5.3 day 2

Use a tree diagram to model a chance process involving a sequence of outcomes and to calculate probabilities.

When appropriate, use the multiplication rule for independent events to calculate probabilities.

Pick Up the Warm Up

In the first part of section 5.3 there was a Conditional Probability Formula:

Calculating Conditional Probabilities

To find the conditional probability P(A | B), use the formula

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{both events occur together})}{P(\text{given event occurs})}$$

The choice of which event is A and which is B is arbitrary, so it is also true that:

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{P(B \cap A)}{P(A)}$$

If you take the last version, $P((B|A) = \frac{P(A \cap B)}{P(A)}$ and re-arrange using a little algebra to solve for $P(A \cap B)$, you would get a new formula known as the General Multiplication Rule:

For any chance process, the probability that events A and B both occur can be found using the **general multiplication rule**:

P(A and B) = P(A ∩ B) = P(A) · P (B|A)

This general rule says that for both of two events to occur, first one must occur. Then, given that the first event has occurred, the second must occur.

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Example Hot coffee

Students who work at a local coffee shop recorded the drink orders of all the customers on a Saturday. They found that 64% of customers ordered a hot drink, and 80% of these customers added cream to their drink.

Find the probability that a randomly selected Saturday customer orders a hot drink and adds cream to the drink.

$$P(\text{hot drink and Adds Cream}) = P(\text{hot drink}) \cdot P(\text{adds cream} \mid \text{hot drink}) = (.64)(.80)$$

$$= .512$$

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B

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$$P(A \text{ and } B) = P(A) \cdot P(B \| A)$$

Pull Out
Your AP
formula sheet

(I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\sum\left(x_i - \bar{x}\right)^2}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1}\Sigma\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

(II) Probability

For any chance process, the probability that events A and B both occur ca be found using the **general multiplication rule**:

P(A and B) = P(A ∩ B) = P(A) · P (B|A)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

The general multipl. rule is not here.

$$E(X) = \mu_X = \sum x_i p_i$$

$$\mathrm{Var}(X) = \sigma_x^2 = \Sigma(x_i - \mu_x)^2 p_i$$

This formula can be extended to 3 or more events

For any chance process, the probability that events A and B … be found using the **general multiplication rule**:

$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B|A)$

$$P(A \text{ and } B \text{ and } C)$$

$$= P(A \cap B \cap C)$$

$$= \underbrace{P(A) \cdot P(B|A)}_{} \bullet P(C|A \cap B)$$

The Game

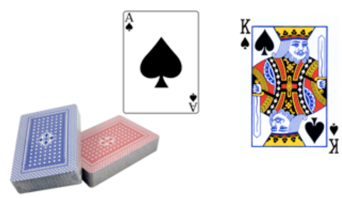
The dealer holds five cards total: 2 aces and 3 Kings.

The player chooses their first card, and then chooses their second card (without replacement).

The player wins if they get a pair of Aces or a pair of Kings. Otherwise the dealer wins.

volunteers to
play me ?

Lesson 5.3: Day 2:
Can you get a pair of Aces or a pair of Kings?



<u>Rules of the game</u>. Five cards total: two aces and three Kings. The player chooses their first card and records the results, and then chooses their second card (without replacement) and records the result. **The player wins if they get a pair of Aces or a pair of Kings.**

1. Choose one person who is the dealer and one who is the player. Play the game 10 times.

First card										
Second card										
Winner?										

Based on your 10 games, what is the probability of winning this game? ___________

Class results:

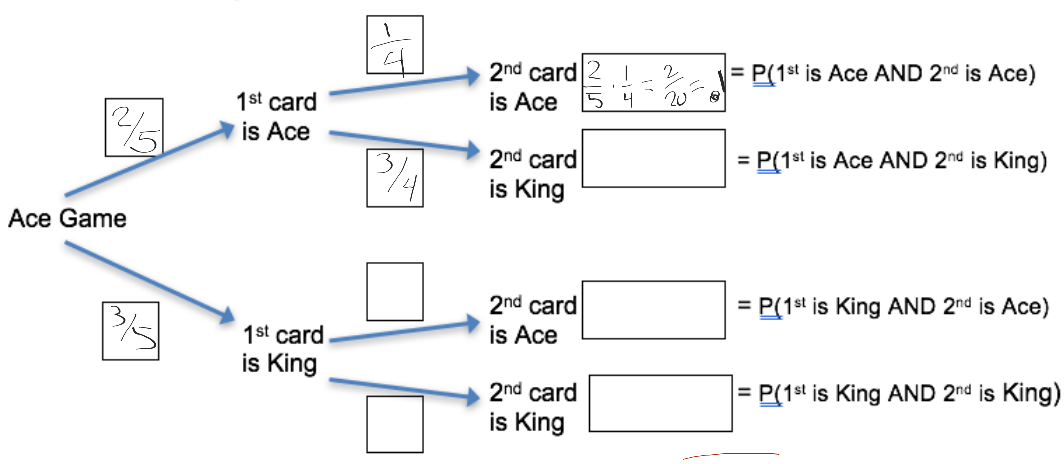
5 5 7 3 4

3 4 4 8 5

record # of wins

Based on the whole class data, what is the probability of winning this game? $\frac{48}{100} = .48$

3. Let's try to use a Tree Diagram to calculate the theoretical probability. Fill in the blank boxes with the correct probabilities.



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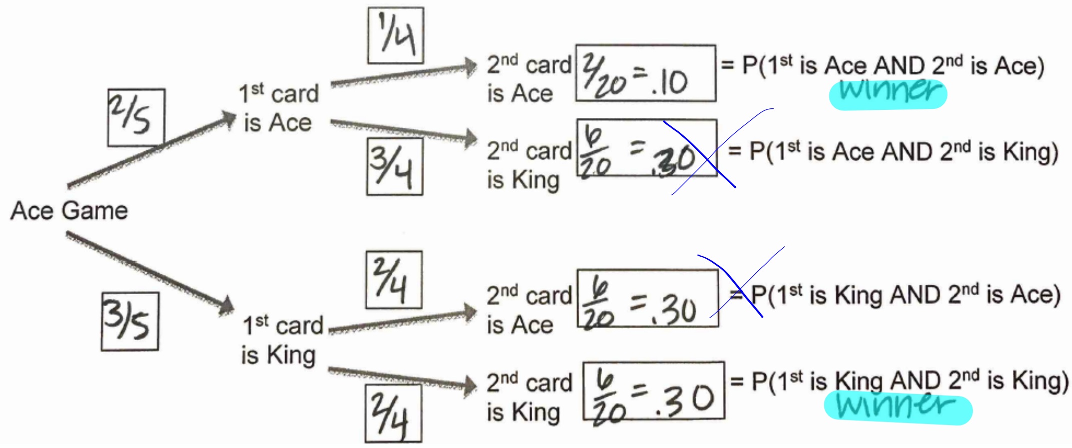

.4

A Tree Diagram is a strategy for solving some types of probability problems that allows you to avoid the dangers of FORMULAS.

Same with Venn Diagrams and Two-Way Tables

4. Find the theoretical probability of winning the game.

$P(\text{2 aces or 2 Kings}) = 0.1 + 0.3 = \underline{\underline{0.4}}$

5. What is the probability that the 1st card was a King, given that the person won the game?

$P(\text{1st King} \mid \text{won}) = \frac{0.3}{0.4} = .75$

Conditional Probability and Independence

Big Ideas:

General Mult. Rule

$P(A \cap B) = P(A) \times P(B|A)$

* if A and B are independent

$P(B) = P(B|A)$ so

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$P(\text{at least } 1) = 1 - P(\text{none})$

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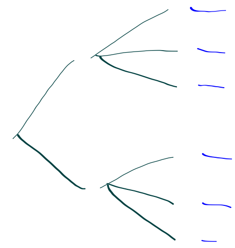
Tree Diagram



label prob. along paths

Find all $\cap$ prob.

Make sure they add to 1

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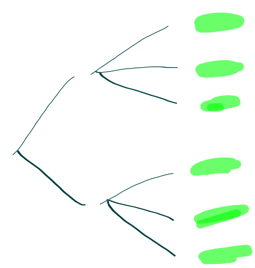

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Find all $\cap$ prob.

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If you calculate all probabilities, you can answer any question

It can be a struggle at times to choose the correct strategy

HINT

Most conditional probability questions can be solved using a tree diagram or a Two-way table.

CYU: A computer company makes desktop, laptop, and tablet computers at factories in two states: California and Texas. The California factory produces 40% of the company's computers and the Texas factory makes the rest. Of the computers made in California, 25% are desktops, 30% are laptops, and the rest are tablets. Of those made in Texas, 10% are desktops, 20% are laptops, and the rest are tablets. All computers are first shipped to a distribution center in Missouri before being sent out to stores. Suppose we select a computer at random from the distribution center and observe where it was made and whether it is a desktop, laptop, or tablet.

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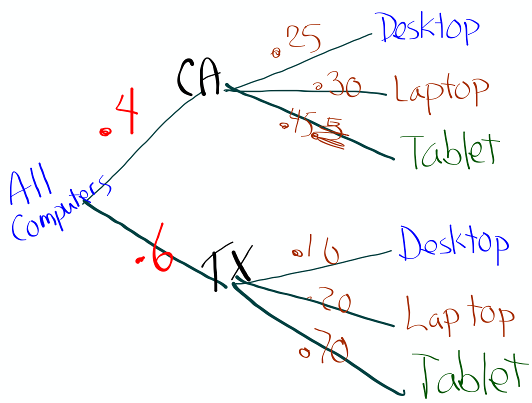

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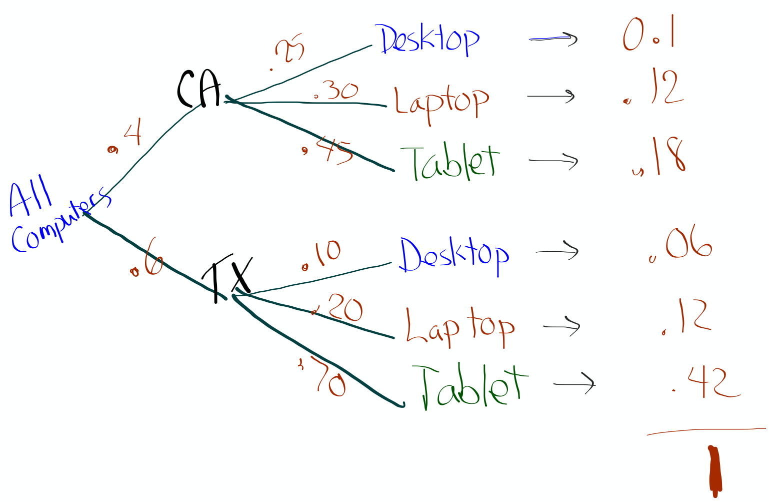

2. Find the probability that the computer is a tablet.

$.18 + .42 = .60$

3. If we select 4 computers at random from the distribution center (with replacement) what is the probability that at least 1 of the computers is a tablet computer?

$P(\text{tablet}) = .18 + .42 = .6$

$P(\text{no tablet}) = 1 - .6 = .4$

$P(\text{all 4 NOT tablets}) = (.4)^4$

$P(\text{at least 1 is a tablet}) = 1 - P(\text{none are tablets})$

$= 1 - (.4)^4$

$= \cancel{.8704}$ .9744

4. Given that a tablet computer is selected, what is the probability that it was made in California?

$P(CA \mid \text{tablet}) = \dfrac{0.18}{0.6} = .30$ ← both, ← given

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Notice you don't really need a formula.

Confusion between

Independence and Mutually Exclusive

may want to take notes

I. Select 1 card from a deck of cards

A: red card B: club

A and B are mutually exclusive

$P(A) = .5$

$P(A \mid B) = 0$

II. Select 1 card

A: red B: card is a 7

because there are red 7's, NOT mutually exclusive

$P(A) = 0.5$

$P(A \mid B) = 0.5$

Knowing it's a 7

these events are independent because knowing it was a 7 didn't matter.

III

Select 1 card

A: red B: heart

← NOT mutually Exclusive because you can have a red heart

Because $P(A) = 0.5$ and $P(A|B) = 1$

heart

these events are not independependent

because it does matter that the card was a heart.

LCQ

can use your AP formula sheet

5.3 ....81, 83, 87, 89, 91,

93, 99, 103-106

and study pp. 338-347

including the Mammogram Example on p.342