Check HW first

with the solutions

then

~~**Warm - up**~~

~~*a handout*~~

HW

questions

$\boxed{136c}$

$(y-1)\quad x = \frac{y+1}{y-1} \cdot (y-1)$

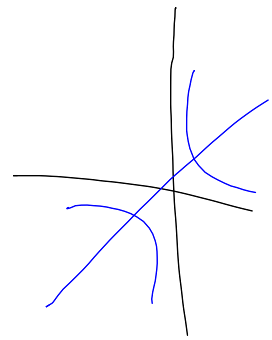
$(y-1)(x) = y+1$

$yx - x = y+1$

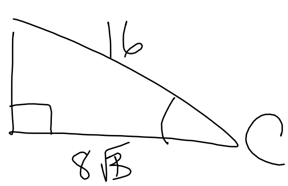
$yx - y = x+1$

$y(x-1) = x+1$

$y = \frac{x+1}{x-1}$



$\boxed{146}$



Soh Cah Toa

$\cos C = \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$

$\cos^{-1}(\cos C) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$C = \cos^{-1}\left(\sqrt{3}/2\right)$

14le answer

54

Warm
Up

•

1) Use the compound interest formula:
Suppose you invest your \$5,000 savings to save for a car. You find a bank that pays 5.8% *annual interest*. Find out how much you would be in your account 7 years from now if you bank pays you interest compounded quarterly. $(n = 4)$

$FV = PV(1 + \frac{r}{n})^{nt}$ or $F = P(1+\frac{r}{n})^{nt}$

$$5000\left(1 + \frac{.058}{4}\right)^{4 \cdot 7}$$

$$\approx \$7482.\underline{17}$$

2) Repeat the calculation, but assume monthly compounding $(n = 12)$

3) Repeat once more, but this time assume compounding daily $(n = 365)$

Compound Interest Formula

$$FV = PV\left(1+\frac{r}{n}\right)^{nt}$$

Invest \$5,000 for 15 years at 7% compound interest quarterly $n=4$

$$= 5000\left(1+\frac{.07}{4}\right)^{4\cdot 15}$$

$$= 14,159.\underline{08}$$

$n = 1$

$n = 4$ $\quad$ $14,159.\underline{08}$

$n = 12$ $\quad$ $14,244.^{73}$

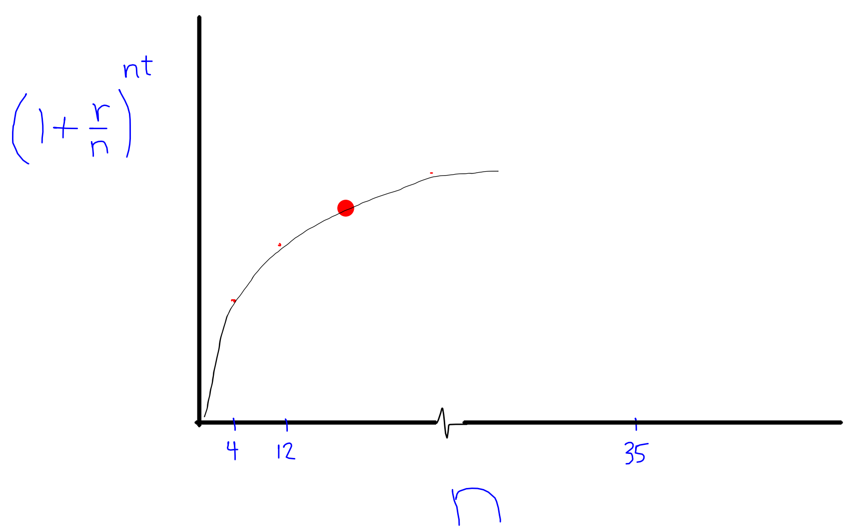
$n = 365$ $\quad$ $14,286.\underline{82}$

the higher the "n"

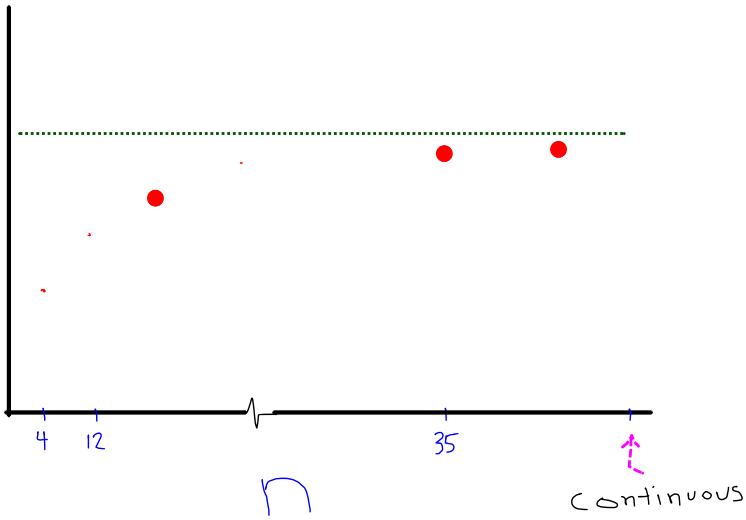
the larger the multiplier

but.....

the increase starts to slow down.



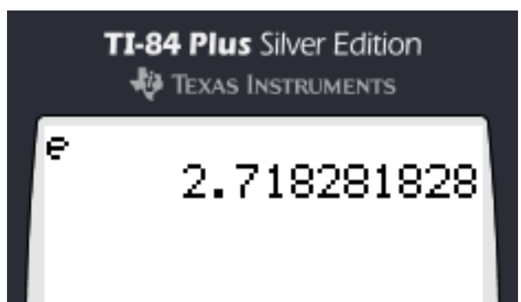
$$\left(1+\frac{r}{n}\right)^{nt}$$



$$\left(1+\frac{1}{n}\right)^{n}$$

As $n \to \infty$

$$\left(1+\frac{1}{n}\right)^{n} \to \boxed{e}$$



π

3.14159. . . .

$$F = P\left(1+\frac{r}{n}\right)^{nt} \longrightarrow F = Pe^{rt}$$

4) Now find the final balance if the bank uses *continuous* compounding.

$FV = PV e^{rt}$

$A = Pe^{rt}$

$= 5000\, e^{.058 \cdot 7}$

CONTINOUS
compound interest
formula

$$A = Pe^{rt}$$

Future Present

Aim

Natural Logarithms

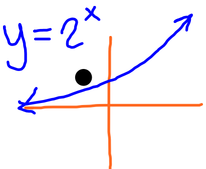
• Solve exponential equations that have the natural base, e.

Notes when you see

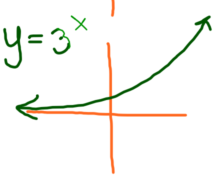
$\boxed{\log}$

$\boxed{LN}$

$y=2^x$

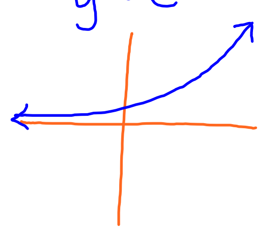


$y=3^x$



etc.

$y=e^x$



The

exponential function

e has many uses

finance, science, etc

Top 4 REASONS WHY $f(x) = e^x$ is KNOWN AS the Exponential Function

#4 $f(x) = e^x$ has special calculus properties that simplify many calculations

#3 e is considered to be the natural base.

#2 $e > 1$ so $f(x) = e^x$ is a growth function and the number one reason why $f(x) = e^x$

is THE natural exponential function ....

#1 Leonhard Euler introduced the notation and he could call it what he wanted to call it!

e

Using logarithms to solve Newton's Law of Cooling

$$\frac{T(t)-T_a}{T_0-T_a} = e^{-kt}$$

Radioactive half-life

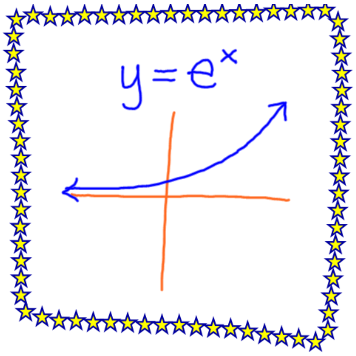
$$P(t) = P_0 e^{-kt}$$

•

Doubling time for an investment

$$2P_0 = P_0(1.0075)^n$$

A spider enjoying his favorite movie.
(and you know which movie that is!)



$y = e^x$

has an inverse which is called the natural log function.

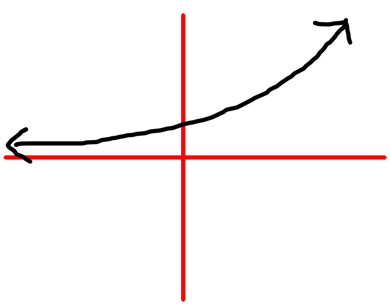
inverse

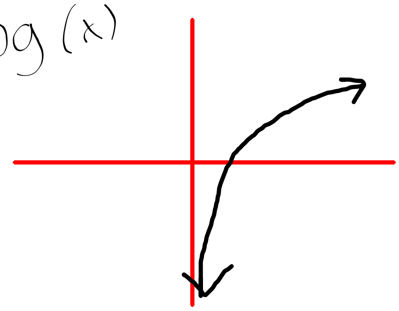
$f(x) = e^x$

$f^{-1}(x) = \cancel{\log_e} x$

$\log_7(x)$

$\log(x)$





e is so prevalent out in the real world its logarithm gets its very own notation

$\cancel{\log_e x} \dashrightarrow \ln x$

$\ln x$

$\ln x = 100$

Monster checking for kids under the bed.

Solve Natural Log Equations

Solve each equation. Check your answers.

$\ln x = 0.1$

convert

$x = e^{.1}$

$x \approx 1.105$

$\log_7(x) = .1$

$$\ln\left(\frac{x+2}{3}\right) = 12$$

convert

$$\frac{x+2}{3} = e^{12} \longrightarrow x+2 = 3e^{12}$$

$$x = 3e^{12} - 2$$

$$x \approx 488,262.374$$

$$\underbrace{\ln 5 - \ln(2x)}_{\text{condense}} = 1$$

$$\ln\left(\frac{5}{2x}\right) = 1$$

convert

$$\frac{5}{2x} = e^{1}$$

multiply by $2x$

$$\frac{\frac{5}{e}}{2} = \frac{\cancel{2}x}{\cancel{2}} \quad x = \frac{5/e}{2}$$

$$\frac{5}{2e} = \frac{\cancel{2x \cdot e}}{\cancel{2e}}$$

$$x = \frac{5}{(2e)} \approx \cancel{6.796} \; .920$$

Shortcut

$\log(10) -$ $\quad \ln(e) = x \quad \log_7(7)$

$\log(10) = x$

$10^x = 10$

$e^x = e$

1

1

and expon. equations with base *e*

$$e^{x+1} = 30$$

take log of both sides

convert directly to log form

$\ln\left(e^{x+1}\right) = \ln(30)$

$(x+1)\ln(e) = \ln(30)$

$x + 1 = \dfrac{\ln(30)}{\ln(e)} \leftarrow 1$

$x = \ln(30) - 1$

$\approx$

$x + 1 = \ln(30)$

$-1 \qquad -1$

$x = \ln(30) - 1$

An initial investment of $200 is now valued at $245.25. The interest rate is 6% compounded continuously. How long has the money been invested?

$$A = Pe^{rt}$$

$$245.25 = 200e^{.06t}$$

$$\frac{245.25}{200} = e^{.06t}$$

$$\ln\left(\frac{245.25}{200}\right) = \ln\left(e^{.06t}\right)$$

$$.06t \cdot \underbrace{\ln(e)}_{1} = \ln\left(\frac{245.25}{200}\right)$$

$$.06t = \ln\left(\frac{245.25}{200}\right)$$

$$t = \frac{\ln\left(\frac{245.25}{200}\right)}{.06}$$

$t = 3.399$ years

Assignment

Worksheet 6242