Agenda

One project presentation

Lesson on Combining Randcom Variables

Second project presentation, more if time

Combining Random Variables

(pages 388-390)

$Z = X + Y$ $Z = X - Y$

When Combining Random Variables:

The means "play nice"

The standard deviations don't "play nice".

Lesson 6.2: Day 2: How much will you make next year?

After much thought Mr. Cedarlund has finally decided on permanent employee wages which are randomly assigned using the probability distribution *X* given below. Additionally, at the end of every year he gives his employees an hourly raise. The bonuses are assigned randomly according to the probability distribution *Y* given below. Assume *X* and *Y* are independent.

1. **Find the mean, variance and standard deviation of the probability distribution of *X*, the hourly wages.**

X	9	12	15
Probability	0.30	0.45	0.25

Mean: ________ Variance: ________ Standard Deviation: ___________

2. **Find the mean, variance and standard deviation of the probability distribution of *Y*, the annual hourly raise.**

Y	$1	$3
Probability	0.70	0.30

Mean: _________ Variance: _______ Standard Deviation: ___________

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$\sum x_i p_i$

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$\sum (x_i - \mu)^2 p_i$

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Mean: _________ Variance: _______ Standard Deviation: __________

$$E(X) = \mu_x = \sum x_i p_i$$

11.85

$$\mathrm{Var}(X) = \sigma_x^2 = \sum \left(x_i - \mu_x\right)^2 p_i$$

L1 L2

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Mean: 11.85 Variance: 4.93 Standard Deviation: 2.22

2. Find the mean, variance and standard deviation of the probability distribution of *Y*, the annual hourly raise.

Y	\$1	\$3
Probability	0.70	0.30

Mean: 1.60 Variance: .839 Standard Deviation: .917

3. Let *N* = the new hourly wage for the upcoming year (*X* + *Y*).

a. What are all the possible new hourly wages for the new year?

b. What is the probability of an employee being assigned a \$9 wage **AND** a \$1 raise? Show your work.

c. Complete the table below for the probability distribution of $N = X + Y$ and find the mean and standard deviation.

N						
Probability						

Mean: ________ Variance: _________ Standard Deviation: _________

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a. What are all the possible new hourly wages for the new year?

9+1 12+1 15+1
9+3 12+3 15+3 so 10, 12, 13, 15, 16, 18

b. What is the probability of an employee being assigned a $9 wage **AND** a $1 raise? Show your work.

$P(\$9 \cap 1) = P(\$9) \times P(1) = (.3)(.7) = .21$

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N	10	12	13	15	16	18
Probability	.21	.09	.315	.135	.175	.075

Mean: ________ Variance: ________ Standard Deviation: ________

NORMAL FLOAT AUTO REAL DEGREE

L1	L2	L3	L4
10	.21	------	------
12	.09		
13	.315		
15	.135		
16	.175		
18	.075		
------	------		

$\mu = 10(.21) + 12(.09) + \cdots$
$= 13.45$

$VAR(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$

$(10 - 13.45)^2(.21) + (12 - 13.45)^2(.09)$
$+ \cdots = 5.76$

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N	10	12	13	15	16	18
Probability	.21	.09	.315	.135	.175	.075

Mean: 13.45 Variance: 5.76 Standard Deviation: 2.40

$\sqrt{5.76}$

11.85 + 1.60
= 13.45

4.93 + .839
= 5.76

$\sqrt{2.22^2 + .917^2}$

d. If $N = X + Y$, complete the following in terms of X and Y:

$\mu_N = \mu_x + \mu_y$

$\sigma_N = \sqrt{\sigma_x^2 + \sigma_y^2}$

Combining Probability Distributions

Important ideas:

Adding & Subtracting Random Variables

Normal Probab. Distribution

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$\mu_{X+Y} = \mu_X + \mu_Y$

$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$

$\mu_{X-Y} = \mu_X - \mu_Y$

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Normal Probab. Distribution

Notice

CAUTION:

When we subtract two independent random variables, their variances **add**.

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Notice

Normal Probab. Distribution

Find new mean and standard deviation

Example

A large auto dealership keeps track of sales and lease agreements made during each hour of the day. Let *X* = the number of cars sold and *Y* = the number of cars leased during the first hour of business on a randomly selected Friday. Based on previous records, the probability distributions of *X* and *Y* are as follows:

Cars sold x_i	0	1	2	3
Probability p_i	0.3	0.4	0.2	0.1

Mean: $\mu_X = 1.1$ Standard deviation: $\sigma_X = 0.943$

Cars leased y_i	0	1	2
Probability p_i	0.4	0.5	0.1

Mean: $\mu_Y = 0.7$ Standard deviation: $\sigma_Y = 0.64$

Define *T* = *X* + *Y*. *Assume that X and Y are independent.*

1. Find and interpret μ_T.

2. Calculate and interpret σ_T.

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Define *T* = *X* + *Y*. *Assume that X and Y are independent.*

1. Find and interpret μ_T. $\mu_T = 1.1 + 0.7 = 1.8$ cars

   Over many many Fridays, the dealer expects to sell ~~or~~ and lease about 1.8 cars on average.

2. Calculate and interpret σ_T. $\sigma_T = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{.943^2 + .64^2} = 1.14$ cars

   The number of cars sold and leased typically vary by 1.14 cars from the mean (1.8 cars).

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Transformation
(multiplier 500)

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$$= 550 + 210$$
$$= \$760$$

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$$\mu_B = 500(1.1) + 300(.7)$$
$$= 550 + 210$$
$$= \$760$$

$$\sigma_B = \sqrt{(500 \cdot .943)^2 + (300 \cdot .64)^2}$$
$$= \$509.09$$

Hoop
Fever

Hoop Fever 1 *(Mean of a sum or difference of random variables)*

Hoop Fever is an arcade basketball game in which a player has 60 seconds to make as many baskets as possible. Morgan and Tim play head-to-head every Tuesday. Let M = the number of baskets made by Morgan and T = the number of baskets made by Tim in a randomly selected match.

A. Based on previous matches, we know that $\mu_M = 39.8$ and $\mu_T = 31.2$. Let $D = M - T$. Calculate and interpret the mean of D.

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A. Based on previous matches, we know that $\mu_M = 39.8$ and $\mu_T = 31.2$. Let $D = M - T$. Calculate and interpret the mean of D.

SOLUTION:

$$\mu_D = \mu_M - \mu_T = 39.8 - 31.2 = 8.6 \quad \text{baskets}$$

The difference (Morgan – Tim) in the number of baskets made is 8.6 baskets, on average, over many randomly selected matches.

B. Based on previous matches, we know also that the *standard deviations* are $\sigma_M = 5.7$ and $\sigma_T = 10.3$. Assume that these two random variables are independent. Define $D = M - T$. Earlier, we found that $\mu_D = 8.6$.

Calculate and interpret the standard deviation of *D*.

B. Based on previous matches, we know also that the *standard deviations* are $\sigma_M = 5.7$ and $\sigma_T = 10.3$. Assume that these two random variables are independent. Define $D = M - T$. Earlier, we found that $\mu_D = 8.6$.

Calculate and interpret the standard deviation of *D*.

Because *M* and *T* are independent random variables,

$$\sigma^2_D = \sigma^2_M + \sigma^2_T = 5.7^2 + 10.3^2 = 138.58$$

Variance

$$\sigma_D = \sqrt{138.58} = 11.77 \text{ baskets}$$

The difference (Morgan – Tim) in the number of baskets made typically varies by about 11.77 baskets from the difference in means of 8.6 baskets.

Now

Normal Distribution

C. Suppose that T = the number of baskets made by **Tim** in a randomly selected match follows an **approximately Normal distribution** with $\mu_M = 31.2$ and $\sigma_T = 10.3$. Assume that these two random variables are independent and define $D = M - T$.

(a) Describe the distribution of D.

Already ready for this

(b) What is the probability that Morgan will make more baskets than Tim in a randomly selected match?

(a) Describe the distribution of *D*.

SOLUTION:

(a) *Shape*: Approximately Normal

Center: $\mu_D = 39.8 - 31.2 = 8.6$ baskets

Variability: $\sigma_D = \sqrt{5.7^2 + 10.3^2} = 11.77$ baskets

(b) What is the probability that Morgan will make more baskets than Tim in a randomly selected match?

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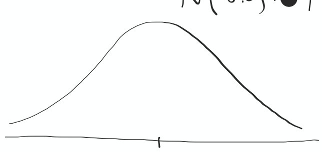
Morgan will win if $D = M - T > 0$

N(8.6, 1[illegible])

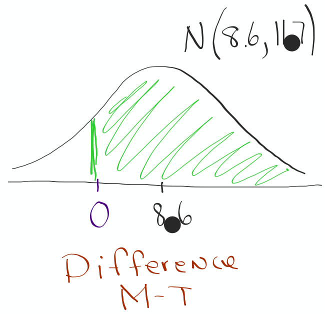


8.6

Difference
M-T

(b) What is the probability that Morgan will make more baskets than Tim in a randomly selected match?

Morgan will win if $D = M - T > 0$



$$Z = \frac{0 - 8.6}{11.77} = -.73$$

normalcdf(Low: -.73 upper: mean: SD:)

normalcdf(Low -.73 upper: 1000 mean: 0 SD: 1)

≈ 0.7673 :)

short
LCQ

Presentation

6.2 ....49, 51, 55, 57, 59,

65, 67, 73-74

388-397

study pp. ~~381-387~~ including the example on p. 387