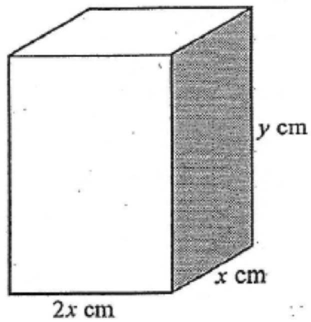
HW Questions

Agenda

1. Discuss HW
2. Warm Up
3. Information about Test *
4. See your LCQ
5. Work on study questions.

P4



$$SA = 2(\text{Top}) + 2(\text{Side}) + 2(\text{Front})$$

$$300 = 2(2x \cdot x) + 2(x y) + 2(2x \cdot y)$$

$$300 = 4x^2 + 2xy + 4xy$$

$$300 = 4x^2 + 6xy$$

(a) Show that $4x^2 + 6xy = 300$. (2)

$$4x^2 + 6xy = 300 \Rightarrow 6xy = 300 - 4x^2$$

(b) Find an expression for y in terms of x. (2)

$$y = \frac{300 - 4x^2}{6x}$$

(c) Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$.

Volume $= (2x)(x)y$ $\qquad y = \frac{300-4x^2}{6x}$

$$= \cancel{2x} \cdot x \cdot \frac{300 - 4x^2}{3\,\cancel{6x}} = \frac{x(300-4x^2)}{3} = \frac{300x - 4x^3}{3}$$

$$= \frac{300x}{3} - \frac{4x^3}{3}$$

(d) Find $\frac{dV}{dx}$. $= 100 - \frac{4}{3} \cdot 3x^2$

$$= 100 - 4x^2$$

$$\frac{dV}{dx} = 100 - 4x^2$$

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(ii) Calculate the maximum volume.

(5)

(Total 13 marks)

maximum Volume ocurrs when tangent is flat (gradient = 0)

Set $\frac{dV}{dx}$ equal to 0

$100 - 4x^2 = 0$

$4x^2 = 100$

$x^2 = \pm 5$

ignore negative dimension

$x = 5$ is the optimum dimension

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(ii) Calculate the maximum volume.

(5)

(Total 13 marks)

maximum Volume ocurrs when tangent is flat (gradient = 0)

Set $\frac{dV}{dx}$ equal to 0

$100 - 4x^2 = 0$

$4x^2 = 100$

$x^2 = \pm 5$

ignore negative dimension

$x = 5$ cm

MAX Volume

$V = 100x - \frac{4}{3}x^3$

$= 100(5) - \frac{4}{3}(5)^3$

$= 333 \text{ cm}^3$

Volume $= 2x^2 y$

$333 = 2(5)^2 y$

$y = 6.66$ cm

2

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x} = x + 100x^{-1}$$

(a) Find $C'(x)$. $= 1 + 100(-1)x^{-2} = \boxed{1 - \frac{100}{x^2}}$ **(3)**

(b) Show that the cost per person is a minimum when 10 people are invited to the party. **(2)**

(c) Calculate the minimum cost per person. **(2)**

(Total 7 marks)

2

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x} = x + 100x^{-1}$$

(a) Find $C'(x)$. **(3)**

(b) Show that the cost per person is a minimum when 10 people are invited to the party. **(2)**

(c) Calculate the minimum cost per person. **(2)**

(Total 7 marks)

(a) $C'(x) = 1 - 100x^{-2} = 1 - \frac{100}{x^2}$

(b) To find minimum cost (where tangent is flat)

set $C'(x) = 0$ and solve

$1 - \frac{100}{x^2} = 0$ multiply by x^2

$$\boxed{1 - \frac{100}{x^2} = 0}$$

$x^2 - 100 = 0$

$x^2 = 100$

$\sqrt{\ }$ $\sqrt{\ }$

$x = \pm 10$

$x = 10$

(b) To find minimum cost (where tangent is flat)

set $c'(x) = 0$ and solve

$1 - \frac{100}{x^2} = 0$ multiply by x^2

$x^2 - 100 = 0$

$x^2 = 100$

$x = \pm 10$ ignore negative

so, 10 people would give the minimum cost

(c) Minimum Cost $= C(10) = x + \frac{100}{x}$

$= 10 + \frac{100}{10} = 20$

So the minimum cost per person is \$20
(which occurs when 10 people are invited)

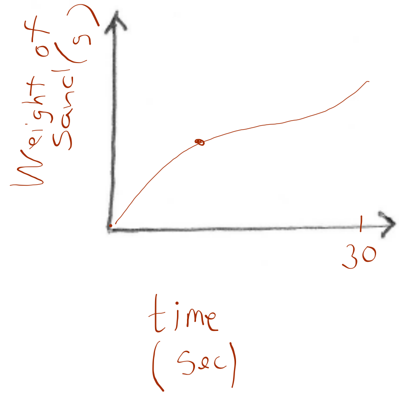
WARM UP

Sand is being poured into a bucket for 30 seconds. After t seconds the weight of the sand is

$$S(t) = 0.3t^3 - 18t^2 + 550t \text{ grams}$$

a) Make a sketch of the graph and label it.

(Adjust your window to match the situation)



b) Find $\boxed{S(12)}$ and interpret its meaning

$= 4526\text{g}$ is the weight of the sand after 12 seconds

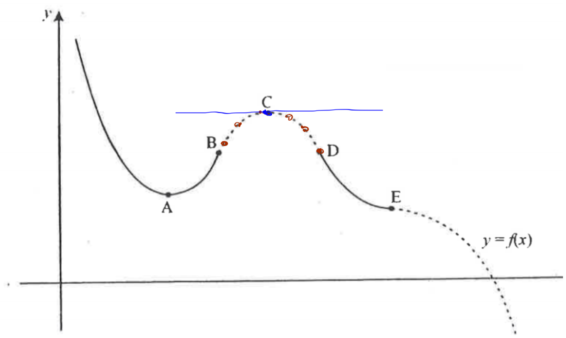
c) Find $S'(12)$ and interpret its meaning.

can use GDC directly

$S'(12) = 247.6$

12 seconds into the trial sand is being poured at a rate of 247.6 g/sec

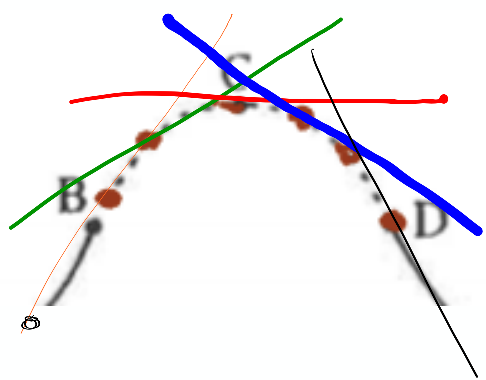
2 The letters A to E are placed at particular points on the curve $y = f(x)$.



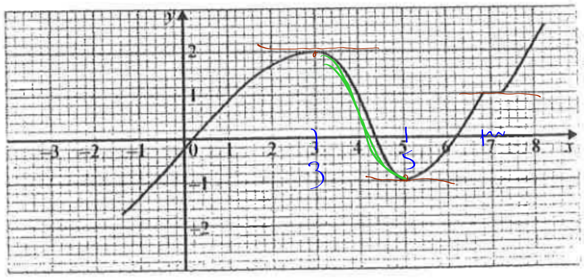
(a) What is the gradient of the curve $y = f(x)$ at the point marked C?

gradient = 0

(b) In passing from point B, through point C, to point D what is happening to $\frac{dy}{dx}$? Is it decreasing or increasing?



3 The diagram shows a part of the curve $y = f(x)$.



(a) For what values of x is $f'(x) = 0$?

$x = 3 \quad x = 5 \qquad 6.8 \le \cancel{x} \le 7.2$

(b) For what range of values of x is $f'(x) < 0$?

$3 < x < 5$

The Quiz on Introductory Calculus will be Monday

List of Quiz Items

NOTATION

$f(x)$

$f'(x)$

✓ Calculator skills: On typical or non-typical functions.... use GDC to:

- Caclulate the gradient at a given location
- b) Calculate the equation of a tangent line at a given location

at $x=5$

$$f(x) = -x^2 + 2^x - \sqrt{x}$$

a) $f'(7) =$

- Study Problems

Review Set B

1 - 6

7, 8 ← nice challenge questions for those going for a 7

I'll be posting solutions