Have your HW out and ready

HW questions →

① Consider the geometric sequence: 2, 6, 18, 54.....

a) what is the common ratio? ___ b) List the next 3 terms _____

c) Calculate the 30th term (show work) ...using IB NOTATION

d) Find the sum of the first 10 terms, showing IB NOTATION.

(1) Find the n^{th} term formula, U_n, for each sequence below

a) 7, 14, 28, ...

b) 80, 86, 92, 98, ...

c) 80, 40, 20, ...

d) 5, -10, 20, -40 ....

(3) Find the sum of each sequence (showing work, etc.) of the first 11 terms.

a) 1000, 500, 125, ...

b) 10, 6, 2, -2, ...

(4) A geometric sequence has $u_1 = 8$ and $u_4 = 216$. What is the common ratio? (show work)

and find the general term, u_n.

and find S_7

(5) Find the sum of each series (show details for all steps)

(a) $10 + 7 + 4 + \ldots\ldots -50$

(1) $u_n = u_n + d(n-1)$

$-50 = 10 - 3(n-1)$

$n = 21$

(2) $S_{21} = \frac{n}{2}\left[u_1 + u_n\right]$

$= \frac{21}{2}\left[10 + -50\right]$

(b) $\frac{1}{4} + \frac{1}{2} + 1 + \ldots + 64$

1 $U_n = U_1 \cdot r^{n-1}$

$64 = \frac{1}{4}(2)^{n-1}$

$216 = 2^{n-1}$

$\log(216) = \log(2^{n-1})$

$\log(216) = (n-1) \cdot \log(2)$

$\frac{\log(216)}{\log(2)} = n-1$

$n = 9$

$S_9 = U_1\left[\frac{r^n - 1}{r-1}\right]$

(6) Find K given that a geometric sequence has consecutive terms of

$4 \quad , \quad K \quad , \quad K^2 - 1$

$\frac{U_2}{U_1} = \frac{U_3}{U_2}$

$\frac{K}{4} = \frac{K^2-1}{K}$

7 The figure shows two adjacent triangular fields ABC and ACD where AD = 30 m, CD = 80 m, BC = 50m. m∠ADC = 60° and m∠BAC = 30°

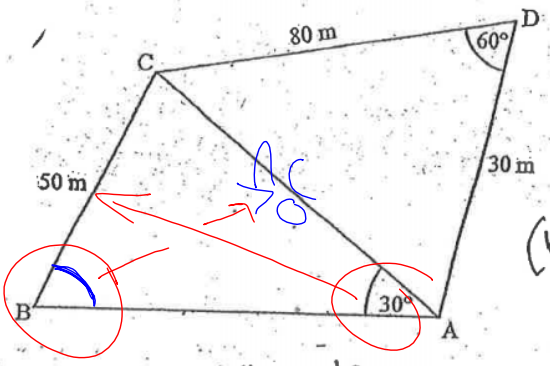

diagram not to scale

(a) Using △ACD calculate AC

* $AC^2 = 80^2 + 30^2 - 2(80)(30)\cos 60$

(b) then calculate m∠ABC

* $\frac{\sin 30°}{50} = \frac{\sin B}{70}$

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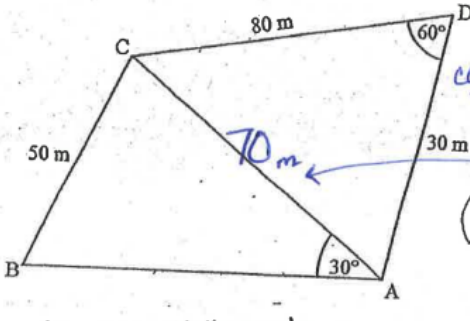

diagram not to scale

(a) Using △ACD calculate AC

COSINE RULE * $AC^2 = 80^2 + 30^2 - 2(80)(30)\cos 60°$

$AC^2 = 4900$

$AC = 70$ m

(b) then calculate m∠ABC

* $\frac{\sin B}{70} = \frac{\sin 30}{50}$

$\sin B = 0.7 \rightarrow B = 44.4°$

(c) Calculate the area of the field ACD.

$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(80)(30)\sin 60^\circ$

$= 1039.23\ m^2$

$= 1040\ m^2$ to nearest 3 sf.

Aim Today

Apply the geometric sequence formula to Financial Investing.

$$U_n = U_1 r^{n-1}$$

↑ if we know

$$U_n = U_0 r^{n}$$

Increase 200 by 50%	1.5	300
Increase 500 by 9%	1.09	545
Decrease 80 by 15%	0.85	68
Decre 2000 by 1.3%	.987	1974
Increase 2000 by 100%	2	400

TODAY: Finance

Have your graphing calculator out.

You will be following a sequence of steps.

Type 1000

ENTER

this represents the amount of money you saved from a job.

you then deposit it in a bank that pays 4% annual interest. You plan to invest this $ for 8 years.

1000
enter

x 1.04
ENTER (8 times)

You have just calculated the future value of your initial investment. This amount is:

$1,368.57

Instead, what if you invested your $1000 at an annual interest rate of 6.5% for 11 years.

to get $1,999.15

which means you almost doubled your money

Hold it, I meant 30 years at 7%!

(just kidding)

Because we are applying a constant percent over and over, we can write an exponential function

$$y = a\,b^{x}$$

$$= 1000(1.08)^{11}$$

x ← # of years

$y = 1000(1 + .08)^x$

$= 1000(100\% + 8\%)^x$

money that grows this way is growing with interest compounded annually.

But wait! Some banks pay you interest multiple times per year

for example: Semi-annually (twice a year)

this means your $ is being compounded twice a year

example: 8% annual interest

4% after six months

4% after next six months

$$1000\left(1+\frac{8\%}{2}\right)^{22}$$ ← 11 years at 2 compo per year

$$1000(1+4\%)^{22}$$

$$1000(1+.04)^{22} = 1000(1.04)^{22}$$

$$=$$

Some banks pay

quarterly

monthly

semi-annually

annually

daily

continuously

NOTES

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{Kn}$$

Future Value

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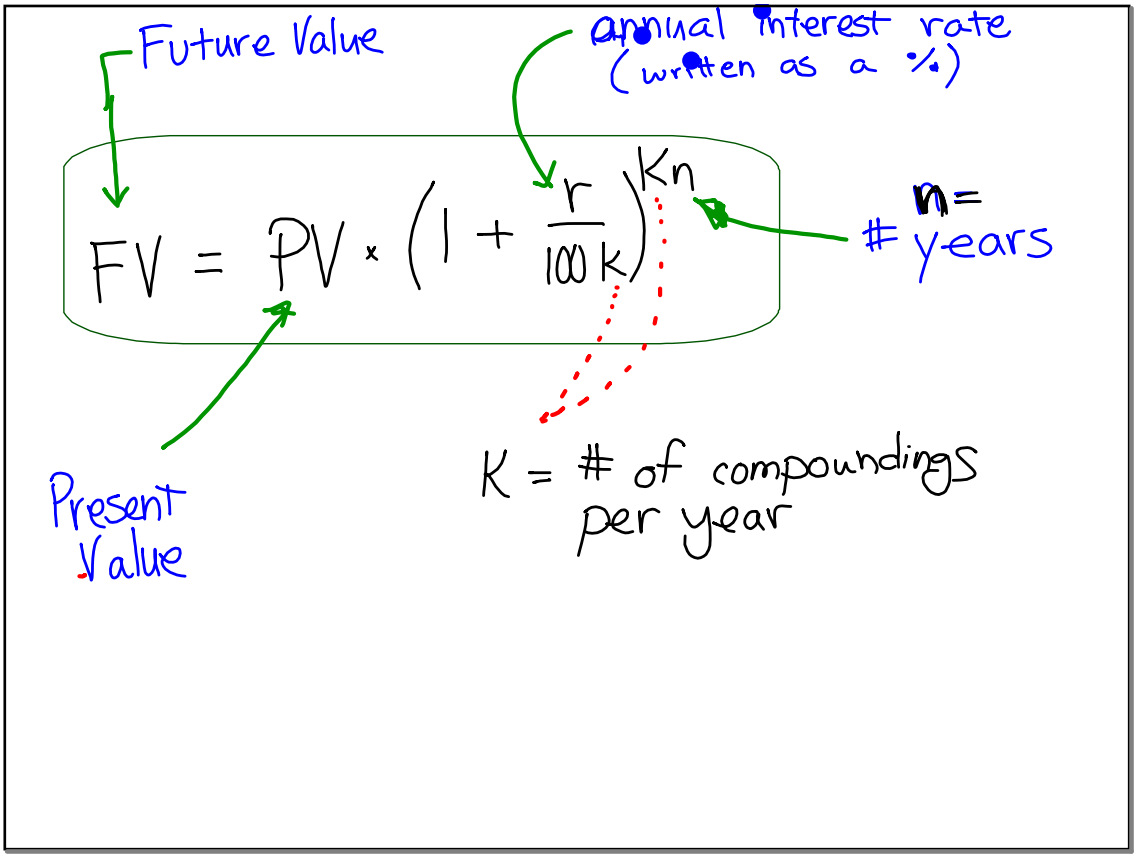
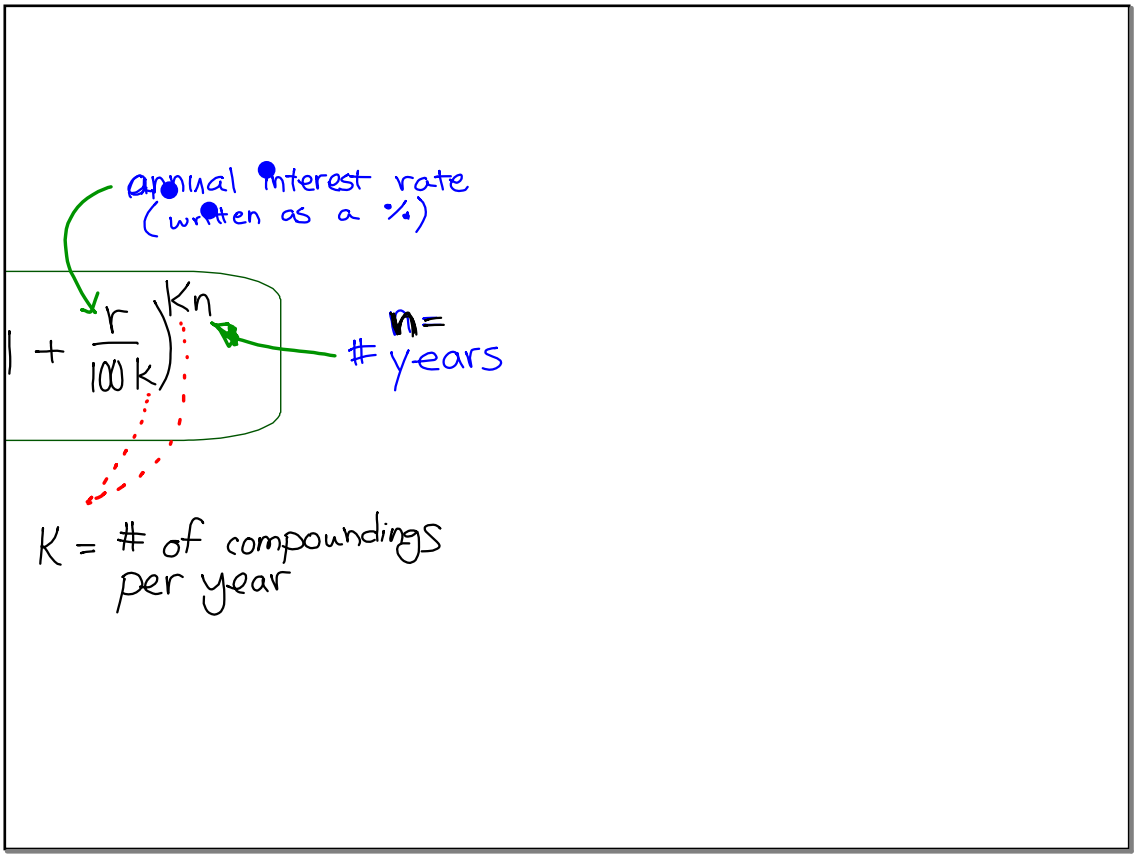
Present Value


Future Value
annual interest rate (written as a %)
$FV = PV \times \left(1 + \frac{r}{100k}\right)^{Kn}$
n = # years
Present Value
K = # of compoundings per year


annual interest rate (written as a %)
$\left(1 + \frac{r}{100k}\right)^{Kn}$
n = # years
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handout

1

Calculate the future value of the following situations.

a) $800 invested at 5% interest for 3 years, compouned monthly

$FV =$

b) $15,000 at 2.5% interest for 20 years, compouned quarterly

c) $4,000 at $6\frac{1}{3}$% interest for 20 years, comp. semi-annually

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$$FV = 15000\left(1 + \frac{2.5}{100(4)}\right)^{(4\times 20)} = \$24,692.37$$

c) $4,000 at $6\frac{1}{8}$% interest for 20 years, comp. semi-annually

$$FV = 4000\left(1 + \frac{6.125}{100(2)}\right)^{(2\times 20)} = \$13,368.63$$

13368.72

13 368.64

2 Finding the Present Value (or Capital)

How much does Nicole need to deposit into an account to collect $50,000 at the end of 6 years if the account is paying 6.8% p.a. compounded quarterly?

② Finding the Present Value (or Capital)

How much does Nicole need to deposit into an account to collect $50,000 at the end of 6 years if the account is paying 6.8% p.a. compounded quarterly?

$$50000 = PV\left(1 + \frac{6.8}{100(4)}\right)^{(4\times 6)}$$

$$PV = \frac{50000}{\left(1 + \frac{6.8}{400}\right)^{24}} \simeq \$33,363.16$$

③ Finding the interest rate

Calculate the interest rate that Tus would need in order to accumulate $25,000 in 5 years time, if the initial amount to invest is $19,971 (assume monthly compounding)

(3)

Finding the interest rate

Calculate the interest rate that Tus would need in order to accumulate $25,000 in 5 years time, if the initial amount to invest is $19,971 (assume monthly compounding)

$$25000 = 19971\left(1 + \frac{r}{100(12)}\right)^{(12\times 5)}$$

$$\frac{25000}{19971} = \left(1 + \frac{r}{1200}\right)^{60}$$

$20 = (1+x)^3$

$$\sqrt[60]{\frac{25000}{19971}} = 1 + \frac{r}{1200}$$

$$\frac{25000}{19971} = \left(1 + \frac{r}{1200}\right)$$

$$\sqrt[60]{\frac{25000}{19971}} = 1 + \frac{r}{1200}$$

-1 $\quad -1$

$$\sqrt[60]{\frac{25000}{19971}} - 1 = \frac{r}{1200}$$

$r = 4.50\%$

(4)

Finding the Time Period

For how long must Jamie invest 4000 euro at 6.4% p.a. compounded half-yearly if is to amount to 10,000 euro?

(4)

Finding the Time Period

For how long must Jamie invest 4000 euro at 6.4% p.a. compounded half-yearly if is to amount to 10,000 euro?

$$10000 = 4000\left(1 + \frac{6.4}{100(2)}\right)^{2\times n}$$

$$\frac{10000}{4000} = \left(1 + \frac{6.4}{200}\right)^{2n}$$

$$\log\left(\frac{10}{4}\right) = \log\left(1 + \frac{6.4}{200}\right)^{2n}$$

$$\frac{10000}{4000} = \left(1 + \frac{6.4}{200}\right)^{2n}$$

$$\log\left(\frac{10}{4}\right) = \log\left(1 + \frac{6.4}{200}\right)^{2n}$$

$$\log\left(\frac{10}{4}\right) = 2n \log\left(1 + \frac{6.4}{200}\right)$$

$$n = \frac{\log\left(\frac{10}{4}\right)}{2\left(\log\left(1 + \frac{6.4}{200}\right)\right)} = 14.5 \text{ years}$$

B.B.

HH Textbook
page 418.....

Review Set 12A... 2 - 5 and
Review Set 12B... 1 , 6 , 8

Your TI-
has a Financial App

✓ For IB students:
in the past, knowledge of this App was not required.

✓ Starting on this year's exams going forward, they recommend it!

Handout

Word of warning: be able to get all answers algebraically, except for any problem involving monthly payments.

Assignment:

Worksheet: Compound Interest Practice

and learn how to use the App program