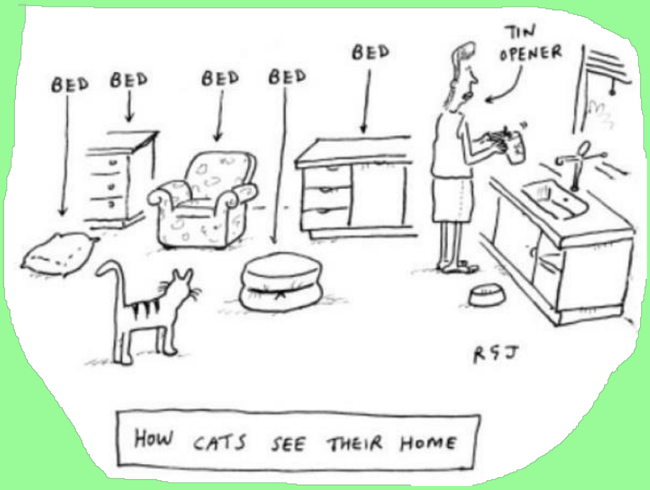


You have **5** minutes to check your HW with the solutions (ask any questions afterwards)

Schedule

Th NORMALS + OPTIMIZATION ✓ ✓

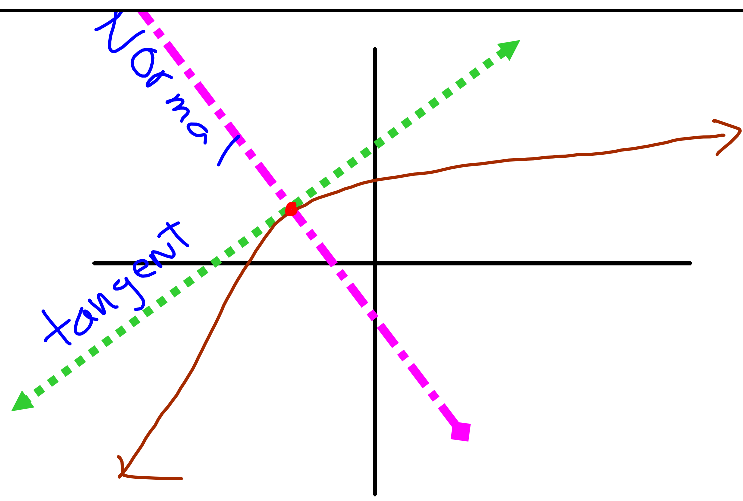
Fri Review ✓

Mon QUIZ ON CALCULUS ✓

Using the derivative:

1. Find the equation of a NORMAL

2. Optimize a situation.



Then, do the first problem

on the Notes 4.0

handout

1 Find the equation of the **Tangent** and the **Normal** for the equation $\mathbf{y = x^3 - 5x + 2}$ at the location $x = -2$

P.O.T. $f(-2)$

$f(-2) = (-2)^3 - 5(-2) + 2$

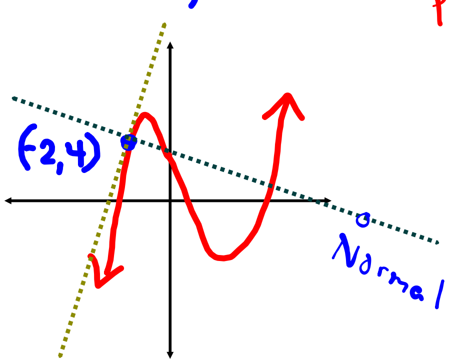
$= 4$

$(-2, 4)$ ← P.O.T.

$f'(x) = 3x^2 - 5$

$f'(-2) = 3(-2)^2 - 5 = 7$

point slope $\boxed{y - 4 = 7(x + 2)}$ → $y = 7x + 18$



slope of tangent 7

slope of Normal $-\frac{1}{7}$

Normal $\boxed{y - 4 = -\frac{1}{7}(x + 2)}$

$y = -\frac{1}{7}x + \frac{26}{7}$

① Find the equation of the **Tangent** and the **Normal** for the equation $\mathbf{y = x^3 - 5x + 2}$ at the location $x = -2$

P.O.T. $f(-2)$

$f(-2) = (-2)^3 - 5(-2) + 2$

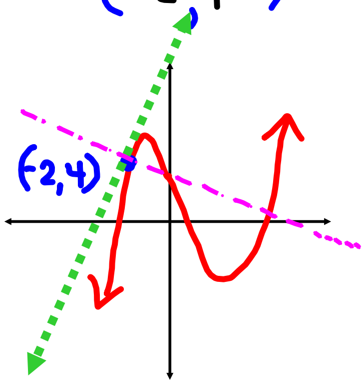
$= 4$

$(-2, 4)$

GRADient $f'(x) =$

$f'(-2) =$

equation of Tangent



equation of Normal

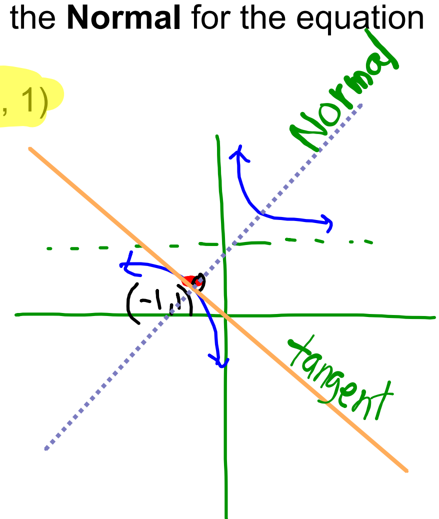
② Find the equation of the **Tangent** and the **Normal** for the equation

$$y = \frac{1}{x} + 2$$ at the location (-1, 1)

$y = x^{-1} + 2$

$f'(x) = -x^{-2} = \boxed{-\frac{1}{x^2}}$

$f'(-1) = -\frac{1}{(-1)^2} = -1$



tangent -1 $-\frac{1}{1}$

Normal 1

Tangent

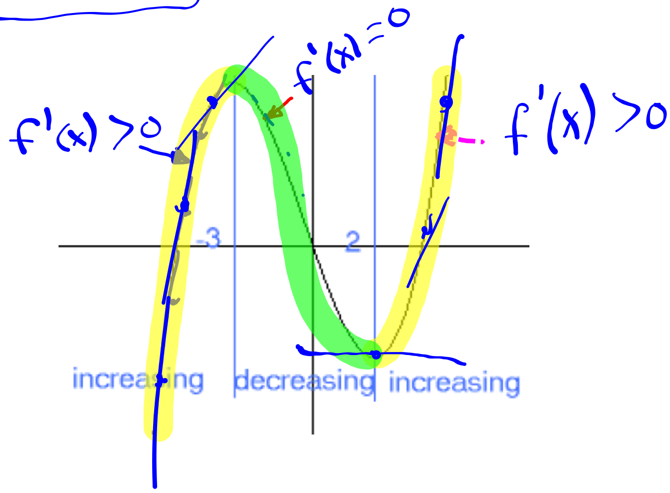
$y - 1 = -(x + 1)$

Normal

$y - 1 = 1(x + 1)$

A

$f'(x) < 0$ $\boxed{f'(x) = 0}$ $f'(x) > 0$

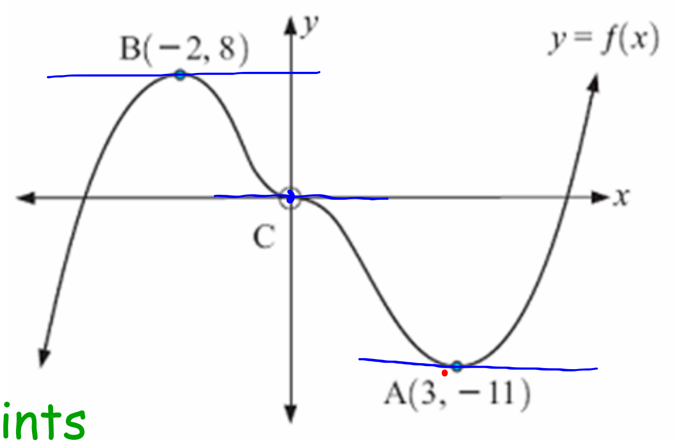


$f'(x)$

any point on a curve where the tangent line is horizontal is a **STATIONARY point**

B

local maximums

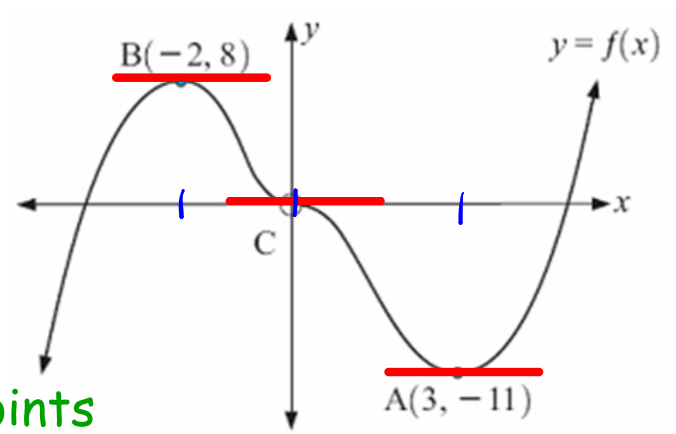


horizontal inflection points

local minimums

$f'(x)=0$

local maximums



horizontal inflection points

local minimums

Horizontal tangents have a gradient of zero

To find all of those places on any given function:

(1) find the gradient function and

(2) set it equal to zero

same as "find the derivative"

$$f'(x) = 0$$

(3) Solve to find x-values (if any) which are the locations where the tangents have a gradient of zero !

Skip 3

3. Find the ~~equation(s)~~ locations of any horizontal tangents of

$f(x) = \frac{1}{3}x^3 - x + 2$

$f'(x) = x^2 - 1$

4. **Find the stationary points of the curve $y = x^3 - 3x^2 - 9x + 10$**

a) $f'(x) = 3x^2 - 6x - 9$

b) $3x^2 - 6x - 9 = 0$

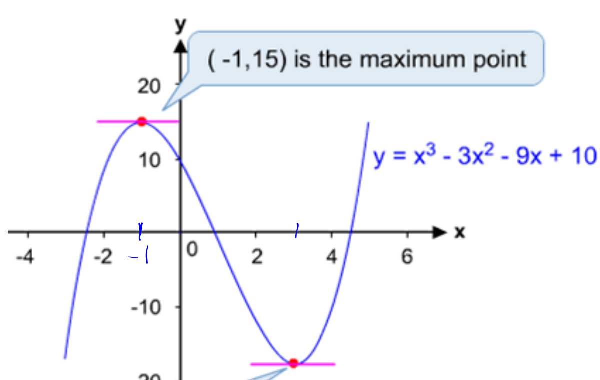
$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

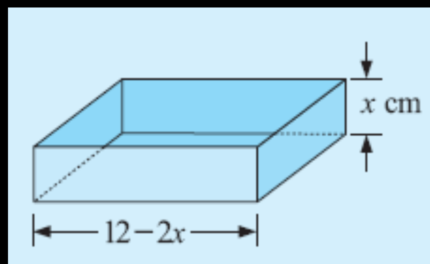
$x - 3 = 0 \quad x + 1 = 0$

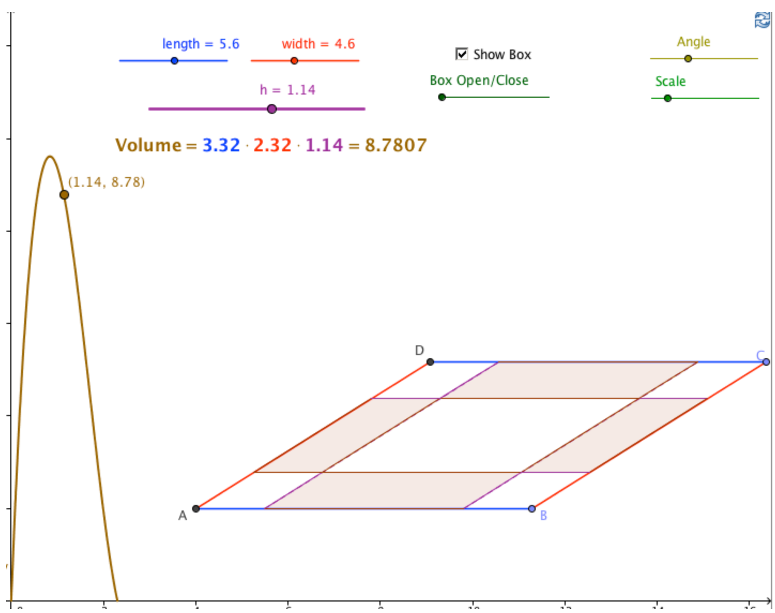
$x = 3 \quad x = -1$



(3, -17)

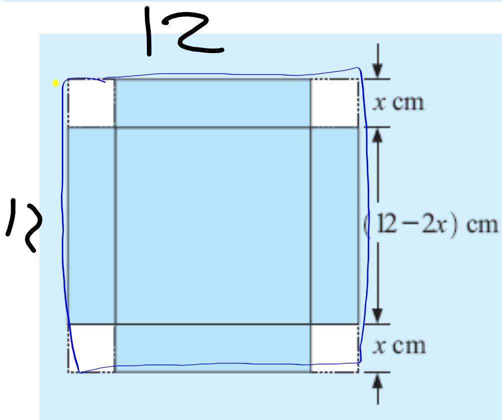
"the Box Problem"

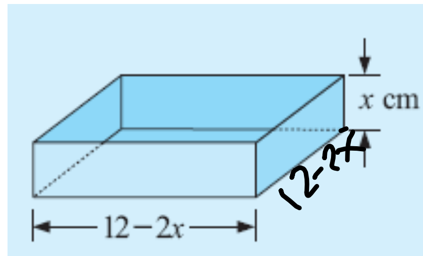




5

Square corners are cut from a piece of 12 cm by 12 cm tinplate which is then bent into the form of an open dish. What size squares should be removed if the volume is to be a maximum?





Note: $0 \leqslant x \leqslant 6$

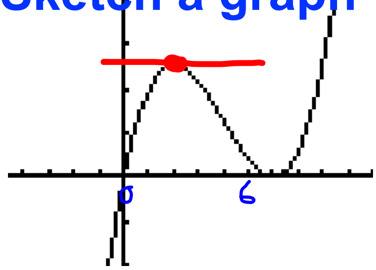
$$
\begin{aligned}
V(x) &= (12-2x)^2 \times x \\
&= (144 - 48x + 4x^2) \times x \\
&= 144x - 48x^2 + 4x^3
\end{aligned}
$$

$$V(x) = 4x^3 - 48x^2 + 144x$$

We can graph to let our GDC find the maximum volume

or we can use calculus !

Sketch a graph



The max volume will occur where the gradient is zero. Therefore, we need to ***find the derivative*** and ***set it to 0***

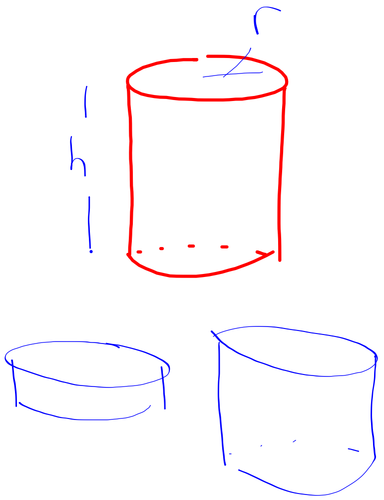
$$\therefore \quad V'(x) = 144 - 96x + 12x^2$$
$$0 = 12(x^2 - 8x + 12)$$
$$0 = 12(x-2)(x-6)$$

$x-2=0$ $\quad$ $x-6=0$

$x=2$ $\quad$ $x=6$

$\therefore$ maximum volume occurs when $x = 2$ cm

$\therefore$ cut out 2 cm squares.



Create a cylindrical package that can hold a ½ liter

but wait.....

there are two variables?

What's the best radius?

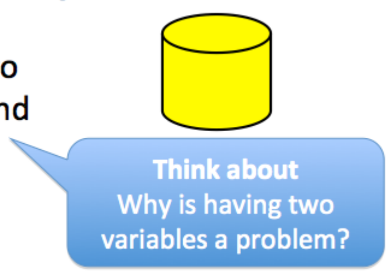
Optimization

skip to the last side.

Now skip back to the last side of the sheet

What if you wanted the minimum material to make a cylinder with a required volume?

In this case you would have two variables (radius and height) and one fixed quantity (volume)



In order to differentiate, you need an expression for the quantity you want to minimise (or maximise) in terms of **just one variable**

Working with a cylinder

First, use the fixed volume to eliminate one of the variables (either the height or radius)

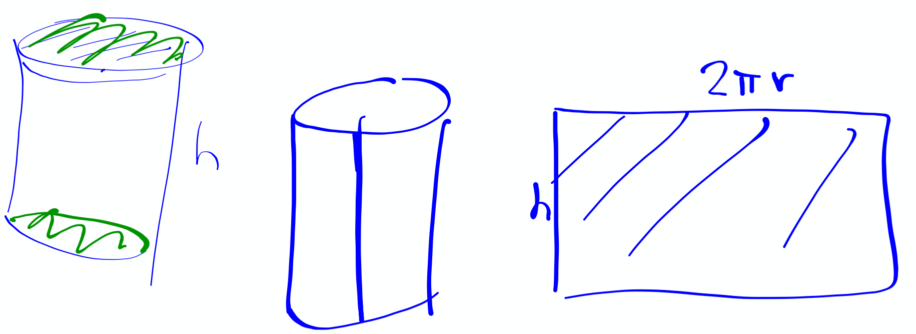
When you have an expression for the quantity of material needed to make the cylinder in terms of just one variable, differentiate it and put the derivative = 0

Solve this equation to find the value of the variable that gives a minimum (or maximum)

Then find the value of the other variable and the minimum (or maximum) that you require

$\overset{500}{\cancel{V}} = \pi r^2 h$

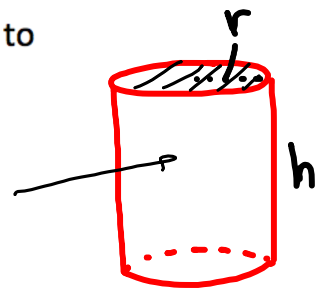
$SA = 2\pi r^2 + 2\pi r h$



Minimum material to make a can

Say you want to find the minimum metal needed to make a can to hold 500 ml (the same as 500 cm^3)

$$500 = \pi r^2 h \qquad h = \frac{500}{\pi r^2}$$



SA of metal $\quad M = 2\pi r^2 + 2\pi r h$

500

$$M = 2\pi r^2 + 2\pi r h \qquad 500 = \pi r^2 h$$

$$M = 2\pi r^2 + 2\pi r h \qquad 500 = \pi r^2 h$$

$$h = \frac{500}{\pi r^2}$$

$$M = 2\pi r^2 + 2\cancel{\pi} r\left(\frac{500}{\cancel{\pi} r^2}\right)$$

$$M = 2\pi r^2 + \frac{1000}{r} \qquad 1000 r^{-1}$$

$$\frac{dM}{dr} = 4\pi r - \frac{1000}{r^2} \qquad -1000 r^{-2}$$

$$4\pi r - \frac{1000}{r^2} = 0$$

multiply by r^2 to clear out the fractions!!

$$4\pi r^3 - 1000 = 0$$

$$4\pi r^3 = 1000$$

$$r^3 = \frac{1000}{4\pi}$$

$$\sqrt[3]{\ } \quad \sqrt[3]{\ } \qquad r = \sqrt[3]{\ }$$

$r = 4.30$ cm would minimize the surf. area

$$r^2(4\pi r) - \cancel{r^2}\left(\frac{1000}{\cancel{r^2}}\right) = (0) r^2$$

$$4\pi r^3 - 1000 = 0$$

$$4\pi r^3 = 1000$$

$$r^3 = \frac{1000}{4\pi} \qquad r = \sqrt[3]{\frac{1000}{4\pi}}$$

Assignment

1 Day 4 Worksheet (both sides)

2 Calculus packet:

and p. 582...Review Set A..... 1-8

B.B.