HW Questions →

Agenda

1. Discuss HW
2. Warm Up
3. Quiz information for tomorrow's Quiz
4. See your LCQ

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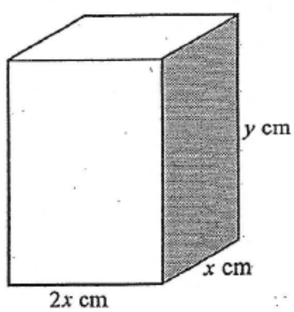
1 A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of 300 cm^2.



$SA = 2(\text{Top}) + 2(\text{Side}) + 2(\text{Front})$

$300 = 2(2x \cdot x) + 2(x y) + 2(2x \cdot y)$

$300 = 4x^2 + 2xy + 4xy$

$300 = 4x^2 + 6xy$

(a) Show that $4x^2 + 6xy = 300$. **(2)**

(b) Find an expression for y in terms of x. **(2)**

$4x^2 + 6xy = 300 \Rightarrow 6xy = 300 - 4x^2$

$y = \dfrac{300 - 4x^2}{6x}$

(c) Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. **(2)**

$\text{Volume} = (2x)(x)y = 2x^2\left(\dfrac{300 - 4x^2}{6x}\right) = \dfrac{x(300 - 4x^2)}{3} = 100x - \dfrac{4}{3}x^3$

(d) Find $\frac{dV}{dx}$ $= 100 - \left(\frac{4}{3}\right)(3x^2) = 100 - 4x^2$ **(2)**

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(max, where tangent is flat, $f'(x) = 0$)

(ii) Calculate the maximum volume. **(5)**

(Total 13 marks)

$100 - 4x^2 = 0$

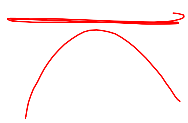
$4x^2 = 100$

$x^2 = 25$

$x = \pm 5$ (ignore −)

optimum size = 5 cm

$f(5) = 100(5) - \frac{4}{3}(5)^3 = \frac{200}{3}$ cm³ or 66.67 cm³ ← maximum volume



2

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x} = x^1 + 100x^{-1}$$

(a) Find $C'(x)$. $C'(x) = 1 + 100(-1)x^{-2} = 1 - \frac{100}{x^2}$ **(3)**

(b) Show that the cost per person is a minimum when 10 people are invited to the party. **(2)**

(c) Calculate the minimum cost per person. **(2)**

(Total 7 marks)

2

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x} = x + 100x^{-1}$$

(a) Find $C'(x)$. (3)

(b) Show that the cost per person is a minimum when 10 people are invited to the party. (2)

(c) Calculate the minimum cost per person. (2)

(Total 7 marks)

(a) $C'(x) = 1 - 100x^{-2} = 1 - \frac{100}{x^2}$

(b) To find minimum cost (where tangent is flat)

set $C'(x) = 0$ and solve

$1 - \frac{100}{x^2} = 0$ multiply by x^2

$(x^2)1 - \frac{100(x^2)}{x^2} = 0 \cdot x^2$

$x^2 - 100 = 0$

$x^2 = 100$

$x = \pm 10$

10

2d

$f(x) = \frac{7x^3 + 2x^4}{x^2} \rightarrow \frac{7x^3}{x^2} + \frac{2x^4}{x^2}$

$\downarrow$

$7x + 2x^2$

$f'(x) = 7 + 4x$

(b) To find minimum cost (where tangent is flat)

set $C'(x) = 0$ and solve

$1 - \frac{100}{x^2} = 0$ multiply by x^2

$x^2 - 100 = 0$

$x^2 = 100$

$x = \pm 10$ ignore negative

so, 10 people would give the minimum cost

(c) Minimum Cost $= C(10) = x + \frac{100}{x}$

$= 10 + \frac{100}{10} = 20$

So the minimum cost per person is \$20 (which occurs when 10 people are invited)

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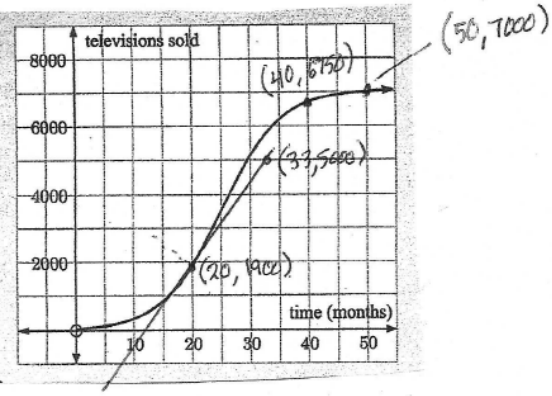
p582 Review Set 20A

① The total number of televisions sold over many months is shown on the graph alongside.

Estimate the rate of sales:

- **a** from 40 to 50 months
- **b** from 0 to 50 months
- **c** at 20 months.



a) 40 to 50 $\frac{\Delta y}{\Delta x} = \frac{7000 - 6750}{50 - 40}$

$\doteq 25$ televisions/month

b) 0 to 50 $\frac{\Delta y}{\Delta x} = \frac{7000 - 0}{50 - 0}$

$= 140$ TV's/month

c) $\frac{\Delta y}{\Delta x} \doteq \frac{5000 - 1900}{33 - 20} \doteq 239$ TV's/mo.

(2) a) $7x^3$

$f'(x) = 21x^2$

b) $x^2 - x^3$

$f'(x) = 2x - 3x^2$

c) $(2x-3)^2 = (2x-3)(2x-3)$

$= 4x^2 - 12x + 9$

$f'(x) = 8x - 12$

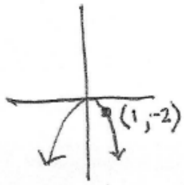
(3) $f(x) = x^4 - 3x - 1$

(a) $f'(x) = 4x^3 - 3$

(b) $f'(2) = 4(2)^3 - 3 = 29$

(c) $f'(0) = 4(0)^3 - 3 = -3$

(4) $y = -2x^2$ at $x = 1$



$f'(x) = -4x$

$f'(1) = -4(1)$

$= -4$ ← gradient

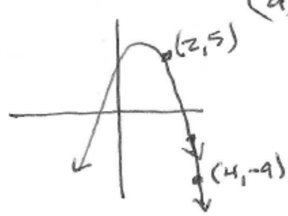
equation: $y - -2 = -4(x-1)$

or

$y = -4x + 2$

Review Set A

(5) $f(x) = -2x^2 + 5x + 3$



(a) Avg rate of change

$\frac{\Delta y}{\Delta x} = \frac{5 - -9}{2 - 4} = \frac{14}{-2} = -7$

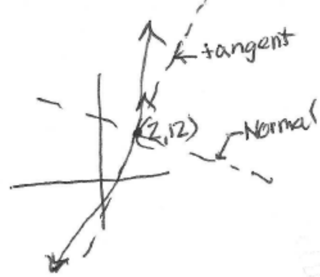
(b) instantaneous rate at $x = 2$

$f'(x) = -4x + 5$

$f'(2) = -4(2) + 5 = -3$

(6) $y = x^3 + 3x - 2$ at $x = 2$

Find NORMAL



$f'(x) = 3x^2 + 3$

$f'(2) = 3(2)^2 + 3 = 15$ ← gradient of tangent

gradient of normal $= -\frac{1}{15}$

Equation of Normal using (2,12) and $m = -\frac{1}{15}$

$y - 12 = -\frac{1}{15}(x - 2)$

$y - 12 = -\frac{1}{15}x + \frac{2}{15}$

$\boxed{y = -\frac{1}{15}x + \frac{182}{15}}$

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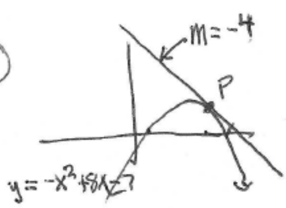
(7)



Find where tangent has gradient $= -4$

$f'(x) = -2x + 8$

$-4 = -2x + 8$

$2x = 4$ (in red: 12)

~~$x = 2$~~ 6

(2, 5) ← f(6)

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(8) $y = ax^3 - 3x + 3$

when $x = 2$

$f'(x) = 21$

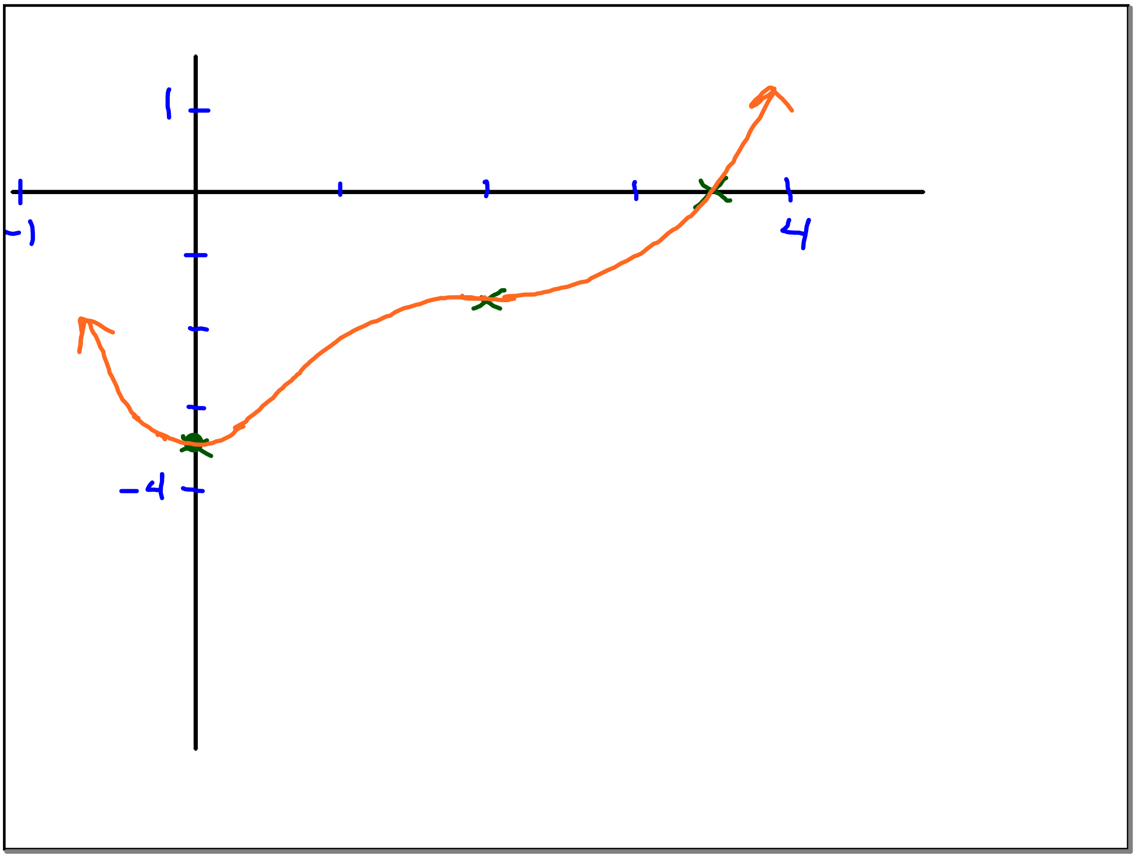
$f'(x) = 3ax^2 - 3$

$21 = 3ax^2 - 3$

$21 = 3a(2)^2 - 3$

$24 = 12a$

$a = 2$



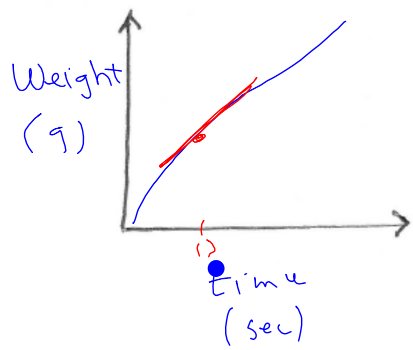
- WARM UP

Sand is being poured into a bucket for 30 seconds. After t seconds the weight of the sand is

$$S(t) = 0.3t^3 - 18t^2 + 550t \text{ grams}$$

a) Make a sketch of the graph and label it.

(Adjust your window to match the situation)



$S(12) = 4526.4$

4530 g

b) Find S(12) and interpret its meaning

c) Find S'(12) and interpret its meaning.

can use GDC directly

$$S'(t) = 0.3(3)t^2 - 36t + 550$$

$.9t^2$

$$S'(12) = 247.6$$

at 12 seconds the sand is increasing at rate of 248 g/sec

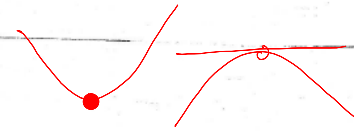
(2)

Consider the quadratic function $f(x) = 2x^2 - 5x + 1$

a Use quadratic theory to find the equation of the axis of symmetry → $x = \frac{-b}{2a}$

b Find $f'(x)$ and hence find x when $f'(x) = 0$. Explain!

↖ This formula is in your IB Formula packet in the Algebra section.



a) $f(x) = 2x^2 - 5x + 1$

$a = 2$

$b = -5$

~~$c = 1$~~

axis of symmetry is:

$x = -\frac{b}{2a}$

$x = \frac{-(-5)}{2(2)}$

$= 5/4$

$x = 1.25$

b) $f'(x) = 4x - 5$

$0 = 4x - 5$

$4x = 5$

$x = 1.25$ is the location where the tangent is flat. In other words $x = 1.25$ is where there must be a min or max of the quadratic, which is the x-coordinate of the vertex, or middle of the graph.

Cool !!!

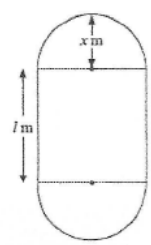
Below is an optimization problem. There won't be any questions on tomorrow's quiz as challenging as this. However tough questions show up on IB exams on occasion usually no more than one per year. If you think you might strive for a 7 on the exam, then I suggest you work on this problem later in class.

An athletics track consists of two 'straights' of length l m and two semicircular ends of radius x m. The perimeter of the track is to be 400 m.

a Show that $l = 200 - \pi x$, and hence write down the possible values that x may have.

b Show that the area of the rectangle inside the track is given by $A = 400x - 2\pi x^2$.

c What values of l and x produce the largest area of the rectangle inside the track?



You will not be turning in your HW tomorrow.

FYI

The next unit on Sequences will be added to the current HW sheet.

The Quiz on Introductory Calculus will be tomorrow.

List of Quiz Items

NOTATION

$f(x)$

$f'(x)$

✓ Calculator skills: On typical or non-typical functions.... use GDC to:

- Caclulate the gradient at a given location
- Calculate the equation of a tangent line at a given location

$f(x) = -x^2 + 2^x - \sqrt{x}$

- Tonight's Assignment is to do Review Set B

1 - 6

7, 8 ← nice challenge questions for those going for a 7

Your Probability Test is available if you want additional time to look at it. ☺

REVIEW SET 20B

1 **a** **i** 5 **ii** $4\frac{1}{2}$ **iii** 4.1

b $f'(x) = 2x + 2$ **c** gradient $= 4$, as $x \to 1$, $f'(x) \to 4$

2 **a** $\frac{dy}{dx} = 6x - 4x^3$ **b** $\frac{dy}{dx} = 1 + x^{-2}$

c $\frac{dy}{dx} = 2 - x^{-2} + 6x^{-3}$

3 $y = 9x - 11$ **4** $\left(-\frac{1}{\sqrt{2}},\ -2\sqrt{2}\right)$ and $\left(\frac{1}{\sqrt{2}},\ 2\sqrt{2}\right)$

5 **a** -17 **b** -17 **6** $(10.1,\ -13.0)$ **7** $a = 2,\ b = 3$

8 **a** P(2, 5) **b** $y = x + 3$ **c** $(-3,\ 0)$ **d** $y = -x + 7$

3 $y = 9x - 11$ **4** $\left(-\frac{1}{\sqrt{2}},\ -2\sqrt{2}\right)$ and $\left(\frac{1}{\sqrt{2}},\ 2\sqrt{2}\right)$

5 **a** -17 **b** -17 **6** $(10.1,\ -13.0)$ **7** $a = 2,\ b = 3$

8 **a** P(2, 5) **b** $y = x + 3$ **c** $(-3,\ 0)$ **d** $y = -x + 7$

REVIEW SET 20C

1 **a** $f'(x) = 4x^3 + 6x^2 + 6x$ **b** $f'(x) = -6x^{-4} - 4x^{-5}$

c $f'(x) = -x^{-2} + 8x^{-3}$

2 **a** -5 **b** -12 **c** $\frac{7}{9}$ **d** -1 **3** $y = -24x + 36$

4 $S'(t) = 0.9t^2 - 36t + 550$ g sec^{-1}

This gives the instantaneous rate of change in weight, in grams per second, for a given value of t.

4 $S'(t) = 0.9t^2 - 36t + 550$ g sec^{-1}

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5 $y = -\frac{1}{2}x + \frac{13}{2}$ **6** $a = 3,\ b = 7$

7 $(-1.32,\ -0.737)$ and $(1.32,\ -1.26)$

8 **a** $f'(x) = 3x^2 - 8x + 4$

b $f'(1) = -1$. This is the gradient of the tangent to the curve at the point $x = 1$.

c **i** 0 **ii** $y = 1$