Section 5.2

# Probability Models

(pages 314 - 315)

Get a partner.

Decide who will be ODD and who will be EVEN

Give a probability model for a chance process with equally likely outcomes and use it to find the probability of an event.

Give a probability model for a chance process with equally likely outcomes and use it to find the probability of an event.

Use basic probability rules, including the complement rule and the addition rule for mutually exclusive events.

**AP Stats Section 5.2 Day 1 – Odds or Evens, Who Will Win?**


We're going to play a game to answer this question. You and your partner must decide who will be "Odds" and who will be "Evens". Then you will roll two dice and **multiply** the numbers. If the product is odd, the odds person wins and vice versa for evens. Play 20 times, keeping track of how many wins each person has.

Pick up the handout when finished

Whole Class Results - Record Number of times odds won

5, 7, 7, 7, 6, 6, 6, 6

Maybe the odds just had a run of bad luck. Let's see how the rest of the class did with odds. Write the number of odds wins for your group in the table on the board.

2. Find the total percent of rolls that were odd products for the whole class. 31.3%

$\frac{50}{160}$

How does this compare to your group's results?

3. To determine the true probability of rolling an odd product, we should list out all possible products that we could get. Complete the table below to show all possible products (multiply).

4. Use your table to find the probability of rolling an odd product.

5. Which was closer to the percentage you found in #4, your group data or the classroom data? Why do you think that is?

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |

6. Use the table to find the probability of rolling each of the following products:

4 or a 5 b) Number besides 6 c) Number from 1 to 36

3. To determine the true probability of rolling an odd product, we should list out all possible products that we could get. Complete the table below to show all possible products (multiply).

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 |

4. Use your table to find the probability of rolling an odd product.

$\frac{9 \text{ odds}}{36 \text{ outcomes}} = \frac{1}{4} = 0.25$

5. Which was closer to the percentage you found in #4, your group data or the classroom data? Why do you think that is?

The class data is closer because there are more rolls. Long term is predictable with probability.

6. Use the table to find the probability of rolling each of the following products:

a) 4 or a 5 — "OR"

$\frac{3}{36} + \frac{2}{36} = \boxed{\frac{5}{36}}$

b) Number besides 6 — Complement

$1 - \frac{4}{36} = \boxed{\frac{32}{36}}$

c) Number between 1 and 36 — Prob. must add to 1 — Inclusive

$= \boxed{1}$

1. Simulation
2. **Sample Space**
3. Two-Way Table
4. Venn Diagram
5. Tree Diagram
6. Formulas

In Section 5.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.


Pick up the 2nd handout

In Section 5.1, we used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.


A **probability model** is a description of some chance process that consists of two parts: a list of all possible outcomes and the probability for each outcome.
The list of all possible outcomes is called the **sample space.**

Same Spaces can vary from Simple

Tossing two coins
(4 possible outcomes)

HH HT TT
TH

to →

Uber complicated

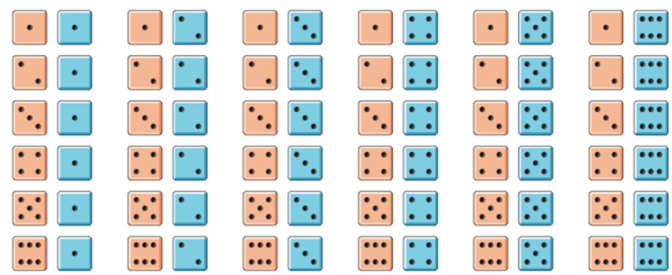
When the Gallop poll takes a random sample of 1532 U.S. adults from the entire population (240 million)

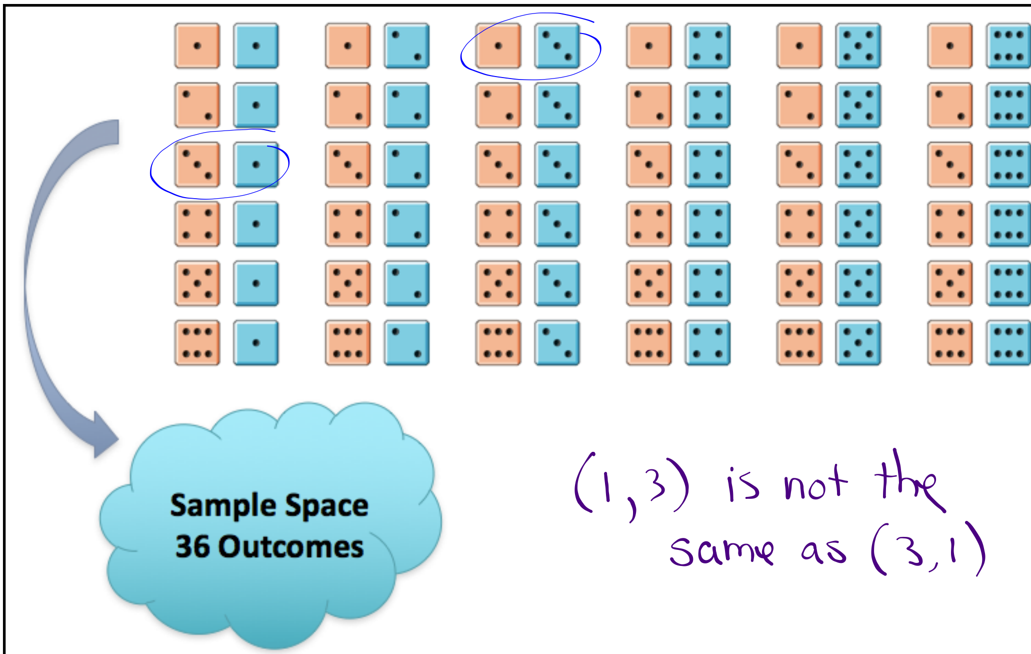
How many combinations?

$1.8 \times 10^{8575}$ possible samples of size 1532 when choosing from 240 mill.

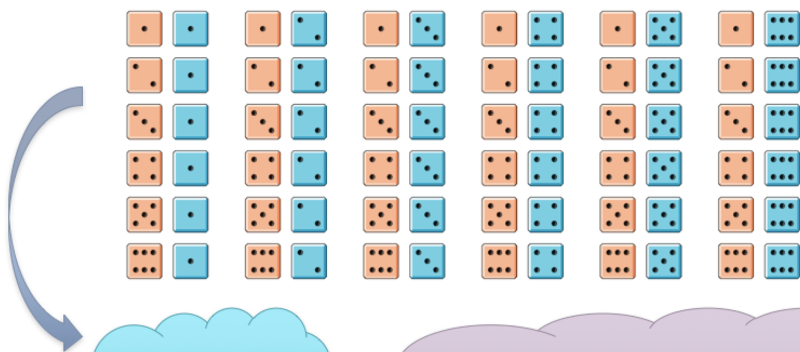
## Probability Models

the classic sample space






(1, 3) is not the same as (3, 1)



Sample Space
36 Outcomes

Since the dice are fair,
each outcome is equally likely.
Each outcome has probability 1/36.

An **event** is any collection of outcomes from some chance process.

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Let A = getting a sum of 5 when two fair dice are rolled


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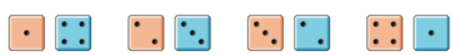
Let A = getting a sum of 5 when two fair dice are rolled


There are 4 outcomes that result in a sum of 5.



Complement - Probability of not getting a sum of 5.

Since $P(A) = \frac{4}{36}$

$P(A^c) = 1 - \frac{4}{36} =$

$\frac{36}{36} - \frac{4}{36} = \frac{32}{36}$

# Basic Probability Rules
(pages 316-318)

These rules may very well be "common sense"

Volunteers to read
top of p. 316

AP Tip

Students will lose any chance for partial credit if you conclude a probability is less than 0 or greater than 1.

Other common notation for Compliment $A^c$

$A'$

$\sim A$

$\rightarrow$

incorrect $P(B)^c$

correct $P(B^c)$

OK ~~not~~ not to reduce fractions

(in fact it is better not to)

$\frac{32}{36}$ ← easier to compare probabilities (and add/subtract fractions)

## Finding Probabilities: Equally Likely Outcomes

If all outcomes in the sample space are equally likely, the probability that event A occurs can be found using the formula

$$P(A) = \frac{number\ of\ outcomes\ in\ event\ A}{total\ number\ of\ outcomes\ in\ sample\ space}$$

If we roll two fair dice, what is the probability that the sum is 6?

P(sum is 6) = $\frac{5}{36}$

If we roll two fair dice, what is the probability that the sum is 5 or 6?

P(sum is 5 or sum is 6) = P(sum is 5) + P(sum is 6)= $\frac{4}{36} + \frac{5}{36} = \frac{9}{36} = 0.25$

Two events A and B are **mutually exclusive** if they have no outcomes in common and so can never occur together—that is, if P(A and B) = 0.
The **addition rule for mutually exclusive events** A and B says that P(A or B) 5= P(A) + P(B)

We can summarize the basic probability rules in symbolic form.

**Basic Probability Rules**

- For any event *A*, $0 \leq P(A) \leq 1$.
- If *S* is the sample space in a probability model, $P(S) = 1$
- In the case of equally likely outcomes,

$$P(A) = \frac{number\ of\ outcomes\ in\ event\ A}{total\ number\ of\ outcomes\ in\ sample\ space}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If *A* and *B* are mutually exclusive, $P(A\ or\ B) = P(A) + P(B)$

Rock Paper Scissors

## Rock Paper Scissors

There is a website where humans can play paper, scissors, rock with a computer. Irresistibly drawn to it, you play the game 2 times. Assume that the computer is randomly choosing its moves for both games.

(a) Give a probability model for the computer's chance process.

(b) Define event A as the computer chooses the same move for both games. Find P(A).

## Rock Paper Scissors

There is a website where humans can play paper, scissors, rock with a computer. Irresistibly drawn to it, you play the game 2 times. Assume that the computer is randomly choosing its moves for both games.

(a) Give a probability model for the computer's chance process.

Sample Space

PP SP RP
PS SS RS
PR SR RR

Because the computer is randomly choosing each move, each of these outcomes will be equally likely.

(b) Define event A as the computer chooses the same move for both games. Find P(A).

There are 3 outcomes with the computer choosing the same move for both games

PP SS RR

so... $P(A) = \frac{3}{9} = 0.333$

M and M's

2. **Suppose you tear open the corner of a bag of M&M'S® Milk Chocolate Candies, pour one candy into your hand, and observe the color. According to Mars, Inc., the maker of M&M'S, the probability model for a bag from its Cleveland factory is:**

| Color | Blue | Orange | Green | Yellow | Red | Brown |
|---|---|---|---|---|---|---|
| Probability | 0.207 | 0.205 | 0.198 | 0.135 | 0.131 | 0.124 |

= 1

(a) Explain why this is a valid probability model.

✓ The probabilities add up to 1

✓ All prob. are between 0 and 1.

(b) Explain why events Red and Blue are mutually exclusive

M&M's cannot blue and red.

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(a) Explain why this is a valid probability model.

The probabilities add up to 1.

Each prob. is between 0 & 1.

(b) Explain why events Red and Blue are mutually exclusive

An M & M cannot be both red & blue.

**For each of the following write the event using proper notation and find the probability:**

(c) Find the probability that you don't get a blue M&M.

$P(\text{Blue}^c) = 1 - .207 = .793$

(d) What's the probability that you get an orange or a brown M&M?

$P(\text{Or or brown})$

$= P(\text{Or}) + P(\text{br}) = .205 + .124$

$= .329$

See your test.

Exit Ticket

page 318.... Check for Learning.

TURN IN
on separate paper

5.2..... 31, 33, 35, 37, 39

and Study p.314-318