

You can lay your posters down on the back desk.

Starting tomorrow, wall do 2 or 3 presentations a day through next week.

Find $\mathrm{P}(\mathrm{Y} \leq 63)$. Interpret this value.


$$
\begin{aligned}
z & =\frac{63-64}{2.7} \\
& =-.37
\end{aligned}
$$

TABLE $A \rightarrow, 3557$

There is a prob. of 0.3557 that a randomly selected female is less than. or equal to 63 in .

Find $P(68 \leq Y \leq 70)$. Interpret this value.
$\square$

## Reminder

A z-score tells us the number of standard deviations above or below the mean that a value falls in a distribution.

## Transforming a Random Variable (pages 382-388)

## DESCRIBE the effect of: adding or subtracting a constant or multiplying or dividing by a constant

on the probability distribution of a random variable.


1. Copy the data collected from yesterday's lesson below.

| $X$ | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability |  |  |  |  |  |  |

Mean: $\qquad$ Standard Deviation: $\qquad$

| Lesson 6.2: Day 1: Time for a Raise |
| :--- |
| Mr. Cedarlund's employees have been working very hard and it's time he gives them a <br> raise. He is trying to decide if he should give everyone a $\$ 10$ raise (add $\$ 10$ per hour) or <br> double everyone's wage (multiply by 2 ). |

1. Copy the data collected from yesterday's lesson below.

田

| $x$ | 1 | 2 | 5 | 3 | 7 | 7 | 10 | 3 | ${ }^{15} 0$ | 25 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | $2 / 17$ | $3 / 17$ | $7 / 17$ | $3 / 17$ | $0 / 17$ | $2 / 17$ |  |  |  |  |

Mean: $\mu={ }^{\$ 18.590}$
2. To make a decision about what raise should be given, complete the tables below and calculate the new mean and standard deviation using your calculator.
a. Option 1: Add $\mathbf{\$ 1 0}$ per hour to all employees


New Mean $(\mu+10)$ : $\qquad$ New Standard Deviation: $\qquad$ $\leftarrow$

How did adding a constant affect the mean and standard deviation?


$$
\sigma=\sqrt{\sum(x-\mu)^{2} P}
$$

2. To make a decision about what raise should be given, complete the tables below and calculate the new mean and standard deviation using your calculator.
a. Option 1: Add \$10 per hour to all employees

| $X-$ Old <br> Wage | $\mathbf{1}$ | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $Y-$ New <br> Wage |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Probability |  |  |  |  |  |  |

Same as
previous
New Mean $(\mu+10)$ : $\qquad$ New Standard Deviation: $\qquad$ $\leftarrow$

How did adding a constant affect the mean and standard deviation?

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\sigma=\sqrt{\sum(x-\mu)^{2} \cdot P}
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a. Option 1: Add $\$ 10$ per hour to all employees

| $X-$ Old <br> Wage | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $Y-$ New <br> Wage | $1 / 1$ | 15 | 17 | 20 | 25 | 35 |
| Probability | $2 / 17$ | $3 / 17$ | $7 / 17$ | $3 / 17$ | $0 / 17$ | $2 / 17$ |

Same as
previous
table
New Mean $(\mu+10)$ :


New Standard Deviation:


How did adding a constant affect the mean and standard deviation?
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same as
previous
New Mean $(\mu+10): \$ / 8.59$
New Standard Deviation: table

How did adding a constant affect the mean and standard deviation?
The new mean is 10 more than the original.
The standard Lev. did not change.

Adding or subtracting a constant: Think about a histogram for some random variable. If we added 12 to each value, this would simply slide the histogram 12 units to the right,

increasing the measures of center (mean, median, quartiles, percentiles)

- but it would not change the variability or shape.
b. Option 2: Double the wage of all employees

| $X$ - Old <br> Wage | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $Z$ - New <br> Wage |  |  |  |  |  |  |
| Probability |  |  |  |  |  |  |

New Mean( $2 \mu$ ): $\qquad$ Standard Deviation: $\qquad$
How did multiplying by a constant affect the mean and standard deviation?
b. Option 2: Double the wage of all employees


How did multiplying by a constant affect the mean and standard deviation?

Multiplying or dividing by a constant:
Think about a histogram for a rand. variable. that takes values between 1 and 8. If we multiplied each value by 10 the new histogram would go from 10 to 80


This would multiply the measures of center, location, and variability by 10 , but it would not change the shape.

These are the same results we got with transformation of summary statistics back in Ch .2

Transforming Probability Distributions
Important ideas:

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## Transforming Probability Distributions

Important ideas: Multiplying the same constant, $b$,
Adding the same constant, to each value.
C, to each value
Shape stays the same
adds $C$ to the center
variability stays the
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Transforming Probability Distributions

Important ideas:
Adding the same constant, C, to each value.

- Shape stays the same adds $C$ to the center variability stays the same.

Multiplying the same constant, $b$, to each value.

- Shape stays the same
- multiplies center by b
- Variability gets multiplied by b

$$
\begin{aligned}
& S D=\sigma \rightarrow b \sigma \\
& \text { variance }=\text { Squoot of SD } \\
& \quad \text { Var }=(b \sigma)^{2}=b^{2} \sigma^{2}
\end{aligned}
$$

Same with Normal Distributions

same shape!

While the standard deviation is multiplied by $b$, the variance is multiplied by $b^{2}$.

Adding the same positive number $a$ to (subtracting a from) each value of a random variable:

- Adds $a$ to (subtracts $a$ from) measures of center and location (mean, median, quartiles, percentiles).
- Does not change measures of variability (range, IQR, standard deviation).
- Does not change the shape of the probability distribution.


## The Effect of Multiplying or Dividing by a Constant

Multiplying (or dividing) each value of a random variable by the same positive number $b$ :

- Multiplies (divides) measures of center and location (mean, median, quartiles, percentiles) by $b$.
- Multiplies (divides) measures of variability (range, IQR, standard deviation) by b.
- Does not change the shape of the distribution.


## Check Your Understanding \#1 -- Everyone gets a bonus

A large corporation has thousands of employees. The distribution of annual salaries for the employees is skewed to the right, with a mean of $\$ 68,000$ and a standard deviation of $\$ 18,000$. Because business has been good this year, the CEO of the company decides that every employee will receive a $\$ 5000$ bonus. Let $X$ be the current annual salary of a randomly selected employee before the bonus and $Y$ be the employee's salary after the bonus. Describe the shape, center, and variability of the probability distribution of $Y$.

## Check Your Understanding \#1 -- Everyone gets a bonus

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Shape: Skewed Right
Center: $\mu_{Y}=\mu_{X}+5000=68,000+5,000=\$ 73,000$
Variability: $\sigma_{y}=\sigma_{x}=\$ 18,000$


## Check Your Understanding \#2

A large auto dealership keeps track of sales made during each hour of the day Let $X=$ the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of $X$ is as follows:

| Cars sold | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.3 | 0.4 | 0.2 | 0.1 |

The random variable X has mean $\mu_{\mathrm{x}}=1.1$ and standard deviation $\sigma_{\mathrm{x}}=0.943$. Suppose the dealership's manager receives a $\$ 500$ bonus from the company for each car sold. Let $\mathbf{Y}=$ the bonus received from car sales during the first hour on a randomly selected Friday.

1. Sketch a graph of the probability distribution of $X$ and a separate graph of the probability distribution of Y. How do their shapes compare?

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| Cars sold | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.3 | 0.4 | 0.2 | 0.1 |


| 0 | 500 | 1000 | 1500 |
| :---: | :---: | :---: | :---: |
| .3 | .4 | .2 | .1 |

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$$
\text { DISTRibution af } Y
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$$
\text { Distribution af } Y
$$




$$
\begin{aligned}
& 370 \\
& \text { Instructions to } \mathrm{m} \\
& \text { with frequencies }
\end{aligned}
$$

2. Find the mean of $\mathbf{Y}$.
3. Calculate and interpret the standard deviation of $\mathbf{Y}$.
4. The manager spends $\$ 75$ to provide coffee and doughnuts to prospective customers each morning. So, the manager's net profit $T$ during the first hour on a randomly selected Friday is $\$ 75$ less than the bonus earned. Describe the shape, center, and variability of the probability distribution of $T$.
5. Find the mean of $Y$. $\mu_{Y}=1.1 \times 500=\$ 550$
6. Calculate and interpret the standard deviation of $\mathbf{Y}$.

$$
\sigma_{r}=0.943 \times 500=\$ 471.50
$$

The bonuses typically vary by $\$ 471.50$ from the mean (6550)
4. The manager spends $\$ 75$ to provide coffee and doughnuts to prospective customers each morning. So, the manager's net profit $T$ during the first hour on a randomly selected Friday is $\$ 75$ less than the bonus earned. Describe the shape, center, and variability of the probability distribution of $T$.
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The shape will remain the same. The mean will be subtracted by 75 . $\overline{(\mu=550-75}=\$ 475)$ The SD does not change

$$
(\sigma=471.70)
$$

Employees selling refrigerators at an appliance store make money on commission based on how many refrigerators they sell. The number of refrigerators $R$ sold in a randomly selected hour has the following probability distribution:

| Number of <br> refrigerators | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.22 | 0.31 | 0.12 | 0.25 | 0.08 | 0.02 |

Here is a histogram of the probability distribution along with the mean and standard deviation.


At this appliance store, the commission earned is $\mathbf{\$ 3 0}$ for each refrigerator sold. That is, if C= total commission earned for a randomly selected hour, $C=30 R$.

| Number of <br> refrigerators | 0 | 1 | 2 | 3 | 4 | 5 |
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| Probability | 0.22 | 0.31 | 0.12 | 0.25 | 0.08 | 0.02 |

Here is a histogram of the probability distribution along with the mean and standard deviation.


At this appliance store, the commission earned is $\mathbf{\$ 3 0}$ for each refrigerator sold. That is, if $C=$ total commission earned for a randomly selected hour, $C=30 R$.
(a) What shape does the probability distribution of C have?
(a) What shape does the probability distribution of C have?

- The same shape as the prob. distrib. of $R$ - slightly skewed right with two peaks
(b) Find the mean of C .

$$
\mu_{c}=30 \mu_{R}=30(1.72)=451.60
$$

(c) Calculate the standard deviation of C .

$$
\sigma_{c}=30 \sigma_{R}=30(1.36)=\$ 40.80
$$

## See your <br> ch. 5 Test

$$
6.2 \text {....37, 39, 41, 43, 47 }
$$

