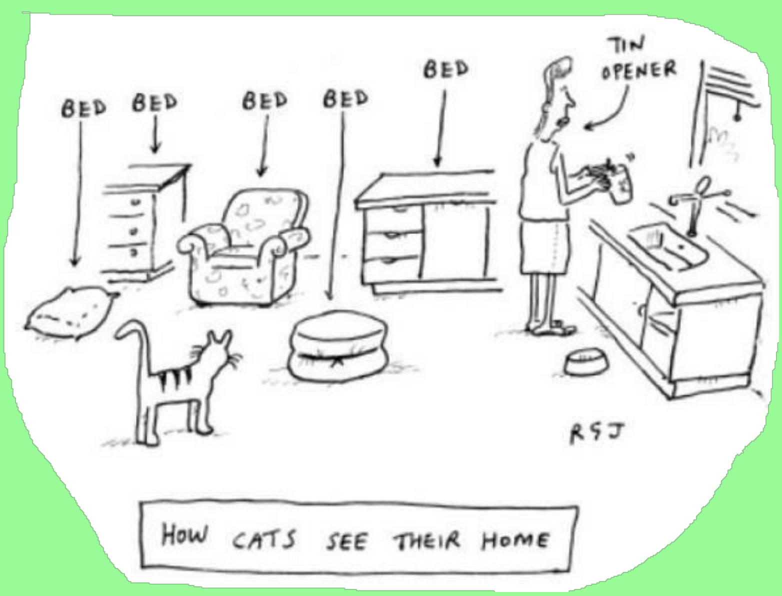
http://www.mathcasts.org/mtwiki/InterA/BoxFolding

Eliminate Box Prob. ?

You have 5 minutes to check your HW with the solutions. Be sure to fill out the new weekly recording sheet.

later you can ask me questions.



3d Dark Br

$$f(x) = \frac{10+3x^2}{x} = \frac{10}{x} + \frac{3x^2}{x} = 10x^{-1} + 3x$$

$$f'(x) = -10x^{-2} + 3$$

$$= -\frac{10}{x^2} + 3$$

Then, do the first problem

on the Notes 4.0

handout

1 Find the equation of the **Tangent** and the **Normal** for the equation **y = x³ - 5x + 2** at the location x = -2

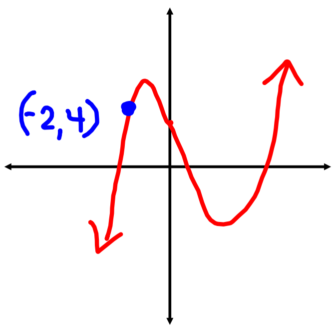
P.O.T. $f(-2)$

$f(-2) = (-2)^3 - 5(-2) + 2$

$= 4$

$(-2, 4)$

$f'(x) =$



1 Find the equation of the **Tangent** and the **Normal** for the equation **y = x³ - 5x + 2** at the location x = -2

P.O.T. $f(-2)$

$f(-2) = (-2)^3 - 5(-2) + 2$

$= 4$

$(-2, 4)$

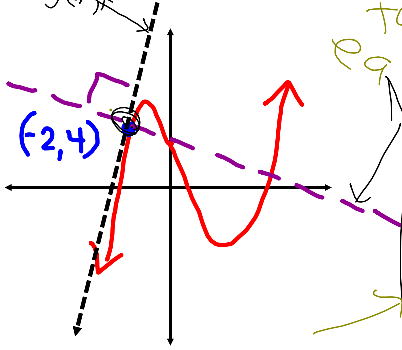
GRADient $f'(x) = 3x^2 - 5$

$f'(-2) = 3(-2)^2 - 5 = 7$

equation of tangent

$\boxed{y - 4 = 7(x + 2)}$

tangent



Normal has a gradient of $-\frac{1}{7}$

eq

$\boxed{y - 4 = -\frac{1}{7}(x + 2)}$

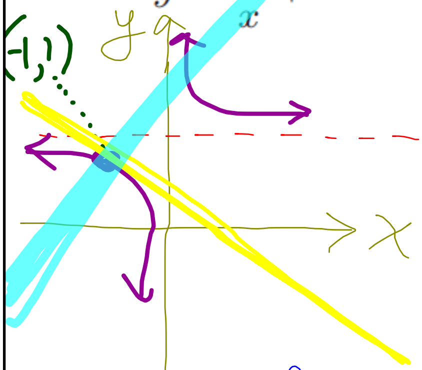
2 Find the equation of the **Tangent** and the **Normal** for the equation

$$y = \frac{1}{x} + 2$$ at the location (-1, 1)

$$y = x^{-1} + 2$$

$$f'(x) = (-1)x^{-2} = \boxed{-\frac{1}{x^2}}$$

$$f'(-1) = \frac{-1}{(-1)^2} = -1$$



gradient of Normal = 1

equation of tangent

$$y - 1 = -1(x+1)$$

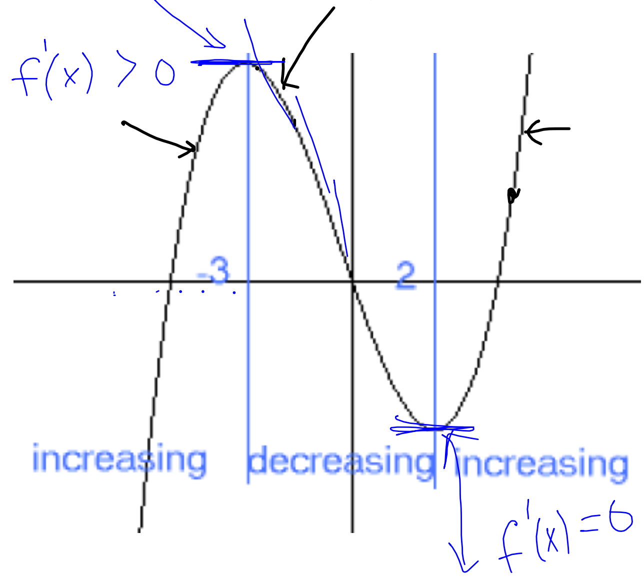
$$\boxed{y - 1 = 1(x+1)}$$

equation of Normal

A

$f'(x) < 0$ $f'(x) = 0$ $f'(x) > 0$

Stationary Points

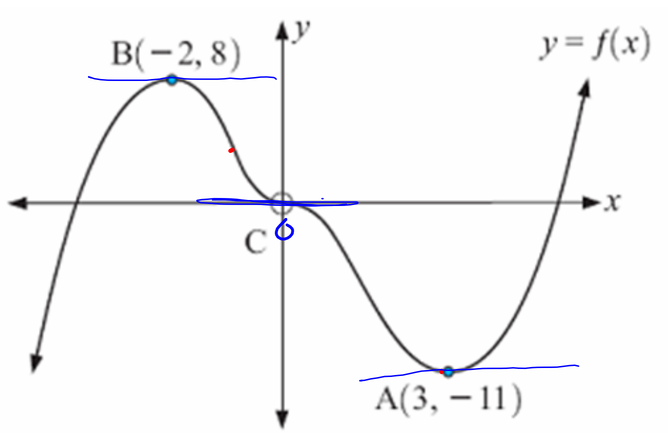


$f'(x) < 0$

any point on a curve where the tangent line is horizontal **is a STATIONARY point**

B

local maximums

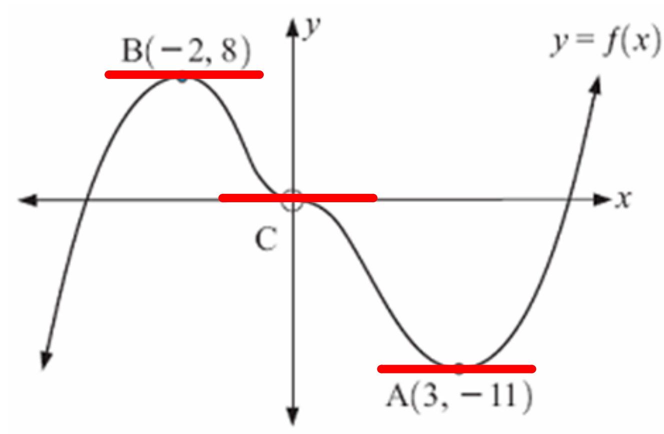


horizontal inflection points

local minimums

$\frac{dy}{dx} = 0$

local maximums



horizontal inflection points

local minimums

Horizontal tangents have a gradient of zero $\boxed{C}$

To find all of those places on any given function:

(1) find the *gradient function* and ✓

(2) set it equal to zero ✓

same as "find the derivative"

$$f'(x) = 0$$

(3) Solve to find x-values (if any) which are the locations where the tangents have a gradient of zero !

3. Find the equation(s) of any horizontal tangents of

$f(x) = \frac{1}{3}x^3 - x + 2$

$f'(x) = x^2 - 1$

Set $f'(x) = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$\sqrt{\ } \quad \sqrt{\ }$

$x = \pm 1$

$\boxed{\begin{array}{l} y = \frac{4}{3} \\ y = \frac{8}{3} \end{array}}$

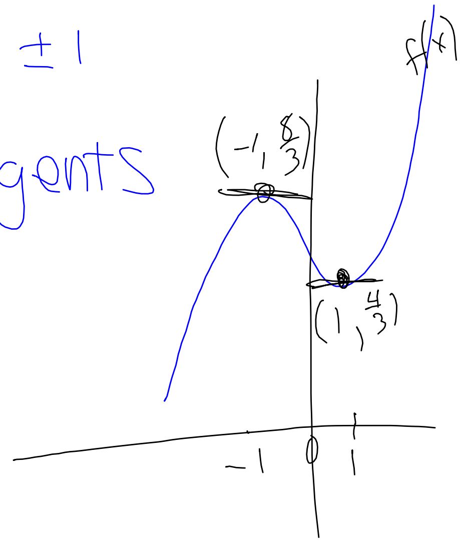
$f(x)$

$f(1) =$

So the horizontal tangents are at $x = 1$ and $x = -1$

$2^{2/3}$

$-2^{2/3}$



4. **Find the stationary points of the curve $y = x^3 - 3x^2 - 9x + 10$**

$f'(x) = 3x^2 - 6x - 9$ $\qquad (-1, 15) \quad (3, -17)$

Set $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

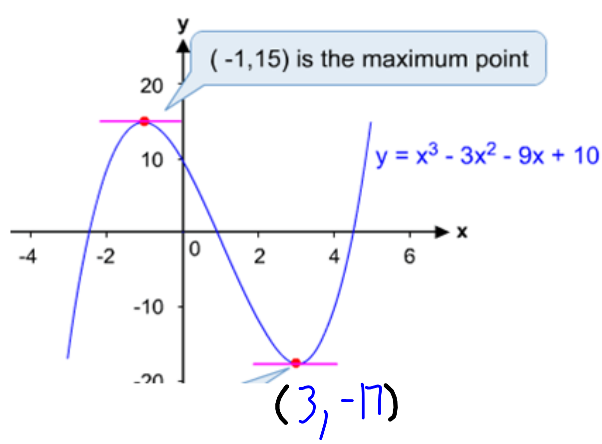
$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x + 1 = 0 \qquad x - 3 = 0$

$x = -1 \qquad x = 3$



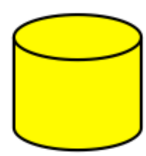
Schedule

Tu NORMALS + OPTIMIZATION

W Review

Thur QUIZ ON CALCULUS

What if you wanted the minimum material to make a cylinder with a required volume?



In this case you would have two variables (radius and height) and one fixed quantity (volume)

Think about
Why is having two variables a problem?

In order to differentiate, you need an expression for the quantity you want to minimise (or maximise) in terms of **just one variable**

Working with a cylinder

First, use the fixed volume to eliminate one of the variables (either the height or radius)

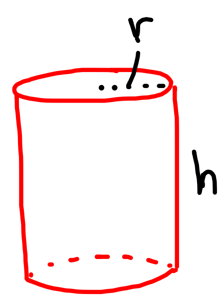
When you have an expression for the quantity of material needed to make the cylinder in terms of just one variable, differentiate it and put the derivative = 0

Solve this equation to find the value of the variable that gives a minimum (or maximum)

Then find the value of the other variable and the minimum (or maximum) that you require

Minimum material to make a can

Say you want to find the minimum metal needed to make a can to hold 500 ml (the same as 500 cm^3)



$V = \pi r^2 h$

$\pi r^2 h = 500$

$h = \frac{500}{\pi r^2}$

SA of metal $= M = 2\pi r^2 + 2\pi r h$

500

$$M = 2\pi r^2 + 2\pi r h \qquad 500 = \pi r^2 h$$

$$h = \frac{500}{\pi r^2}$$

$$M = 2\pi r^2 + 2\cdot\pi\cdot r\left(\frac{500}{\pi r^2}\right)$$

$$M = 2\pi r^2 + \frac{1000}{r}$$

$$M = 2\pi r^2 + 2\pi r h \qquad 500 = \pi r^2 h$$

$$h = \frac{500}{\pi r^2}$$

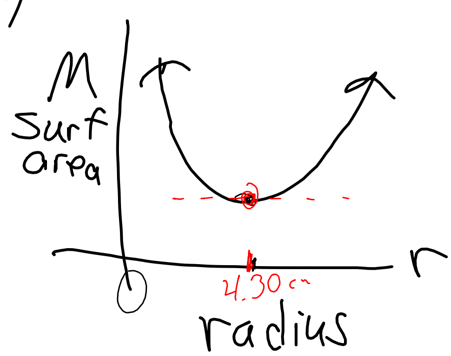
$$M = 2\pi r^2 + 2\cancel{\pi r}\left(\frac{500}{\cancel{\pi r^2}_{\,r}}\right)$$

$$M = 2\pi r^2 + \frac{1000}{r}$$

$$M = 2\pi r^2 + 1000 r^{-1}$$

$$\frac{dM}{dr} = 4\pi r - 1000 r^{-2}$$

$$\text{or } 4\pi r - \frac{1000}{r^2}$$



$$4\pi r - \frac{1000}{r^2} = 0$$

multiply by r^2

$$4\pi r^3 - 1000 = 0$$

$$4\pi r^3 = 1000$$

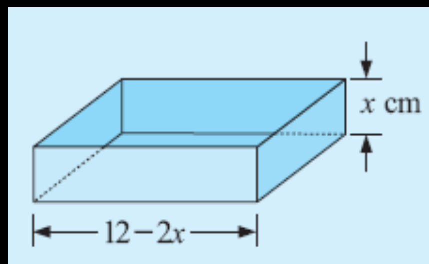
$$r^3 = \frac{1000}{4\pi}$$

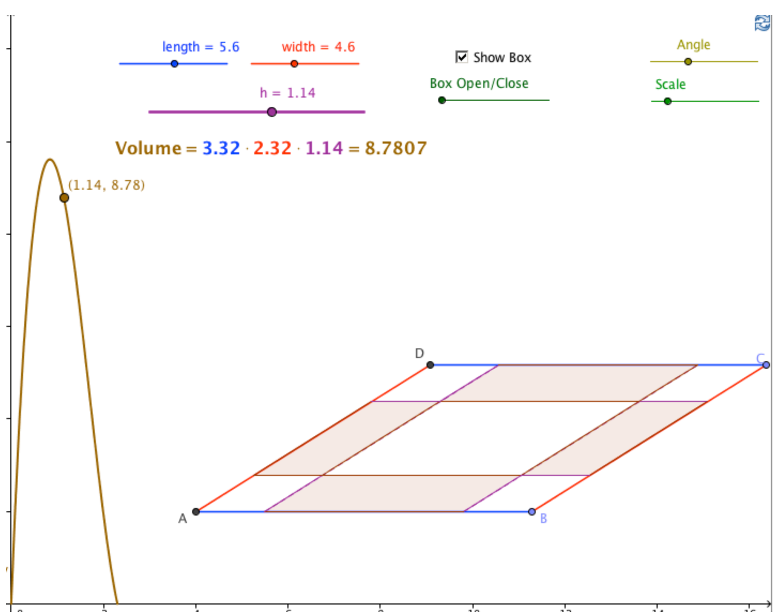
$$r = \sqrt[3]{\frac{1000}{4\pi}} = 4.30$$

So a radius of 4.30 cm will minimize the surface area.

B.B.

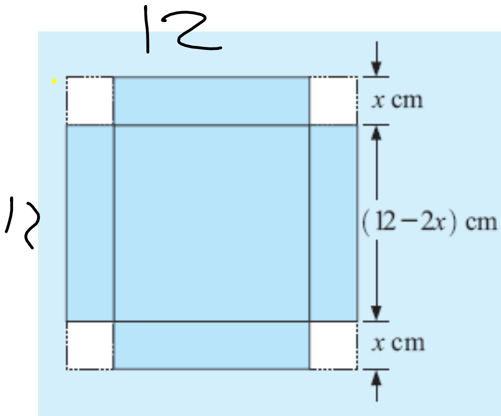
"the Box Problem"

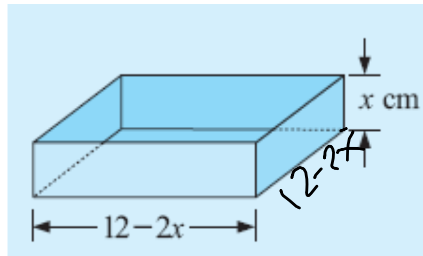




5

Square corners are cut from a piece of 12 cm by 12 cm tinplate which is then bent into the form of an open dish. What size squares should be removed if the volume is to be a maximum?





Note: $0 \leqslant x \leqslant 6$

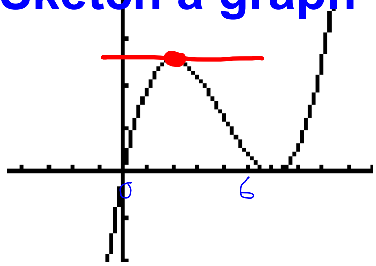
V(x) $$\begin{aligned} &= (12 - 2x)^2 \times x \\ &= (144 - 48x + 4x^2) \times x \\ &= 144x - 48x^2 + 4x^3 \end{aligned}$$

$V(x) = 4x^3 - 48x^2 + 144x$

We can graph to let our GDC find the maximum volume

or we can use calculus !

Sketch a graph



The max volume will occur where the gradient is zero. Therefore, we need to ***find the derivative*** and ***set it to 0***

$$\therefore \quad V'(x) = 144 - 96x + 12x^2$$
$$0 = 12(x^2 - 8x + 12)$$
$$0 = 12(x-2)(x-6)$$

$x - 2 = 0$ $\quad$ $x - 6 = 0$

$x = 2$ $\quad$ $x = 6$

$\therefore$ maximum volume occurs when $x = 2$ cm

$\therefore$ cut out 2 cm squares.

Assignment

1 Day 4 Worksheet (both sides)

2 Calculus packet:

and p. 582...Review Set A..... 1-8

SAVE AS PDF , Mr.C !!!