Pick Up the
Warm Up

Warm Up 6.3 Day 3

In a survey of 500 U.S. teenagers aged 14 to 18, subjects were asked a variety of questions about personal finance. One question asked was whether teens had a debit card. Suppose that exactly 12% of teens aged 14 to 18 have debit cards.

Let X = the number of teens in a random sample of size 500 who have a debit card.

1. Explain why **X** can be modeled by a binomial distribution even though the sample was selected without replacement.

When taking a random sample of size n from a population of size N, we can use a binomial distribution to model the count of successes in the sample as long as $n < 0.10N$. We refer to this as the **10% condition**.

1. Explain why **X** can be modeled by a binomial distribution even though the sample was selected without replacement.

10% Condition As long as $n < 0.10N$ we can use a binomial distribution

500 < 10% of Population of teenagers, so ok.

2. Since a binomial distribution can be used, estimate the probability *(using binomial probability)* that exactly 60 teens in the sample have debit cards.

3. Use a binomial distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

2. Since a binomial distribution can be used, estimate the probability *(using binomial probability)* that exactly 60 teens in the sample have debit cards.

$P(X=60) = {}_{500}C_{60}\,(.12)^{60}(.88)^{440} = .055$

or binompdf(n: 500, p: .12, K: 60) = same

3. Use a binomial distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

$P(X \leq 50) =$ binomcdf(n: 500, p: .12, value: 50)

up to 50

$= 0.093$

same as $P(0) + P(1) + P(2) + \ldots\ldots P(50)$

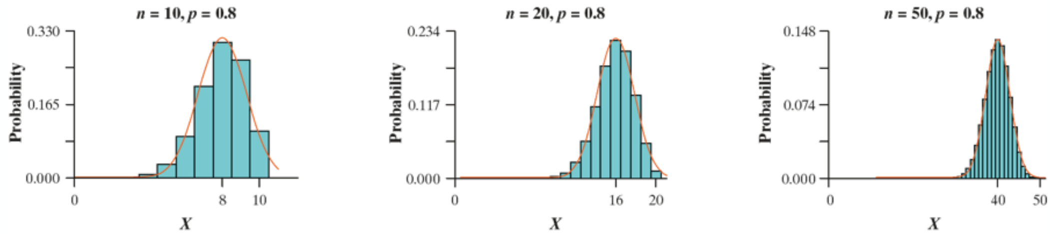
NOT ON AP EXAM

4. Justify why X can be approximated by a Normal distribution.

5. Use a Normal distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

The Normal Approximation to Binomial Distributions*

As the number of observations n becomes larger, the binomial distribution gets close to a Normal distribution.



$np = 8$ $np = 16$ $np = 40$

$n(1-p) = 2$ $n(1-p) = 4$ $n(1-p) = 10$

Large Counts Condition

Suppose that a count X of successes has the binomial distribution with n trials and success probability p. The **Large Counts condition** says that the probability distribution of X is approximately Normal if

$$np \geq 10 \text{ and } n(1-p) \geq 10$$

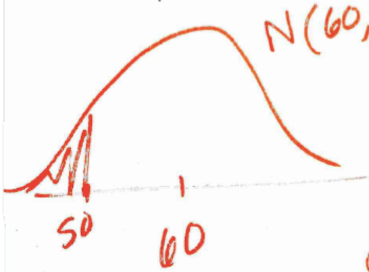
That is, the expected numbers (counts) of successes and failures are both at least 10.

3. Justify why X can be approximated by a Normal distribution.

$500 \times .12 = 60 \geq 10$ ✓ Large counts condition was met.

$500 \times .88 = 440 \geq 10$ ✓

4. Use a Normal distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.



$N(60, 7.27)$

$z = \frac{x - \mu}{\sigma}$

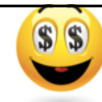
$z = \frac{50 - 60}{7.27}$

$= -1.38$

Area = .084

A game called Greed

Lesson 6.3: Day 4: GREED



We're going to play **Greed**. Each round you must decide if you want to sit or stand. If you sit, you keep all earned points but are no longer playing. If you stand, you must play the round. You earn 1 point for each round you make it through. Mr. Cedarlund is going to roll a die. If the die lands on a number from 1 to 5, the people standing move on to the next round and earn a point. If the die lands on a 6 the people standing lose <u>all</u> their points.

Practice Game

Now a real game

with some added incentive

greed

grēd/

noun

noun: **greed**

intense and selfish desire for something, especially wealth, power, food, or homework points.

For every point you end up with at the end of the game, I will add an extra point to your homework score for the chapter.

This game could end quickly, last a long time, or somewhere in between.

Remember.
Once you sit, I will write down your total at that point.

We keep going as long as at least one person is standing AND a six has not shown up.

2. Let ***X* = the number of rounds played until a 6 occurs.** Is this a binomial setting?

B

I

N

S

2. Let ***X* = the number of rounds played until a 6 occurs.** Is this a binomial setting?

B Success → Roll a 6
Failure → Other than a 6

I Rolls are independ.

N $n = ?$ not a set number of trials

S $p = 1/6$

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it's a Geometric Distribution

3. Use probability rules to calculate the probability for each of the following (knowing that it is not a binomial situation) Show work.

   a. P(X = 1) $= \frac{1}{6} = .167$

   b. P(X = 2) $\frac{5}{6} \cdot \frac{1}{6} =$

   c. P(X = 3)

   d. P(X = 4)

   e. P(X = k)

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   a. P(X = 1) $= \frac{1}{6} = .167$

   b. P(X = 2) $= \frac{5}{6} \times \frac{1}{6} = .139$

   not a 6 → ← a six

   c. P(X = 3)

   d. P(X = 4)

   e. P(X = k)

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not a 6 — a six

c. $P(X = 3)$ $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .116$

not — a six

d. $P(X = 4)$

e. **$P(X = k)$**

•

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c. $P(X = 3)$ $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .116$

not — a six

d. $P(X = 4)$ $\frac{5}{6} \cdot \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .096$

e. **$P(X = k)$**

•

$= (\text{Prob of failure})^{k-1}(\text{prob. of success})$

$= \left(\frac{5}{6}\right)^{k-1}\left(\frac{1}{6}\right)$

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   a. $P(X = 1)$ $= \frac{1}{6} = .167$

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   not a 6 a six

   c. $P(X = 3)$ $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .116$

   not a six

   d. $P(X = 4)$ $\frac{5}{6} \bullet \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .096$

   e. P(X = k) $\left(\frac{5}{6}\right)^{k-1}\left(\frac{1}{6}\right)$

•

4. Write the probability that a 6 is rolled *within the first 4 rolls* in terms of *X* and find the probability. Show your work.

$P(X \leq 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

Now Using technology

4. Write the probability that a 6 is rolled *within the first 4 rolls* in terms of X and find the probability. Show your work.

$$P(X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$.167 + .139 + .116 + .096$$

$$= .518$$

Now Using technology

for all

$$P(X \le 4) = \text{geometcdf}\left(\tfrac{1}{6}, 4\right) = .158 \quad .518 \quad \cancel{.58}$$

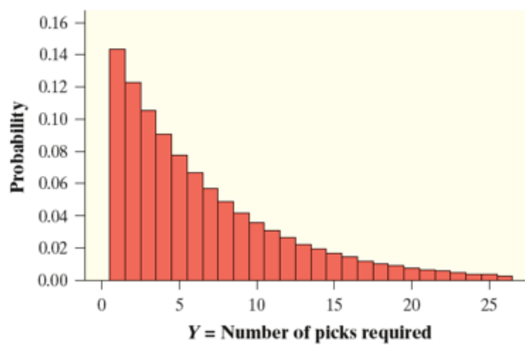
for individuals

ie. $P(X=3) = \text{geometpdf}\left(\tfrac{1}{6}, 3\right) = .116$

5. How many rolls would you **predict** it to take until a 6 is rolled? *(you won't be asked this on an AP Exam or a test in this class, FYI)*

mean $\frac{1}{p} = \frac{1}{\frac{1}{6}} = 6$ rolls

6. What shape would the distribution of *X* have? *(you won't be asked this either)*



skewed right
- Always

Geometric Distributions

Important ideas:

Calculator Commands:

$geometpdf\ (p, k)$ computes $P(Y = k)$

$geometcdf\ (p, k)$ computes $P(Y \leq k)$ $= P(Y = 1) + P(Y = 2) + \cdots \ldots . P(Y = k)$

Geometric Distributions

Important ideas:

B binary
I Independent
T #trials until a success
S same probability

$P(Y=k) = (1-p)^{k-1}p$

mean $= \frac{1}{p}$

shape – skewed right.

Calculator Commands:

$geometpdf\ (p,k)$ computes $P(Y=k)$

$geometcdf\ (p,k)$ computes $P(Y \le k)$ $= P(Y=1) + P(Y=2) + \cdots \ldots . P(Y=k)$

Check Your Understanding

Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot. **Let T = the number of spins it takes until Marti wins.**

1. Show that T is a geometric random variable.

Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot. **Let T = the number of spins it takes until Marti wins.**

1. Show that T is a geometric random variable.

B success → win
failure → lose

I spins are independent

T number of TRIALS until Marti wins

S same prob. $p = \frac{1}{38}$

2. Find P(T = 3). Interpret this result.

3. How many spins do you expect it to take for Marti to win? (not on AP exam)

$(1-\frac{1}{38})^2(\frac{1}{38})$

2. Find P(T = 3). Interpret this result.

$P(T=3) = \frac{37}{38} \cdot \frac{37}{38} \cdot \frac{1}{38} = .025$

or geomet pdf (P: $\frac{1}{38}$, K: 3) = .025

There is a 2.5% probability that Marti wins for the first time on the 3rd spin.

3. How many spins do you expect it to take for Marti to win? (not on AP exam)

$\text{mean} = \frac{1}{\frac{1}{38}} = 38$

4. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

4. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

$$P(T \leq 3) = P(T=1) + P(T=2) + P(T=3)$$

$$= \text{geomet cdf}\left(p: \tfrac{1}{38}, k: 3\right)$$

$$= .077$$

Not too surprised. winning in 3 or fewer spins has a prob of 7.7% which is not that rare.

6.3.....107, 109, 111, 113-116

Study..... pp. 422-426