Last of the presentations

6.3 Day 2 (more on binomial distributions)

✓CALCULATE the mean and standard deviation of a binomial random variable. INTERPRET these values.

✓When appropriate, USE the Normal approximation to the binomial distribution to CALCULATE probabilities.*

Lesson 6.3: Day 2: Will the SHS girls' soccer team win?



When the time runs out in a soccer game and the score is tied, the game will go to a shootout. Each team gets to choose 5 players to kick penalty kicks. Whichever team makes the most penalty kicks wins. Suppose the IRISH soccer team makes 60% of their penalty kicks, what are the chances they will win the game?

1. Is this a binomial setting? Explain.

B — Success (make PK) / Failure (miss PK)

I — shots are Independent

N — fixed, n = 5

S — p = .6

B(5, .6)

When the time runs out in a soccer game and the score is tied, the game will go to a shootout. Each team gets to choose 5 players to kick penalty kicks. Whichever team makes the most penalty kicks wins. Suppose the IRISH soccer team makes 60% of their penalty kicks, what are the chances they will win the game?

1. Is this a binomial setting? Explain.

Binary: Success → make Penalty / Failure → don't make / Each shot Independent / Trials n = 5 / same prob. p = 0.6

So, yes. B(5, 0.6)

2. Fill in the table below showing the probability of making X penalty kicks. Mr. C may show you a shortcut.

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Goals (X)	0	1	2	3	4	5
Probability						

P(0 successes)

$= {}_5C_0 (.6)^0 (.4)^5$

$\binom{5}{0}$

P(3)

${}_5C_3 (.6)^3 (.4)^2$

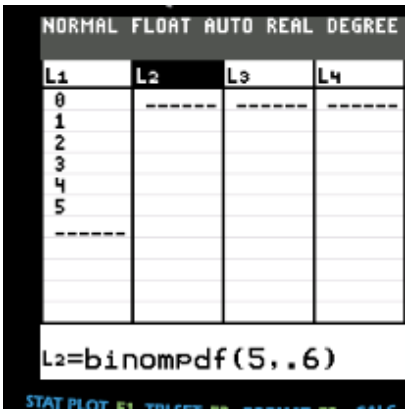
2. Fill in the table below showing the probability of making X penalty kicks. Mr. C may show you a shortcut.

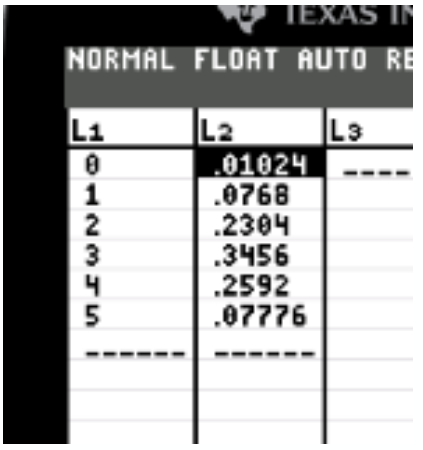
Goals (X)	0	1	2	3	4	5
Probability						

Technology Assist

L_1	L_2
0	
1	
2	
3	
4	
5	

L_2 → binompdf(5, .6)





1. Is this a binomial setting? Explain.

Binary: Success → make Penalty / Each shot Independent / Trials n=5 / same prob. p=0.6 B(5, 0.6)

2. Fill in the table below showing the probability of making X penalty kicks. Mr. C may show you a shortcut.

Goals (X)	0	1	2	3	4	5
Probability	.01024	.0768	.2304	.3456	.2592	.07776

3. Find and interpret the mean of the probability distribution. Show your work.

4. Find and interpret the standard deviation of the distribution.

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for rand var distrib

$\sum x \cdot p$

4. Find and interpret the standard deviation of the distribution.

$\sigma = \sqrt{\sum (x-\mu)^2 p}$

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3

3. Find and interpret the mean of the probability distribution. Show your work.

$\mu = 3$ After many shootouts, we expect the average number of goals made to be 3 of out the 5 attempted.

4. Find and interpret the standard deviation of the distribution.

$\sigma = 1.09$ We expect the number of goals made in a shootout to typically vary by 1.09 goals from the mean of 3 out of 5 goals.

$\sigma = \sqrt{\sum (x-3)^2 p}$

Do You happen to see a shortcut for μ and σ ?

[a shortcut for Binomial Distributions only]

3. Find and interpret the mean of the probability distribution. Show your work.

$\mu = 3$ After many shootouts, we expect the average number of goals made to be 3 of out the 5 attempted.

$\mu = np = 5(.6) = 3$

4. Find and interpret the standard deviation of the distribution.

$\sigma = 1.09$ We expect the number of goals made in a shootout to typically vary by 1.09 goals from the mean of 3 out of 5 goals.

$\sigma = \sqrt{np(1-p)} = \sqrt{5(.6)(.4)} = 1.09$

(II) Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$E(X) = \mu_x = \Sigma x_i p_i$

$\text{Var}(X) = \sigma_x^2 = \Sigma(x_i - \mu_x)^2 p_i$

If X has a binomial distribution with parameters n and

$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

$\mu_x = np$

$\sigma_x = \sqrt{np(1-p)}$

5. What is the probability that the team scores at least one goal?

6. If the other team is expected to make 3 goals, what is the probability that the SHS girls' team wins?

5. What is the probability that the team scores at least one goal?

$P(\text{at least } 1) = 1 - P(\text{none})$
$= 1 - .01024 = .98976$

6. If the other team is expected to make 3 goals, what is the probability that the SHS girls' team wins?

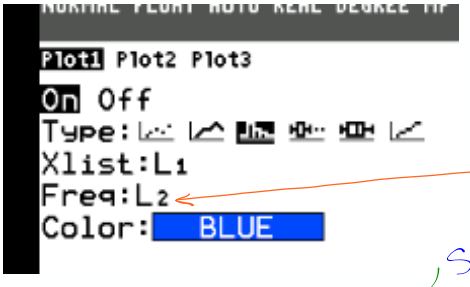
$P(4 \text{ or } 5) = P(4) + P(5)$
$= .2592 + .07776$

$= .337$
$.33696$

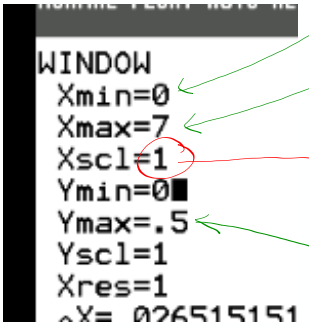
7. Use technology to make a histogram of the probability distribution and then Describe its shape.

L_1	L_2
0	
1	
2	
3	
4	
5	

binompdf



Freq: L_2

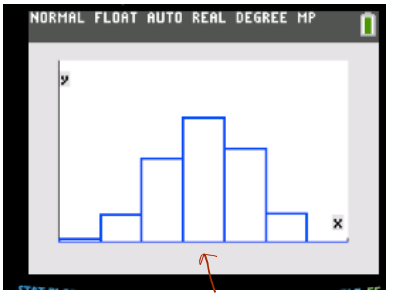


Smallest value of binom. rand. variable

A bit larger than rand. var

Must be 1

Larger than your highest prob



Symmetric
with Single Peak
at 3 shots
made

Textbook Site

Extra Applets
- Probability
→ Binomial Distrib

http://bcs.whfreeman.com/webpub/statistics/spa3e/analyze_data/prob.html

Describing Binomial Distributions

Important ideas:

Describing Binomial Distributions

Important ideas:

Shape:

Center:

Variability:

Describing Binomial Distributions

Important ideas:

Shape: Make a histogram → Quick sketch on your paper

Center: $\mu = np$

Variability: $\sigma = \sqrt{np(1-p)}$

Pop Quiz

Guessing

POP QUIZ GUESSING: Mr. Miller's class is very difficult. It's so hard that when he gave a pop quiz recently, the students just guessed on every question! Each student in the class guesses an answer from A through E on each of the 10 multiple-choice questions. Hannah is one of the students in this class. Let Y = the number of questions that Hannah answers correctly.

Note: B: **Success** (makes a penalty shot) **Failure** (does not make the penalty)
I: Each shot is independent.
N: Set # of trials, $n = 10$
S: Same probability, $p = 0.2$

THEREFORE: _____

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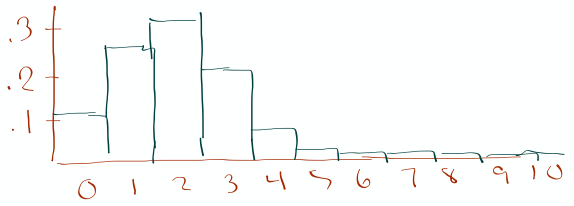
Note: B: **Success** (makes a penalty shot) **Failure** (does not make the penalty)
I: Each shot is independent.
N: Set # of trials, $n = 10$
S: Same probability, $p = 0.2$

THEREFORE: This setting represents a binomial distribution.

1. Use technology to make a histogram of the probability distribution of Y. Describe its shape.



Skewed right with a single peak at 2 questions correct

2. Calculate the mean of Y and then complete the interpretation.

After _____ quizzes, we expect the average number of correct _____ is _____ questions.

3. Calculate the standard deviation of Y and then complete the interpretation.

The number correct on a quiz of 10 questions _____ varies by _____ from the mean of 2 questions.

2. Calculate the mean of Y and then complete the interpretation. $\mu = np$

After _____ quizzes, we expect the average number of correct _____ is _____ questions.

3. Calculate the standard deviation of Y and then complete the interpretation.

$\sigma = \sqrt{np(1-p)}$

The number correct on a quiz of 10 questions _____ varies by _____ from the mean of 2 questions.

2. Calculate the mean of Y and then complete the interpretation. $\mu = np = 10 \cdot 0.2 = 2$

After many many quizzes, we expect the average number of correct out of 10 is 2 questions.

3. Calculate the standard deviation of Y and then complete the interpretation.

$\sigma = \sqrt{np(1-p)} = \sqrt{10(.2)(.8)} = 1.26$

The number correct on a quiz of 10 questions typically varies by 1.26 from the mean of 2 questions.

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Let's read

In practice, the binomial distribution gives a good approximation to situations that don't have replacement (non-independence) as long as we sample less than 10% of the population. This is called the 10% condition.

When taking a random sample of size n from a population of size N, we can use a binomial distribution to model the count of successes in the sample as long as $n < 0.10N$. We refer to this as the **10% condition**.

PEW RESEARCH CENTER: A recent report from the Pew Research Center estimates that 71% of teenagers aged 13–17 use Facebook. Assume this claim is true. Suppose that some researchers are going to contact a random sample of 300 teenagers to find out if they use Facebook. Let X = the number of teens in a random sample of size 300 who use Facebook.

(a) Explain why X can be modeled by a binomial distribution even though the sample was selected without replacement.

(b) Use a binomial distribution to estimate the probability that 200 or more teens in the sample use Facebook.

(a) Explain why X can be modeled by a binomial distribution even though the sample was selected without replacement.

300 is definitely less than 10% of all teenagers aged 13-17

(b) Use a binomial distribution to estimate the probability that 200 or more teens in the sample use Facebook.

... 197 198 199 200 201 202 ...

$P(X \geq 200)$

$= 1 - (X \leq 199)$

$= 1 -$ binomcdf [trials: 300, p: 0.71, X-value: 199]

$= 1 - 0.044$

$= 0.956$

See Take Home LCQ

6.3....91, 93, 95, 99, 101, 105

and study pp. 412-421