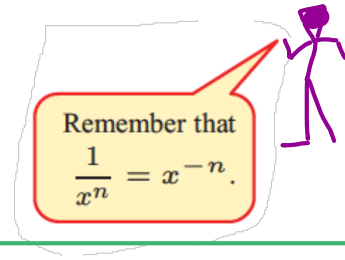
Warm Up

Convert each term of the function below to the form

ax^n where $n \in \mathbb{Z}$



$$f(x) = \frac{10}{x^1} + \frac{5}{x^2} =$$

HW HOTLINE

ax^n

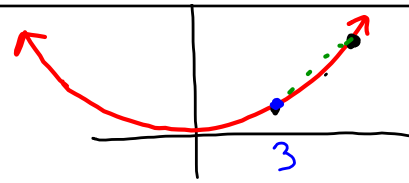
$$g(x) = \frac{3x^2 - 6x + x^3}{x^3} =$$

$$= 3x^{-1}$$

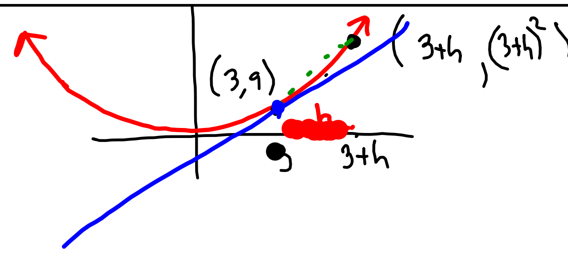
$$g(x) = \frac{3x^2 - 6x + x^3}{x^3} =$$

Questions on HW

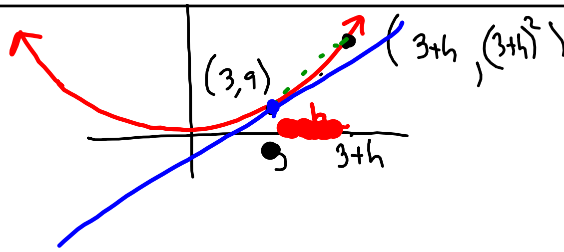
1a $y = x^2$ $x=3$



1a $y = x^2$ $x=3$



1a $y = x^2$ $\underline{x=3}$



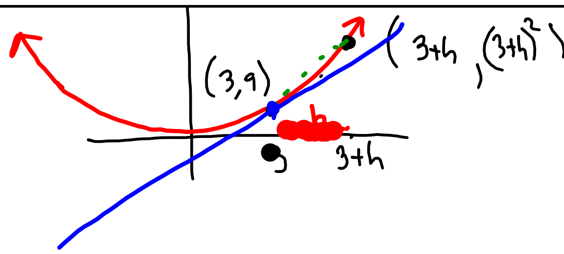
$$m = \frac{(3+h)^2 - 9}{\cancel{3}+h - \cancel{3}}$$

$$= \frac{\cancel{9}+6h+h^2 - \cancel{9}}{h}$$

$$= \frac{6h+h^2}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+h$$

1a $y = x^2$ $\underline{x=3}$

$+2x$



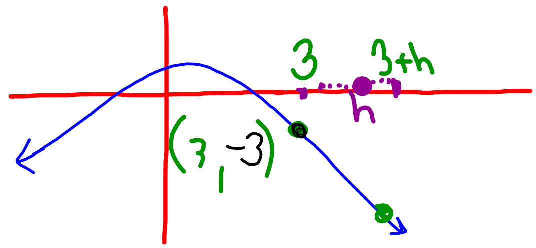
$$m_{gr} = \frac{(3+h)^2 - 9}{\cancel{3}+h - \cancel{3}}$$

$$= \frac{\cancel{9}+6h+h^2 - \cancel{9}}{h}$$

$$= \frac{6h+h^2}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+h$$

As $h \to 0$
then gradient
is 6

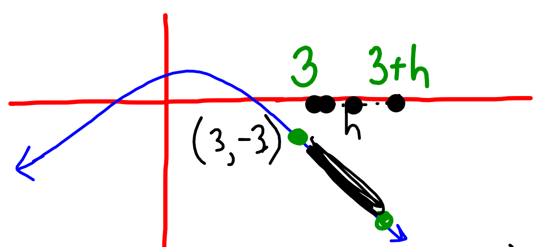
1e $\boxed{y = 2x - x^2}$ at $x=3$



$(3+h, \quad)$

1e $y = 2x - x^2$ at $x=3$

$\uparrow$ 3+h $\quad \uparrow$ 3+h



$(3+h, \quad)$

$-3 - 4h - h^2$

$$2(3+h) - (3+h)^2$$

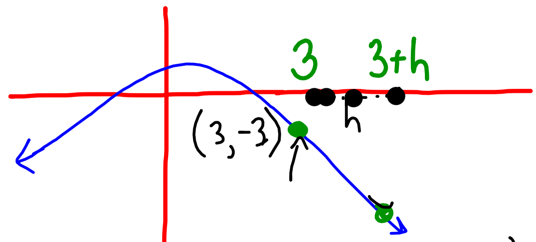
$$6 + 2h - (3+h)(3+h)$$

$$6 + 2h - [9 + 6h + h^2]$$

$$= 6 + 2h - 9 - 6h - h^2$$

$$= -3 - 4h - h^2$$

1e $y = 2x - x^2$ at $x=3$



$(3+h, \quad)$

$-3-4h-h^2$

Gradient =

$$\frac{-3-4h-h^2 \; - \; -3}{3+h \; - \; 3}$$

$$= \boxed{\frac{-4h-h^2}{h}} = \frac{\cancel{h}(-4-h)}{\cancel{h}}$$

$$= -4-h$$

as $h \to 0$

gradient is -4

Last class, we found the gradient of curves at a specific point

ⓐ drawing a tangent and estimating

ⓑ Algebraically

ⓒ with GDC

K. **Basic Differential Calculus**

Calculate the derivative at a specific point.

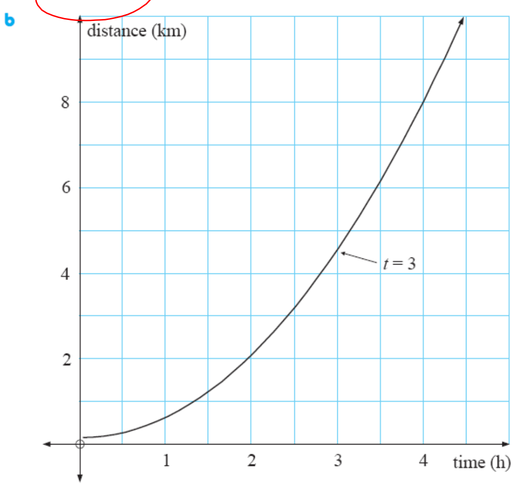
1. Graph the function, $f(x)$, and obtain an appropriate window.
2. Select **2nd** then **TRACE** , the select $\frac{dy}{dx}$,
3. enter the appropriate $x - value$, then **ENTER**

Draw Tangent Line (& calculate it's equation)

1. Graph the function, $f(x)$, and obtain an appropriate window.
2. Select **2nd** then **DRAW,** then **TANGENT**,
3. enter the appropriate $x - value$, then **ENTER**

Find the instaneous rate of change at t=3 hours

K



$f(x) = 0.5(2)^x$

✓ Graph (zoom 6)

✓ 2nd Calc

✓ $\frac{dy}{dx}$

t=3 seconds

at t=1 ?

TODAY'S AIM

Find derivatives directly

for functions in the form

$f(x) = ax^n$

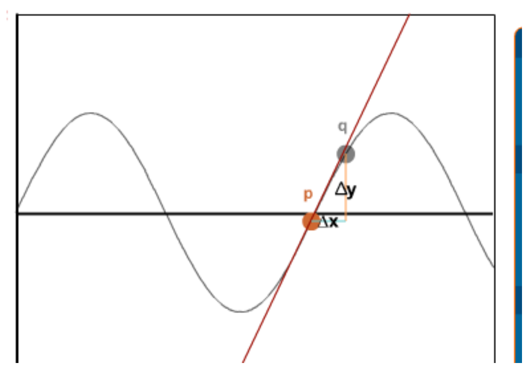
or

$f(x) = ax^n + bx^{n-1} \dots$

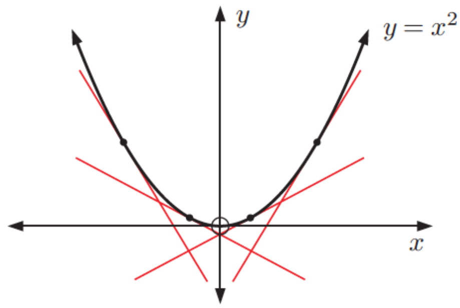
Pick Up

Notes 2,0

What we are about to look at will require you to focus on on the gradients of all of the tangents of a function



A Wouldn't it be cool if there was a **magic function** that could quickly give you the **gradients** at any x-value you want, for any function



That function is called:

THE **Gradient** FUNCTION

or more commonly called

THE **Derivative** function

B

Gradient is a rate of change

The Derivative is a function that that you can use to generate the gradient at any x-value.

other symbols for it:

B

$f'(x)$ or $\frac{dy}{dx}$

$\frac{\Delta y}{\Delta x}$

$\cancel{f^{-1}(x)}$

c

The gradient function (a.k.a derivative function) is created from the original function, f(x).

An example: $f(x) = x^2$

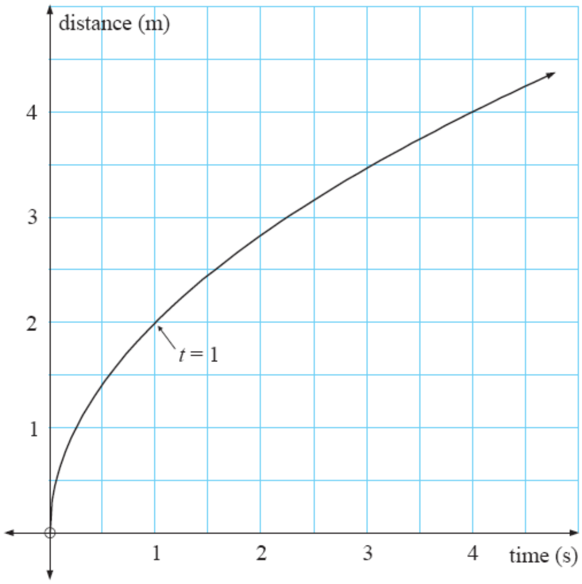
its derivative, $f'(x)$, is $2x$

Differentiation is the process of finding the derivative of a function

- Before we look at some patterns, lets find the gradient one more time with our GDC

D) Find the instaneous rate of change at x = 1 second

$f(x) = 2\sqrt{x}$



✓ Graph (zoom 6)

✓ 2nd Calc $\frac{dy}{dx}$

✓ x=1 (for example) enter

it turns out the derivative function is

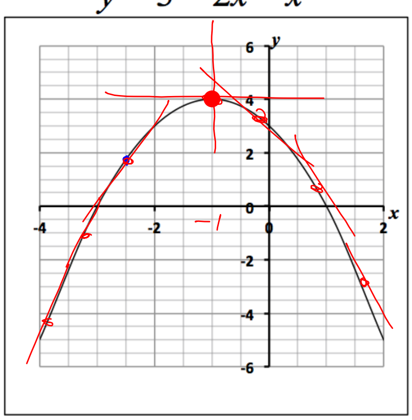
$$f'(x) = \frac{1}{\sqrt{x}}$$

$$f'(1) = \frac{1}{\sqrt{1}} = \frac{1}{1} = 1$$

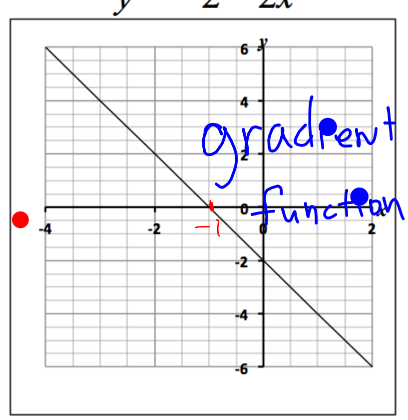
First estimate visually

E) See if you can see a connection between these two graphs?

$y = 3 - 2x - x^2$



$y = -2 - 2x$



F

Simple Patterns of Differentiation

for functions in the form $y = ax^n$ where $n \in \mathbb{Z}$

integers

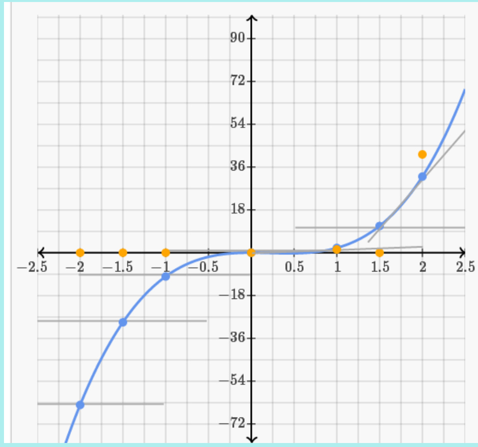
G

f(x)	f'(x)
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
x^5	$5x^4$

$x^{17} = 17x^{16}$

$f(x) = x^n$

$f'(x) = n \cdot x^{n-1}$



tp://www.khanacademy.org/math/calculus/differential-calculus/e/derivative_intuition

H

f(x)	f '(x)
$7x^2$	$7 \cdot 2x = 14x$
$-5x^3$	$-5 \cdot 3x^2 = -15x^2$
$4x^{10}$	$40x^9$

If $y = x^n$ then $\dfrac{dy}{dx} = nx^{n-1}$.

$f(x) = ax^n$

$f'(x) = a \cdot x^{n-1}$

anx^{n-1}

I

$f(x)$	$f'(x)$
$x^4 + 2x^2$	$4x^3 + 4x$
$x^5 - 3x^2$	$5x^4 - 6x$
$\frac{2}{x^1} \to 2x^{-1}$	$f'(x) = 2(-1)x^{-2} = -2x^{-2}$ or $\frac{-2}{x^2}$
$\frac{3}{x^2} \to 3x^{-2}$	$f'(x) = 3(-2)x^{-3} = -6x^{-3}$ or $\frac{-6}{x^3}$

Derivative of simple functions

J

f(x)	f'(x)
x^1	$f'(x) = 1 \cdot x^0 = 1$
$9x^1$	$f'(x) = 1 \cdot 9x^0 = 9$
13 → $13x^0$	$f'(x) = 13(0)x^{-1} = 0$
-50 → $-50x^0$	$f'(x) = 0$

K

Function	$f(x)$	$f'(x)$
a constant	a	0
x^n	x^n	nx^{n-1}
a constant multiple of x^n	ax^n	anx^{n-1}
multiple terms	$u(x) + v(x)$	$u'(x) + v'(x)$

Classwork

p. 571 1-4

work as a group

EXERCISE 20C

1 Find the gradient function $\frac{dy}{dx}$ for:

a $y = x^6$

$\frac{dy}{dx} = 6x^5$

b $y = \frac{1}{x^5}$

$= x^{-5}$

$\frac{dy}{dx} = -5x^{-6}$

$= -\frac{5}{x^6}$

c $y = x^9$

$\frac{dy}{dx} = 9x^8$

d $y = \frac{1}{x^7}$

$= x^{-7}$

$\frac{dy}{dx} = -7x^{-8}$

or

$-\frac{7}{x^8}$

2 For $f(x) = x^5$, find:

a $f(2)$ **b** $f'(2)$ **c** $f(-1)$ **d** $f'(-1)$

$f(2) =$

$= (2)^5$

$= 32$

$f'(x) = 5x^4$

$\therefore f'(2) = 5(2)^4$

$= 80$

$= (-1)^5$

$= -1$

$f'(-1)$

$= 5(-1)^4$

$= 5$

$\frac{dy}{dx}$ ON GDC

3 Consider $f(x) = \frac{1}{x^4}$.

a Find $f'(x)$. **b** Find and interpret $f'(1)$.

$f(x) = x^{-4}$

$f'(x) = -4x^{-5}$

$= \frac{-4}{x^5}$

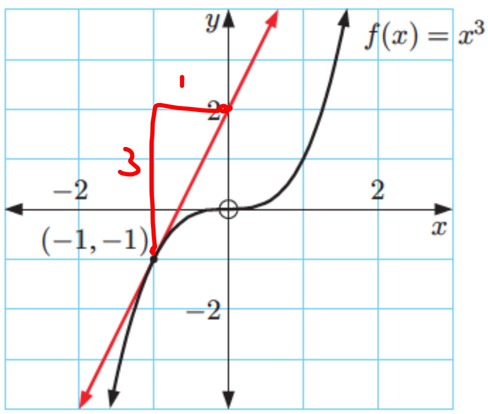
$f'(1) = -\frac{4}{(1)^5} = -4$

-4 is the gradient of the tangent at $x = 1$

4 The graph of $f(x) = x^3$ is shown alongside, and its tangent at the point $(-1, -1)$.

a Use the graph to find the gradient of the tangent. $\frac{3}{1} = \underline{\underline{3}}$

b Check your answer by finding $f'(-1)$.



$f'(x) = 3x^2$

$f'(1) = 3(-1)^2 = \underline{\underline{3}}$

Brain Break

Find $f'(x)$ for............

$$f(x) = 5x^3 + 6x^2 - 3x + 2$$

$$f'(x) = 5(3x^2) + 6(2x) - 3(1) + 0$$

$$f'(x) = 15x^2 + 12x - 3$$

Papa Bear

$$f(x) = 7x - \frac{4}{x} + \frac{3}{x^3}$$

$$= 7x - 4x^{-1} + 3x^{-3}$$

$$f'(x) = 7(1) - 4(-1x^{-2}) + 3(-3x^{-4})$$

$$f'(x) = 7 + 4x^{-2} - 9x^{-4}$$

or $$f'(x) = 7 + \frac{4}{x^2} - \frac{9}{x^4}$$

The Big Kahuna

$$f(x)=\frac{x^2+4x-5}{x}$$

$$f(x)=\frac{x^2}{x}+\frac{4x}{x}-\frac{5}{x}$$

$$f(x)=x+4-5x^{-1}$$

$$f'(x)=1+0+5x^{-2}$$

$$f'(x)=1+\frac{5}{x^2}$$

Assignment

p.573 **Calculus packet**

1aceghj , 2, 4ace , 7, 8

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