

If you were not here yesterday, give me your question 1 and 2 from the Mat Exam

Pick Up  
the  
Review Warm Up

1

The point A has coordinates  $(4, -8)$  and the point B has coordinates  $(-2, 4)$ .

- (a) Write down the coordinates of C, the midpoint of line segment AB.

The point D has coordinates  $(-3, 1)$ .

- (b) Find the gradient of the line DC.
- (c) Find the equation of the line DC. Write your answer in the form  $ax + by + d = 0$  where  $a$ ,  $b$  and  $d$  are integers.

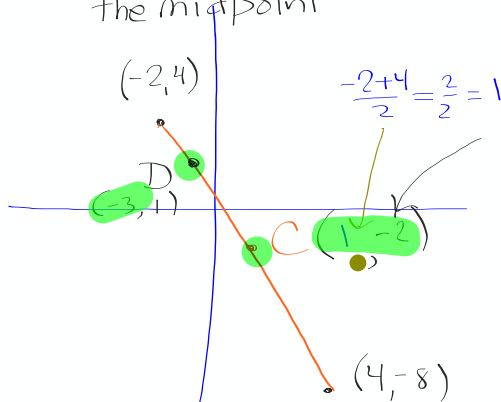
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a) COORDINATES OF the midpoint



b) gradient of DC

$$\text{grad} = \frac{1 - -2}{-3 - 1} = \frac{1+2}{-4} = \frac{3}{-4}$$

$$\left(-\frac{3}{4}\right)$$

c) equation of DC  
in  $ax + by + d = 0$

$$-3x - 4y - 5 = 0$$

$$4y + 8 = -3x + 3$$

$$3x + 4y + 5 = 0$$

$$(1, -2) \quad \text{gr } -\frac{3}{4}$$

$$y = mx + b$$

$$-2 = -\frac{3}{4}(1) + b$$

$$-2 = -\frac{3}{4} + b$$

$$-8 = -3 + 4b$$

$$-5 = 4b$$

$$b = -\frac{5}{4}$$

$$y = -\frac{3}{4}x - \frac{5}{4}$$

$$4y = -3x - 5$$

$$3x + 4y + 5 = 0$$

$$-3x - 4y - 5 = 0$$

$$ax + by + d = 0$$

$$y + 2 = -\frac{3}{4}(x - 1)$$

$$4y + 8 = -3(x - 1)$$

$$4y + 8 = -3x + 3$$

$$3x + 4y + 5 = 0$$

2

Consider the frequency distribution of heights of elementary school students below:

$L_1$   $L_2$

mid-interval	height $h$ (cm)	frequency
132.5	$130 \leq h < 135$	2
137.5	$135 \leq h < 140$	3
142.5	$140 \leq h < 145$	5
147.5	$145 \leq h < 150$	7
152.5	$150 \leq h < 155$	6
157.5	$155 \leq h < 160$	2

a. Determine the midpoint of the  $130 \leq h < 135$  interval.  $132.5 \text{ cm}$

b. Calculate the approximate mean height of the students.

c. Determine the modal class interval.

$$\bar{x} = \frac{\sum f \cdot x}{\sum f} = \frac{3685}{25} = 146.1 \text{ cm}$$

cm

2

Consider the frequency distribution of heights of elementary school students below:

$L_1$   $L_2$

mid-interval	height $h$ (cm)	frequency
132.5	$130 \leq h < 135$	2
137.5	$135 \leq h < 140$	3
142.5	$140 \leq h < 145$	5
147.5	$145 \leq h < 150$	7
152.5	$150 \leq h < 155$	6
157.5	$155 \leq h < 160$	2

a. Determine the midpoint of the  $130 \leq h < 135$  interval.  $132.5$

b. Calculate the approximate mean height of the students.  $146.1 \doteq 146 \text{ cm}$

c. Determine the modal class interval.

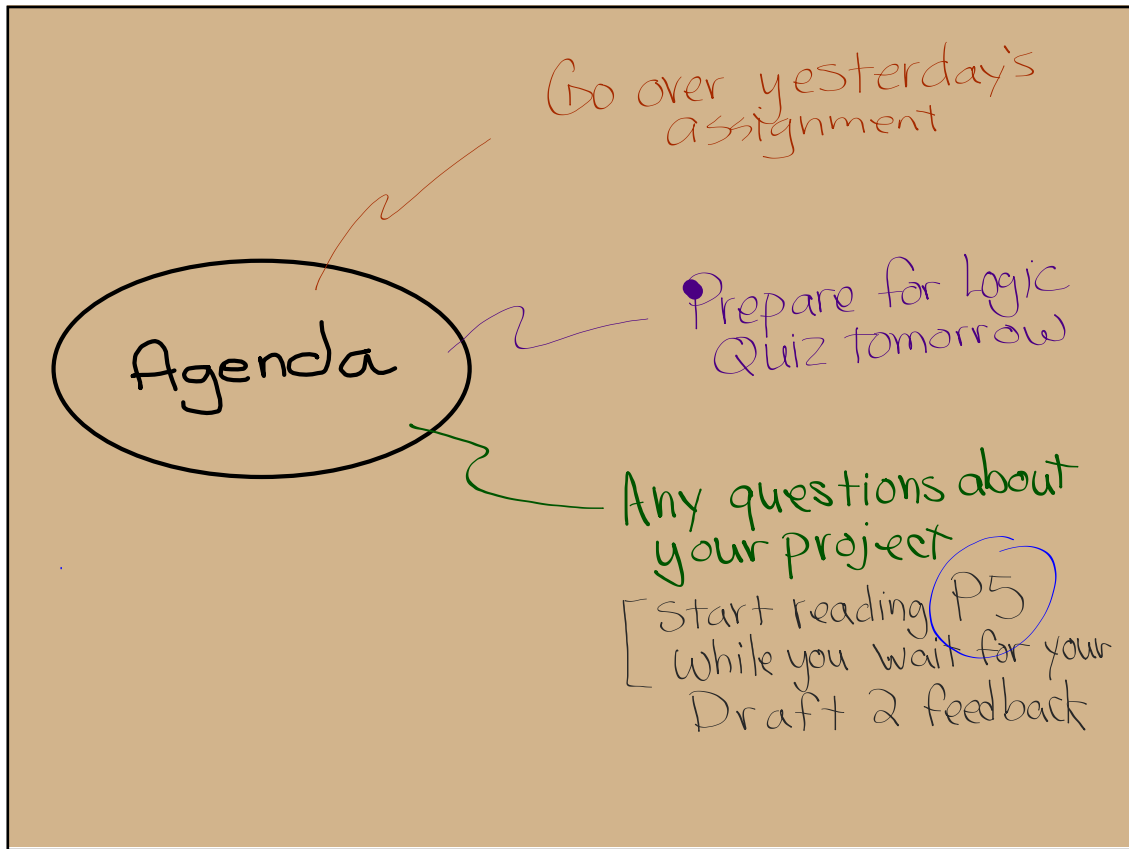
$$145 \leq h < 150$$

- 5 Consider the frequency distribution of heights of elementary school students (cm) shown below:

$x$ midpoint	height $h$ (cm)	frequency
132.5	$130 \leq h < 135$	2
137.5	$135 \leq h < 140$	3
142.5	$140 \leq h < 145$	5
147.5	$145 \leq h < 150$	7
152.5	$150 \leq h < 155$	6
157.5	$155 \leq h < 160$	2

$$\begin{aligned}\bar{x} &= \frac{\sum f \cdot x}{n} \\ &= \frac{\sum f \cdot x}{\sum f} \quad \text{critical total} \\ &= \frac{3652.5}{25} \\ &\doteq 146.1 \\ &\quad \text{146 cm}\end{aligned}$$

- a. Determine the midpoint of the  $130 \leq h < 135$  interval. 132.5
- b. Calculate the approximate mean height of the students. 146 cm
- c. Determine the modal class interval.  $145 \leq h < 150$   
because it occurs the most.



$\neg (a \vee b)$   
or

(i) Logic, symbols and words

Given the statements

$p$ : It is raining

$q$ : I am wearing my coat

- (a) Write down, in words, the meaning of  $q \Rightarrow \neg p$  If I am wearing my coat then it is not raining

- (b) Complete the truth table

$p$	$q$	$\neg p$	$q \Rightarrow \neg p$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

- (c) Write in symbols, the contrapositive of  $q \Rightarrow \neg p$

$$p \rightarrow \neg q$$

(ii) 3 propositions and truth tables

Considering the propositions,  $a$ ,  $b$  and  $c$  and the compound statements

$a$ : a person achieves grade 7 in Mathematics

$b$ : a person achieves grade 7 in English

$c$ : a person has above average IQ

- (i) write the following statements in words

$c \Rightarrow (a \vee b)$  If a person has ↑ IQ, then he achieves a 7 in math OR a 7 in English

$\neg(a \vee b) \Rightarrow \neg c$  If a person doesn't achieve a 7 in math and doesn't achieve a 7 in English then he doesn't have ↑ IQ

(ii) Use the truth tables to establish the truth values of both of these statements

i) write the following statements in words

$c \Rightarrow (a \vee b)$  If a person has ↑ IGCSE, then he achieves a 7 in math or a 7 in English

$\neg(a \vee b) \Rightarrow \neg c$  If a person doesn't achieve a 7 in math and doesn't achieve a 7 in English then he doesn't have ↑ IGCSE

ii) Construct a truth table to establish the truth values of both of these statements

a	b	c	$\neg c$	$a \vee b$	$\neg(a \vee b)$	$c \rightarrow (a \vee b)$	$\neg(a \vee b) \rightarrow \neg c$
T	T	T	f	T	f	T	T
T	T	f	T	T	f	T	T
T	f	T	f	T	f	T	T
T	f	f	T	T	f	T	T
f	T	T	f	T	f	T	T
f	T	f	T	T	f	T	T
f	f	T	f	<del>T</del> f	<del>f</del> T	<del>T</del> f	<del>T</del> f
f	f	f	T	f	T	T	T

3) Truth Tables and Contradiction

Show using truth tables that the following statement is a contradiction

$$\neg b \wedge [a \wedge (b \vee \neg a)]$$

a	b	$\neg a$	$\neg b$	$(b \vee \neg a)$	$[a \wedge (b \vee \neg a)]$
T	T	f	f	T	T
T	f	f	T	f	f
f	T	T	f	T	f
f	f	T	T	f	f

all false so a contradiction



4) Construct a truth table for  $(\neg p \vee q) \leftrightarrow r$  and determine if the statement is a tautology, a contradiction or neither.

p	q	r	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \leftrightarrow r$
T	T	T	f	T	T
T	T	f	f	T	f
T	f	T	f	f	f
T	f	f	f	f	f
f	T	T	T	T	T
f	T	f	T	T	f
f	f	T	T	f	f
f	f	f	T	f	f

Notes: The truth table shows that the statement is neither a tautology nor a contradiction. The result "Neither" is circled in blue. The value "f" in the last row is crossed out with a blue slash and labeled "brok. prom".

5) If I like Irish, then I like Logic. Write the converse, inverse, and contrapositive.

If I like Logic then I like Irish ← converse

If I don't like Irish then I don't like Logic ← inverse

If I don't like Logic, then I don't like ~~Logic~~ ← contrapos.

## For the quiz tomorrow:

You can use the Formula Packet which has the basic truth tables listed

You will need to memorize the symbols and definitions of converse, inverse, and contrapositive.

$\wedge$     $\vee$     $\perp$     $\rightarrow$     $\leftrightarrow$

Know how to prove a tautology  
or a logical contradiction.

Know how to prove two logical  
statements are logically equivalent.

Available  
= Logic Practice with Answers

B.B.

# Logic Assignment 5

$p$ : IHS student  
 $q$ : IB student  
 $r$ : non IHS student

$$p \wedge \neg q$$

$$\neg(a \vee c) \Rightarrow b \wedge c$$

$a$	$b$	$c$	$a \vee c$	$\neg(a \vee c)$	$b \wedge c$	$\neg(a \vee c) \Rightarrow b \wedge c$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

$a$	$b$	$c$	$a \vee c$	$\neg(a \vee c)$	$b \wedge c$	$\neg(a \vee c) \Rightarrow b \wedge c$
T	T	T	T	F	T	T
T	T	F	T	F	F	T
T	F	T	T	F	F	T
T	F	F	T	F	F	T
F	T	T	T	F	T	T
F	T	F	F	T	F	F
F	F	T	T	F	F	T
F	F	F	F	T	F	F

f

November 14, 2018

<i>a</i>	<i>b</i>	<i>c</i>	<i>b̄vc</i>	<i>a</i>	<i>b̄vc</i> ↔ <i>a</i>
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

<i>a</i>	<i>b</i>	<i>c</i>	<i>b̄vc</i>	<i>a</i>	<i>b̄vc</i> ↔ <i>a</i>
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	F	F	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	F	T

(6) Mario has spent \$ 40000 to buy some land. The land increases in value by 5% each year.

(i) What is the value of the land after the end of five years?

At the end of five years, Mario sells the land. He pays 1% tax on the sale and spends the rest of the money on a car. The car then loses value at a rate of \$ 2500 every year.

(ii) How much tax does Mario pay?

(iii) How much is the car worth five years after Mario buys it?

*Handwritten notes:* \$2732 in interest

**Working:**

(i) can use  $FV = C(1 + \frac{r}{100})^n$  or  $y = ab^x$   
 $= 40000(1 + \frac{5}{100})^5$  or  $y = 40000(1.05)^5 = 51,051.26$

(ii) TAX  
 1% of 51,051.26  
 is \$ 510.51

**Answers:**

(i) \$51,051  
 (ii) \$ 510.51 or \$511  
 (iii) \$ 38,040

(iii) 
$$\begin{array}{r} 51,051 \\ - \quad 511 \\ \hline 50,540 \\ \text{after taxes} \end{array}$$

*Handwritten notes:* Buys car valued at \$ 50,540  
 depreciates linearly  
 $FV = 50540 - 5(2500) = \$38,040$

## FINANCE PRACTICE

Name \_\_\_\_\_

You can use your TI-App to check answers, but you are expected to show work.

(1) Takaya invested 1000 JPY at 6.3% ~~comp~~ <sup>monthly</sup> interest for 15 years. Morimi invested 900 JPY at 6.3% interest compounded ~~quarterly~~ <sup>quarterly</sup> for 15 years. Who had more money at the end of the 15th year? Justify your answer clearly.

**Takaya**  $FV = 1000(1 + \frac{6.3}{100(12)})^{(12 \times 15)}$

$= 2566.46$  JPY

**Morimi**  $FV = 900(1 + \frac{6.3}{(100)(4)})^{(4 \times 15)}$

$= 2298.54$  JPY

So Takaya had more.

2 Bobby is spending a year travelling from America to France and Britain. Consider the following exchange rates.

1 US dollar (USD) = 0.983 Euros  
 1 British Pound (GBP) = 1.59 Euros

(a) Bobby changes 500 USD into Euros.

(i) Calculate how many Euros he receives.

He spends 328 Euros in France and changes the remainder into GBP. but this time is charged a 3% commission.

(ii) Calculate how many GBP he receives.

3% of 163.5 = 4.9 euro commission

163.5 - 4.9 = 158.6 Euros

158.6 Euros  $\times \frac{1 \text{ GBP}}{1.59 \text{ Euros}} = 99.75 \text{ GBP}$

500 USD  $\times \frac{0.983 \text{ Euros}}{1 \text{ USD}} = 491.50 \text{ Euros}$

491.50 - 328 = 163.5 Euros left

While in Britain Bobby decides to put this money in a bank that pays 6% per annum, compound yearly and he gets a part-time job to cover his expenses. Bobby remains in Britain for six months.

(b) Calculate how much interest he receives for the six months.

99.75  $(1 + \frac{6}{100(1)})^{(1 \times 0.5)}$

102.70 GBP

Interest 102.70 - 99.75 = 2.95

4 Bob invests 600 EUR in a bank that offers a rate of 2.75% compounded annually. The interest is added on at the end of each year.

(a) Calculate how much money Bob has in the bank after 4 years.

(b) Calculate the number of years it will take for the investment to double.

Ann invests 600 EUR in another bank that offers interest compounded annually. Her investment doubles in 20 years.

(c) Find the rate that the bank is offering.

a)  $FV = 600 (1 + \frac{2.75}{100(1)})^{(4 \cdot 1)} = 668.77 \text{ EUR}$

b) IF REQUIRED, initial investment would be 1200 EUR

$1200 = 600 (1 + \frac{2.75}{100(1)})^{(n \cdot 1)}$

$2 = (1 + \frac{2.75}{100})^n$

Using logs or graphing

$n \approx 25.6 \text{ years}$

c)  $1200 = 600 (1 + \frac{r}{100})^{(20 \cdot 1)}$

$2 = (1 + \frac{r}{100})^{20}$

$\frac{r}{100} = \sqrt[20]{2} - 1$

$r = 3.5$



5 A family in Malaysia received a gift of AUD \$ 4000 from a cousin living in Australia.  
The money was converted to Malaysian Ringgit. One Ringgit can be exchanged for 0.4504 AUD.

(a) Calculate the amount of Ringgit received.

$$4000 \text{ AUD} \times \frac{1 \text{ RINGGIT}}{0.4504 \text{ AUD}} = \underline{\underline{8880 \text{ RINGGIT}}}$$

The money was invested for 2 years and 6 months at 5.2 % p.a. compounding monthly.

(b) Calculate the amount of interest earned from this investment. Give your answer to the nearest Ringgit.

Assuming 8880 RINGGIT

$$FV = 8880 \left(1 + \frac{5.2}{100(12)}\right)^{(2.5 \times 12)}$$

$n = 2 \text{ yrs } 6 \text{ m} = 2.5 \text{ year}$

$$= 10,109.96 \text{ RINGGIT} \rightarrow \text{nearest RINGGIT}$$

$$10,110 \text{ ringgit}$$

– 8880  
+ 2230 interest

6 Mario has spent \$ 40000 to buy some land. The land increases in value by 5 % each year.

(i) What is the value of the land after the end of five years?

7 John invests  $X$  USD in a bank. The bank's stated rate of interest is 6% per annum, compounded monthly.

(a) Write down, in terms of  $X$ , an expression for the value of John's investment after one year.

$$FV = C \left(1 + \frac{r}{100k}\right)^{nk}$$

$$FV = X \left(1 + \frac{6}{100 \cdot 12}\right)^{(1 \times 12)}$$

$r = 6$   
 $k = 12$   
 $n = 1$

$$FV = X \left(1 + \frac{6}{1200}\right)^{12}$$

$$FV = 8880 \left(1 + \frac{5.2}{100(12)}\right)^{(2.5 \times 12)}$$

(b) What rate of interest, when compounded annually (instead of monthly) will give the same value of John's investment as in part (a)? Give your answer correct to three significant figures.

Working: if annually it would be  $X(1 + \frac{r}{100})^1$

set them equal to each other

$$X\left(1 + \frac{6}{1200}\right)^{12} = X\left(1 + \frac{r}{100}\right)^1$$

Solve for  $r$

Answers:

(a) .....  $X\left(1 + \frac{6}{1200}\right)^{12}$

(b) ..... 6.17%

$$\left(1 + \frac{6}{1200}\right)^{12} = 1 + \frac{r}{100}$$

$$\left(1 + \frac{6}{1200}\right)^{12} - 1 = \frac{r}{100}$$

$$r \approx 6.17$$