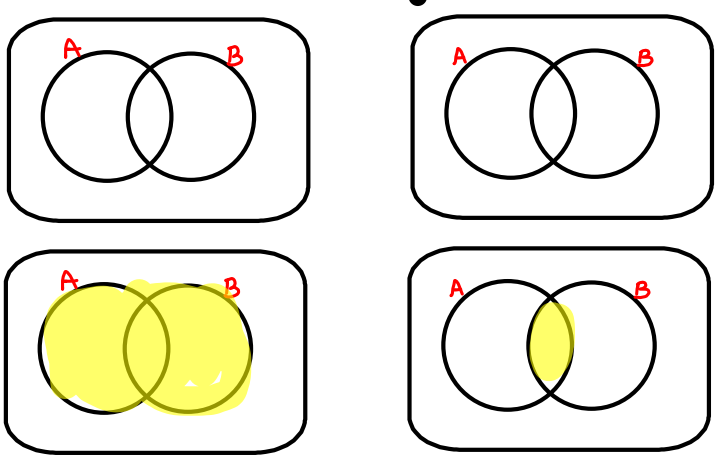
Warm Up - Pick Up the handout.

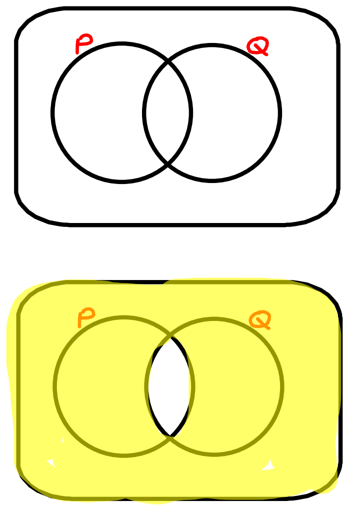
HW questions

1. $A \cup B$

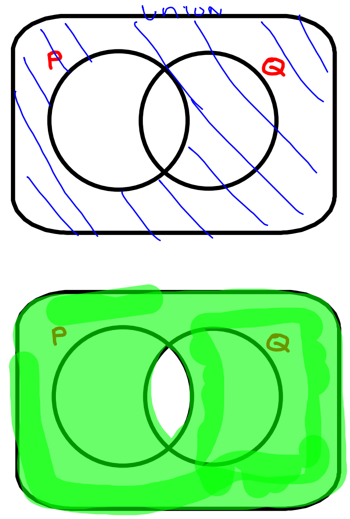
2. $A \cap B$



3 $(P \cap Q)'$



4 $P' \cup Q'$



Logic Assignment 2

Were there any tautologies?

p	q	$\wedge$ $\vee$ $\veebar$ $\Rightarrow$ $\Leftrightarrow$
T	T	
T	F	
F	T	
F	F	

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T			
T	F			
F	T			
F	F			

p	q	$\neg q$	$p \vee \neg q$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T					
T	F					
F	T					
F	F					

7.

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$
T	T	T	T	T	F	F
T	T	F	T	F	T	F
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	T	T	F	F
F	T	F	T	F	T	F
F	F	T	T	T	F	F
F	F	F	T	T	T	T

Y f

A.

The Veterinarian has gathered the following data about the weight of dogs and the weight of their puppies.

		Dog		Total
		Heavy	Light	
Puppy	Heavy	36	27	63
	Light	22	35	57
	Total	58	62	120

$(2-1)(2-1)$

1

$\frac{57 \cdot 62}{120} = 29.45$

$\chi^2 = \sum \frac{(f_e - f_e)^2}{f_e}$

The veterinarian wishes to test the following hypotheses.

H_0: A puppy's weight is independent of its parent's weight.
H_1: A puppy's weight is related to the weight of its parent.

(a) The table below sets out the elements required to calculate the χ^2 value for this data.

	f_o	f_e	$f_e - f_o$	$(f_e - f_o)^2$	$(f_e - f_o)^2 / f_e$
heavy/heavy	36	30.45	−5.55	30.8025	1.012
heavy /light	27	32.55	5.55	30.8025	0.946
light/heavy	22	27.55	5.55	30.8025	1.118
light/light	35	*a* 29.45	*b* -5.55	*c* 30.8025	*d* 1.04159

38.4104

(i) Write down the values of *a*, *b*, *c*, and *d*. **(4)**

(ii) What is the value of χ^2_{calc} for this data? **(1)**

(iii) How many degrees of freedom exist for the contingency table? 1 **(1)**

B.

A rumour spreads through a group of teenagers according to the exponential model

$$N = 2 \times (1.81)^{0.7t}$$

where N is the number of teenagers who have heard the rumour t hours after it first started.

(a) Find the number of teenagers who started the rumour. **(2)**

(b) Write down the number of teenagers who have heard the rumour 5 hours after it first started. **(1)**

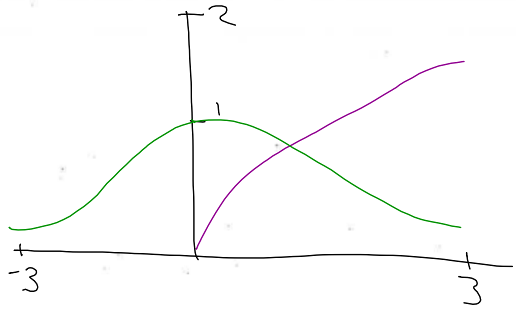
Two functions $f(x)$ and $g(x)$ are given by

$\boxed{C}$

$$f(x) = \frac{1}{x^2+1},$$
$$g(x) = \sqrt{x},\ x \geq 0.$$

$$\frac{1}{x^2+1} = \sqrt{x}$$

(a) Sketch the graphs of $f(x)$ and $g(x)$ together on the same diagram using values of x between -3 and 3, and values of y between 0 and 2. You must label each curve.



(b) State how many solutions exist for the equation $\frac{1}{x^2+1} - \sqrt{x} = 0$.

1

(c) Find a solution of the equation given in part (b).

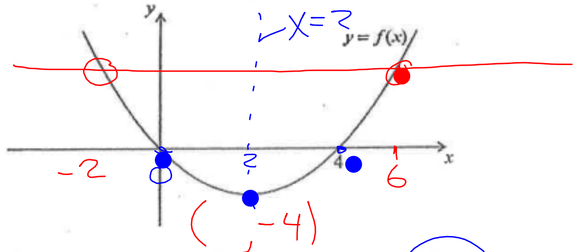
$$\frac{1}{x^2+1} = \frac{\sqrt{x}}{1}$$

$$1^2 = (\sqrt{x})^2 (x^2+1)^2$$

$$1 = x(x^2+1)^2$$

$\boxed{D}$

The following is the graph of the quadratic function $y = f(x)$.



(a) Write down the solutions to the equation $f(x) = 0$.

$x = 0 \quad x = 4$

(b) Write down the equation of the axis of symmetry of the graph of $f(x)$.

$x = 2$

$y = 12$

(c) The equation $f(x) = 12$ has two solutions. One of these solutions is $x = 6$. Use the symmetry of the graph to find the other solution.

$x = -2$

(d) The minimum value for y is -4. Write down the range of $f(x)$.

$-4 \leq y < \infty$ $\quad [-4, \infty]$

Goal today: Use Truth tables to verify logical statements being equivalent or not

including De Morgan's Law

Implication $p \to q$

Converse $q \to p$

Inverse $\neg p \to \neg q$

Contrapositive $\neg q \to \neg p$

Implication: If $2x=6$ then $x=3$

Converse: If $x=3$ then $2x=6$

Inverse If $2x \neq 6$, then $x \neq 3$

Contrapositive If $x \neq 3$, then $2x \neq 6$

Implication: If $2x=6$ then $x=3$

Converse: If $x=3$ then $2x=6$

Inverse If $2x\neq6$, then $x\neq3$

Contrapositive If $x\neq3$, then $2x\neq6$

on a quiz

T T ⊺ ⊺

T F

p	q	$\neg p$	$\neg q$	Implication $\overset{T}{p} \to \overset{F}{q}$	Converse $q \to p$	Inverse $\neg p \to \neg q$	Contrapositive $\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

p	q	$\neg p$	$\neg q$	Implication $p \to q$	Converse $q \to p$	Inverse $\neg p \to \neg q$	Contrapositive $\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Implication: If $2x=6$ then $x=3$

Converse: If $x=3$ then $2x=6$

Inverse If $2x\neq 6$, then $x\neq 3$

Contrapositive If $x\neq 3$, then $2x\neq 6$

We can use deMorgan's laws to help us to negate compound statements

Negate the following compound statement using precise language:

the class sings <u>and</u> Dalton cringes

hey... that statement is not true

The first deMorgan's property

$$\neg(p \wedge q) = \neg p \vee \neg q$$

a) **Negation of:** the class sings (p) and Dalton cringes (q)

is: class not sing OR Dalton doesn't cringe

b) **Negation of:** $10 \le n \le 20$

$n \le 20$ and $n \ge 10$

is:

$n > 20$ OR $n < 10$

The 2nd property

$$\neg(p \vee q) = \neg p \wedge \neg q$$

Negation of: Griffin jumps or Brenda sneezes

is: Griffin doesn't jump And Brenda does not sneeze

Will DeMorgan's Laws always work ?

We can prove that two logical statements are equivalent by showing their truth tables are equivalent

$$\neg(p \vee q) = \neg p \wedge \neg q$$

$$\neg(p \vee q) = \neg p \wedge \neg q$$

$(P \cap Q)' \equiv \quad P' \cup Q'$

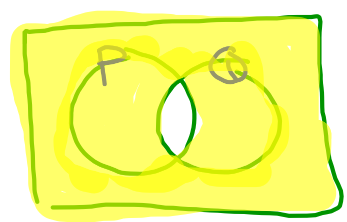
p	q	$p \vee q$	$\neg(p \vee q)$
T	T		
T	F		
F	T		
F	F		

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T			
T	F			
F	T			
F	F			

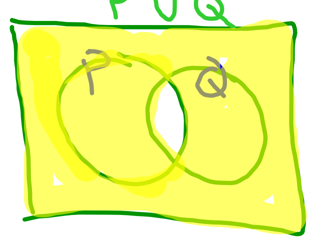
p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$(P \cap Q)'$



$P' \cup Q'$



Logic Assignment #3

- p.509..... 1ad, 3ae, 5b
- p.504...... 3c
- <u>and</u> construct your own truth table for:

$p \vee (\neg p \wedge q)$

pg 504 15C..... 3c

3 Use deMorgan's properties to find the negation of:

c $x < -1$ or $x > 7$

$\neg(p \wedge q) = \neg p \vee \neg q$

$\neg(p \vee q) = \neg p \wedge \neg q$

negation

$x \geq -1$ and $x \leq 7$

$p \vee (\neg p \wedge q)$

	$p \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q)$

p	q		$p \vee (\neg p \wedge q)$

$$p \vee (\neg p \wedge q)$$

p	q		$\neg p \wedge q$	$p \vee (\neg p \wedge q)$

$$p \vee (\neg p \wedge q)$$

p	q	$\neg p$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q)$ $\Longrightarrow$

p	q	$\neg p$	$\neg p \wedge q$	$p \vee (\neg p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F