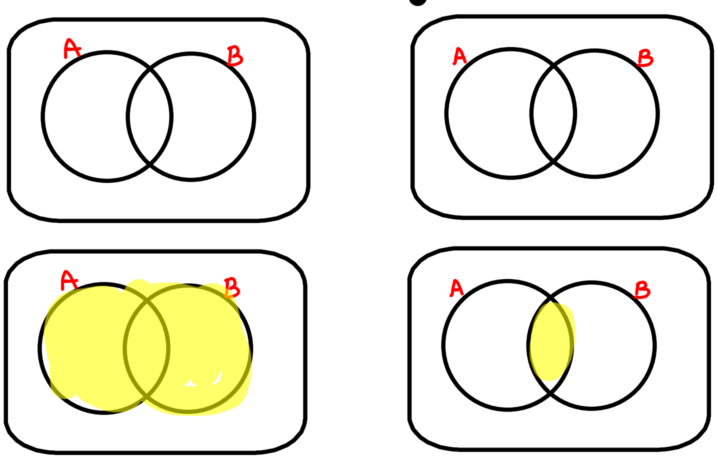
Warm Up - Pick Up the handout.

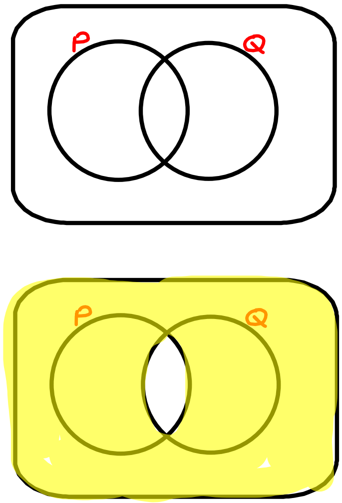
HW questions

1. $A \cup B$

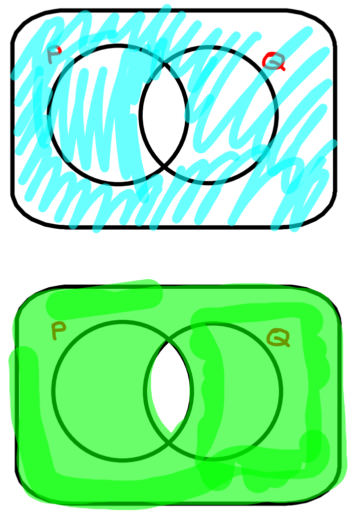
2. $A \cap B$



3 $(P \cap Q)'$



4 $P' \cup Q'$



Logic Assignment 2

Were there any tautologies?

p	q	$\wedge$ $\vee$ $\underline{\vee}$ $\Rightarrow$ $\Leftrightarrow$
T	T	
T	F	
F	T	
F	F	

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T			
T	F			
F	T			
F	F			

p	q	$\neg q$	$p \vee \neg q$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T					
T	F					
F	T					
F	F					

7.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$
T	T	T	T	T	F	
T	T	F	T	F	T	
T	F	T	F	T	F	
T	F	F	F	T	T	
F	T	T	T	T	F	
F	T	F	T	F	T	
F	F	T	T	T	F	
F	F	F	T	T	T	

f T

T T T F

F F

A.

The Veterinarian has gathered the following data about the weight of dogs and the weight of their puppies.

		Dog: Heavy	Dog: Light	Total
Puppy	Heavy	36	27	63
	Light	22	35	57
	Total	58	62	120

$\frac{57 \cdot 62}{120} =$

$$\chi^2 = \sum \frac{(f_e - f_e)^2}{f_e}$$

The veterinarian wishes to test the following hypotheses.

H_0: A puppy's weight is independent of its parent's weight.
H_1: A puppy's weight is related to the weight of its parent.

(a) The table below sets out the elements required to calculate the χ^2 value for this data.

	f_o	f_e	$f_e - f_o$	$(f_e - f_o)^2$	$(f_e - f_o)^2 / f_e$
heavy/heavy	36	30.45	−5.55	30.8025	1.012
heavy /light	27	32.55	5.55	30.8025	0.946
light/heavy	22	27.55	5.55	30.8025	1.118
light/light	35	a	b −5.55	c	d

(i) Write down the values of a, b, c, and d. **(4)**

$a =$

(ii) What is the value of χ^2_{calc} for this data? **(1)**

(iii) How many degrees of freedom exist for the contingency table? **(1)**

(iv) Write down the critical value of χ^2 for the 5% significance level

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$\frac{57 \cdot 62}{120} = 29.45$

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light/heavy	22	27.55	5.55	30.8025	1.118
light/light	35	a 29.45	b −5.55	c	d

(i) Write down the values of a, b, c, and d. **(4)**

$a = 29.45$ 29.5 $b = -5.55$ $c = 30.8025$ 30.8 $d = 1.0459\ldots$ 1.05

(ii) What is the value of χ^2_{calc} for this data? **(1)**

$\boxed{4.12}$

(iii) How many degrees of freedom exist for the contingency table? **(1)**

$(2-1)(2-1) = 1$

(iv) Write down the critical value of χ^2 for the 5% significance level. **(1)**

$\chi^2_{critical} = 5.99$ ← I'll give this to you

(b) Should H_0 be accepted? Explain why. **(2)**

We fail to reject H_0 because the χ^2 value was not greater than the critical value from the table.

(Total 9 marks)

B.

A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

where N is the number of teenagers who have heard the rumour t hours after it first started.

(a) Find the number of teenagers who started the rumour. $t=0$ $2(1.81)^{(.7\cdot 0)} = 2$ teenagers **(2)**

(b) Write down the number of teenagers who have heard the rumour 5 hours after it first started. **(1)**

$N = 2(1.81)^{(.7\cdot 5)}$

$= 15.955\ \ldots\ 16$ teenagers

Two functions $f(x)$ and $g(x)$ are given by

$\boxed{C}$

$$f(x)=\frac{1}{x^2+1},$$
$$g(x)=\sqrt{x},\ x\geq 0.$$

(a) Sketch the graphs of $f(x)$ and $g(x)$ together on the same diagram using values of x between -3 and 3, and values of y between 0 and 2. You must label each curve.

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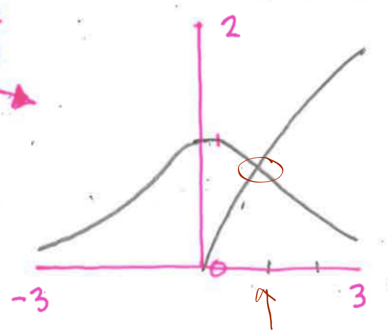
should look very close to this



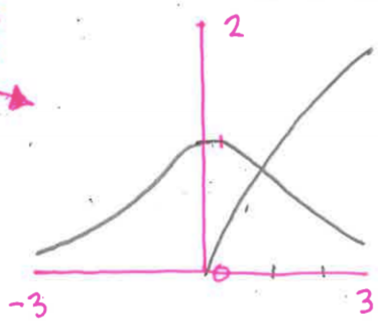
$$\frac{1}{x^2+1}=\sqrt{x}$$

$$\frac{1}{x^2+1}-\sqrt{x}=0$$

is there ~~are~~ only one intersection

(a) Sketch the graphs of $f(x)$ and $g(x)$ together on the same diagram using values of x between -3 and 3, and values of y between 0 and 2. You must label each curve.

should look very close to this



there ~~are~~ is only one intersection

(b) State how many solutions exist for the equation $\frac{1}{x^2+1} - \sqrt{x} = 0$. 1

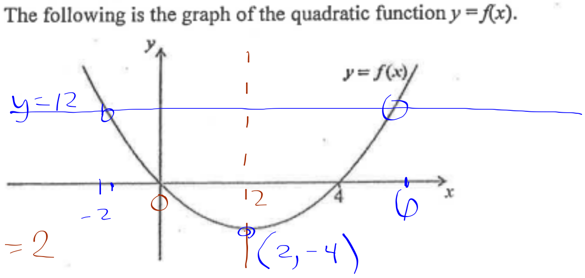
(c) Find a solution of the equation given in part (b).

can do with GDC intersections feature

$\boxed{x = 0.570 \text{ to 3 sf}}$

$\boxed{D}$ The following is the graph of the quadratic function $y = f(x)$.

$f(x) = 12$



$\frac{0+4}{2} = 2$

(a) Write down the solutions to the equation $f(x) = 0$. $x=0$ $x=4$

(b) Write down the equation of the axis of symmetry of the graph of $f(x)$.

$x=2$

$y > -4$

(c) The equation $f(x) = 12$ has two solutions. One of these solutions is $x = 6$. Use the symmetry of the graph to find the other solution.

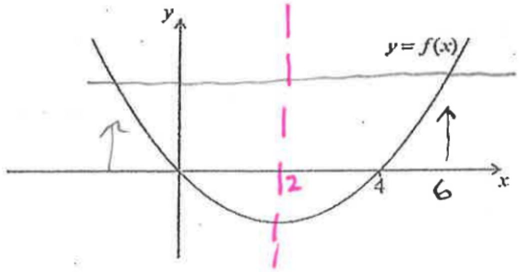
$x = -2$

(d) The minimum value for y is -4. Write down the range of $f(x)$.

$-4 \leq y < \infty$

D

The following is the graph of the quadratic function $y = f(x)$.



(a) Write down the solutions to the equation $f(x) = 0$. $x = 0$ and $x = 4$ **(2)**

(b) Write down the equation of the axis of symmetry of the graph of $f(x)$. $x = 2$ **(2)**

$\frac{0+4}{2} = 2$

(c) The equation $f(x) = 12$ has two solutions. One of these solutions is $x = 6$. Use the symmetry of the graph to find the other solution. $x = -2$ **(1)**

(d) The minimum value for y is -4. Write down the range of $f(x)$. **(1)**

$-4 \leq y < \infty$

(Total 6 marks)

Goal today: Use Truth tables to verify logical statements being equivalent or not

including De'Morgan's Law

handout

Implication $p \to q$

Converse

Inverse

Contrapositive

Implication: If $2x=6$ then $x=3$

Converse: If $x=3$ then $2x=6$

Inverse If $2x\neq 6$, then $x\neq 3$

Contrapositive If $x\neq 3$, then $2x\neq 6$

Implication: If $2x=6$ then $x=3$

Converse: If $x=3$ then $2x=6$

Inverse If $2x\neq 6$, then $x\neq 3$

Contrapositive If $x\neq 3$, then $2x\neq 6$

on a quiz

F T f F

T F

				Implication	Converse	Inverse	Contrapositive
p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

p	q	$\neg p$	$\neg q$	Implication $p \to q$	Converse $q \to p$	Inverse $\neg p \to \neg q$	Contrapositive $\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Implication: If $2x=6$ then $x=3$

Converse: If $\underline{x=3}$ then $\underline{2x=6}$

Inverse If $\underline{2x \neq 6}$, then $\underline{x \neq 3}$

Contrapositive If $\underline{x \neq 3}$, then $\underline{2x \neq 6}$

We can use deMorgan's laws to help us to negate compound statements

Negate the following compound statement using precise language:

p $\wedge$ q

the class sings <u>and</u> Dalton cringes

$(p \wedge q)$

hey... that statement is not true

The first deMorgan's property

$\neg(p \wedge q) = \neg p \vee \neg q$

and or

a) **Negation of:** the class sings and Dalton cringes (p, q)

is: the class doesn't sing or Dalton doesn't cringe

b) **Negation of:** $10 \leq n \leq 20$

$n \geq 10$ and $n \leq 20$ same

is: $n < 10$ or $n > 20$

The 2nd property

$$\neg(p \vee q) = \neg p \wedge \neg q$$

or and

Negation of: Griffin jumps **or** Brenda sneezes

is: Griffin doesnt jump and Brenda doesnt sneeze

Will DeMorgan's Laws always work ?

We can prove that two logical statements are equivalent by showing their truth tables are equivalent

$$\neg(p \vee q) = \neg p \wedge \neg q$$

$$\neg(p \vee q) = \neg p \wedge \neg q$$

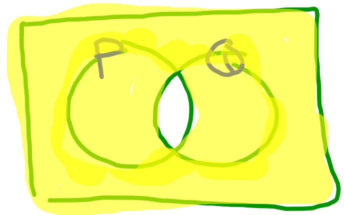
p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	f
T	f	T	f
f	T	T	f
f	f	f	T

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	f	f	f
T	f	f	T	f
f	T	T	f	f
f	f	T	T	T

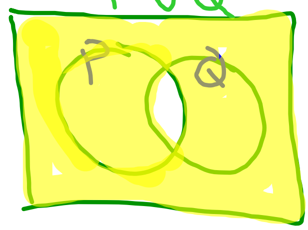
p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$(P \cap Q)'$



$P' \cup Q'$



Logic Assignment #3

- p.509..... 1ad, 3ae, 5b
- p.504...... 3c
- <u>and</u> construct your own truth table for:

$$p \vee (\neg p \wedge q)$$

pg 504 15C..... 3c

3 Use deMorgan's properties to find the negation of:

c $x < -1$ or $x > 7$

$\neg(p \wedge q) = \neg p \vee \neg q$

$\neg(p \vee q) = \neg p \wedge \neg q$

negation

$x \geq -1$ and $x \leq 7$

$p \vee (\neg p \wedge q)$

	$p \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q)$

p	q		$p \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q)$

p	q		$\neg p \wedge q$	$p \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q)$

p	q	$\neg p$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q)$ $\Longrightarrow$

p	q	$\neg p$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F