1

Symbolic Logic

2

Basic Concepts of Symbolic Logic

The word "logic" derives from the Greek word ______ meaning "word". For a mathematician, logic deals with the conversion of word statements to symbolic form, and the use of that symbolic form to make deductions and create proofs.

Mathematical logic deals with basic statements called ____________.

A proposition can ______or______but might be indeterminate.

Questions, exclamations and orders are ________________

In the study of mathematical logic, we concentrate on propositions which have a well-defined __________, that is, they are true or false.

3

Which of the following are propositions? For each proposition identified, discuss whether it is true, false or indeterminate.

Justice has blue eyes.

Today it is snowing in Eugene.

All dogs have tails.

1 is a prime number.

Is this the last week of the trimester ?

For all $x \in \mathbb{R}$, $x^2 \geq 0$

4

To avoid writing all the words in a proposition all the time, we label propositions with letters. Usually we use ________________etc.

e.g. p: The wind is blowing. q: I will lose my hat.

Compound Statements

All relations between two or more propositions are called **compound statements** or **compound propositions.**

Symbols used to connect propositions are called ______________.

We will look at the symbols and meaning of Implication, Equivalence, Negation, Conjunction, Disjunction and Exclusive disjunction.

9 Given w : I am wearing shorts.
s : I am going to swim.
r : I am going to run.

Write in words these compound propositions:

a. $w \wedge s$

b. $w \Rightarrow r \vee s$

c. $s \underline{\vee} r$

d. $\neg s \Leftrightarrow r$

e. $w \wedge \neg (s \vee r)$

10

For example: $p \Rightarrow q \wedge q \Rightarrow p$ is the same as

$\neg (p \vee q)$ is the same as

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To understand why these propositions are equivalent, an example helps.

Take p : The sun is shining. q : It is raining.

Then $\neg p \wedge \neg q$ becomes "the sun is not shining <u>and</u> it is not raining" while $(p \vee q)$ becomes "the sun is shining or it is raining (or both)" and hence $\neg (p \vee q)$ is "neither is the sun shining nor is it raining", which is clearly the same as "the sun is not shining it is not raining".

Warning: The use of brackets is important when a compound proposition could be ambiguous without their use. For example if I write $\neg p \wedge q$, this is different from $\neg (p \wedge q)$.

It is like the difference between $-x + y$ and $-(x + y)$ in algebra. **If in doubt, use brackets!**

11

Police in a town are investigating the theft of mobile phones one evening from three cafés, "Alan's Diner", "Sarah's Snackbar" and "Pete's Eats".

They interviewed two suspects, Matthew and Anna about that evening.

Matthew said:

"I visited Pete's Eats and visited Alan's Diner and I did not visit Sarah's Snackbar"

Let p, q and r be the statements:

p : I visited Alan's Diner
q : I visited Sarah's Snackbar
r : I visited Pete's Eats

(a) Write down Matthew's statement in symbolic logic form. *[3 marks]*

What Anna said was lost by the police, but in symbolic form it was

$$(q \vee r) \Rightarrow \neg p$$

(b) Write down, in words, what Anna said. *[3 marks]*

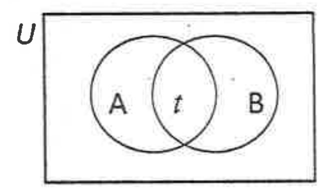
12

The connectives used in symbolic logic are closely related to ideas which you have met when working with sets.

1. If we have: p : Today is a cold day. q : Today is a wet day
then $p \wedge q$ means that today is cold _____ wet.

Suppose that A is the set of days which are cold, and B is the set of days which are wet.

Then $A \cap B$ stands for the set of days which are cold ***and*** wet.
So if 'today' is a member of the set $A \cap B$, i.e. $t \in A \cap B$, it is cold and wet today.



So $t \in A \cap B$ gives the same information as ______________

(1)

Symbolic Logic

(2)

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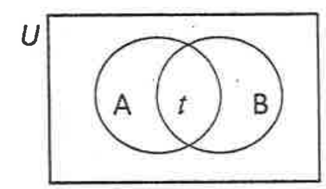

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