## IB Style Solutions - Logic, Sets and Probability

## Logic

## Question 1 - Answers/Solutions

a. (i) It is snowing if and only if I do not wear a hat.
(ii) It is not snowing, and the boys will walk to school.
b. \&He does not see or he does not run.
(ii) Converse: If I visit the Louvre, then I go to Paris Inverse: If I do not go to Paris, then I do not visit the Louvre.

Contrapositive: If I do not visit the Louvre, then I do not go to Paris.
c. $\neg q \Rightarrow p$

Question 2-Answers/Solutions
a. (i) Converse: If $x^{2}=25$, then $x=5$.
(ii) No
(iii) Contrapositive: If $x^{2} \neq 25$, then $x \neq 5$.
[1 mark]
(iv) Yes
[1 mark]
[1 mark]
b. 保
[1 mark]
(ii) False
[1 mark]

## Question 3 - Answers/Solutions

a. (i) $T \underline{T} \equiv \mathrm{~F}$
[1 mark]
(ii) $\mathrm{F} \Rightarrow \mathrm{T} \equiv \mathrm{T}$
[1 mark]
(iii) $\mathrm{F} \Leftrightarrow \mathrm{F} \equiv \mathrm{T}$
[1 mark]
b. continued on next page ...
b.

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ | $(\neg p \vee q) \Rightarrow p$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | $\mathbf{F}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| T | F | $\mathbf{F}$ | $\mathbf{F}$ | $\mathbf{T}$ |
| F | T | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{F}$ |
| F | F | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{F}$ |

[3 marks, 1 mark per column]

## Question 4 - Answers/Solutions

a. (i) Conclysion: The team did not win.
(ii) Conclusion: Carlos studies for his exam.
(i) $\neg p \wedge \not \subset$
(ii) $p \vee q \quad$ This is the negation of part (b) (i) above).

## Question 5 - Answers/Solutions

a. If he plays the game and he hates to lose, then we cannot party after the game.
[2 marks]
b.

| $p$ | $q$ | $r$ | $\neg r$ | $p \wedge q$ | $(p \wedge q) \Rightarrow \neg r$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | F | T | F |
| T | T | F | T | T | T |
| T | F | T | F | F | T |
| T | F | F | T | F | T |
| F | T | T | F | F | T |
| F | T | F | T | F | T |
| F | F | T | F | F | T |
| F | F | F | T | F | T |

[3 marks, 1 mark per column]

## Question 6 - Answers/Solutions

a. If it is not Tuesday, then Marie does not play the violin.
[1 mark]
b. (i)

| $p$ | $q$ | $p \Rightarrow q$ | $\neg p$ | $\neg p \vee q$ | $(p \Rightarrow q) \Rightarrow(\neg p \vee q)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| T | F | $\mathbf{F}$ | $\mathbf{F}$ | $\mathbf{F}$ | $\mathbf{T}$ |
| F | T | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{T}$ |
| F | F | $\mathbf{T}$ | T | $\mathbf{T}$ | $\mathbf{T}$ |

[3 marks, 1 mark per column]
(ii) Yes, the statement is a tautology because it is true for all possible cases of $p$ and $q$.

## Question 7 - Answers/Solutions

a.

| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \wedge \neg q$ | $\neg q \Rightarrow \neg p$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | $\mathbf{F}$ | $\mathbf{F}$ | $\mathbf{F}$ | $\mathbf{T}$ |
| T | F | $\mathbf{F}$ | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{F}$ |
| F | T | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{F}$ | $\mathbf{T}$ |
| F | F | $\mathbf{T}$ | $\mathbf{T}$ | $\mathbf{F}$ | $\mathbf{T}$ |

[4 marks, 1 mark per column]
b. The two statements are not logically equivalent because
[2 marks] corresponding (i.e. each row) truth values are not identical. In fact one is the negation of the other.
[2 marks]

## Question 8 - Answers/Solutions

a. (i) $\neg p \Leftrightarrow r$
(ii) $\neg q \Rightarrow(\neg p \wedge \neg r)$
b. (i)

| $p$ | $q$ | $r$ | $p \wedge q$ | $(p \wedge q) \Rightarrow r$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | T | $\mathbf{T}$ | $\mathbf{T}$ |
| T | T | F | $\mathbf{T}$ | $\mathbf{F}$ |
| T | F | T | $\mathbf{F}$ | $\mathbf{T}$ |
| T | F | F | $\mathbf{F}$ | $\mathbf{T}$ |
| F | T | T | $\mathbf{F}$ | $\mathbf{T}$ |
| F | T | F | $\mathbf{F}$ | $\mathbf{T}$ |
| F | F | T | $\mathbf{F}$ | $\mathbf{T}$ |
| F | F | F | $\mathbf{F}$ | $\mathbf{T}$ |

(ii)

| $p$ | $q$ | $r$ | $\neg r$ | $p \wedge q$ | $(p \wedge q) \vee \neg r$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T | T | T | F | T | T |
| T | T | F | T | T | T |
| T | F | T | F | F | F |
| T | F | F | T | F | T |
| F | T | T | F | F | F |
| F | T | F | T | F | T |
| F | F | T | F | F | F |
| F | F | F | T | F | T |

(iii) No, the statements are not equivalent as the corresponding truth values are not identical (see last column of parts (i) and (ii) shown above.
[2 marks, 1 mark per column]

