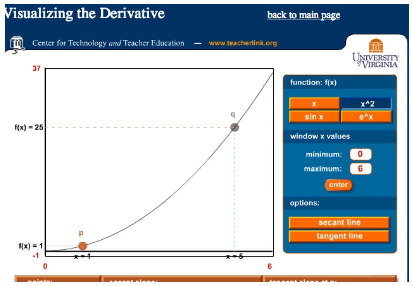
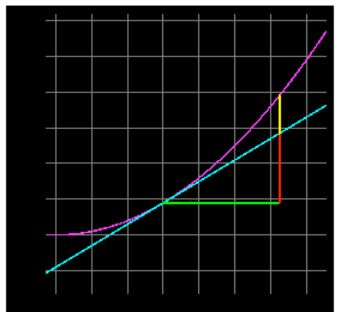
http://www.teacherlink.org/content/math/interactive/flash/derivativetool/derivative.html



http://www.ima.umn.edu/~arnold/calculus/differential/differential-g.html



- Pick up the new HW recording sheet for this week.

You'll need some type of a ruler or straight edge

(a student body card or ATM type card will do)

Calculus

Sequences & Series & Financial Math

Logic

Today we will start a short,6 day,

unit on **Calculus.**

How Steep ?

How Big ?

How Fast ?

All Assignments will be from the Differential Calculus packet

There will be a Quiz on this unit on Friday, Nov. 3

and and one or two LCQ's

to check on your learning.

from now on, the words "slope" and "gradient" are interchangeable.

Start by going over the

Calculus Precursor assignment

(After test worksheet)

From now on, the word "**slope**" and "***gradient***" **mean the same thing** $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

1. Find the equation of the straight line joining each of the following points. Use Point-Slope form (we'll need it for calculus) $y - y_1 = m(x - x_1)$ hint: first find m

   (a) (–2, –4) and (1, –7)

   (b) Then convert to *gradient-intercept* form ($y = mx + b$) a.k.a. slope-intercept form

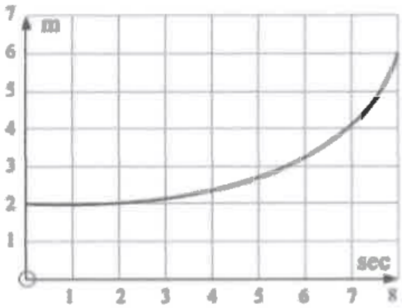
2. Find the equation of the straight lines below, given its gradient and the coordinates of a point on the straight line. *Point-slope form*

$-\frac{1}{2}$, (5, 7)

3. New tires have a tread depth of 8 mm. After driving for 32,178 km the tread depth was reduced to 2.3 mm. What was the wearing rate of the tires in km travelled per mm of depth.

(The value you calculated can also be called the average wear rate)

5. Before answering this question, first go to question #4 on the back side. Then come back. Estimate the **average speed in graph between 2 and 7 seconds** and *average rate of beetle decrease from dose 4 to 14*



Consider a trip from Adelaide to Melbourne. The following table gives places along the way, distances travelled and time taken.

We plot the *distance travelled* against the *time taken* to obtain a graph of the situation. Even though there would be variable speed between each place we will join points with straight line segments.

Place	taken (min)	travelled (km)
Adelaide tollgate	0	0
Tailem Bend	63	98
Bordertown	157	237
Nhill	204	324
Horsham	261	431
Ararat	317	527
Midland H/W Junction	386	616
Melbourne	534	729

We can find the average speed between any two places.

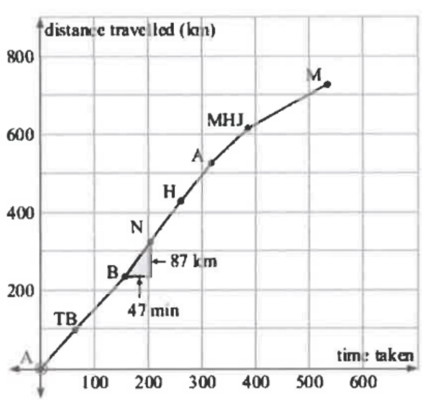
For example, the average speed from Bordertown to Nhill is:

$$\frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{324 - 237 \text{ km}}{204 - 157 \text{ min}}$$

$$= \frac{87 \text{ km}}{\frac{47}{60} \text{ h}}$$

$$\doteqdot 111 \text{ km/h}$$

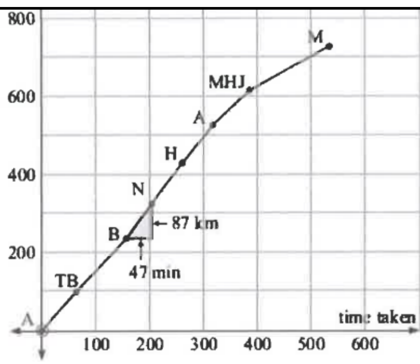


We notice that the average speed is the $\dfrac{y\text{-step}}{x\text{-step}}$ on the graph.

So, the average speed is the **gradient of the line segment** joining the two points which means that the faster the trip between two places, the greater the gradient of the graph.

If $s(t)$ is the distance travelled function then the average speed over the time interval from $t = t_1$ to $t = t_2$ is given by:

$$\textbf{Average speed} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}.$$



Place	taken (min)	travelled (km)
Adelaide tollgate	0	0
Tailem Bend	63	98
Bordertown	157	237
Nhill	t_1 204	324 $s(t_1)$
Horsham	261	431
Ararat	317	527
land H/W Junction	386	616
Melbourne	t_2 534	729 $s(t_2)$

(7) Calculate the average speed between *Nhill* and *Melbourne. Then go back and answer question #4.*

Pick up:

Calculus 1.0 Notes

start with #2

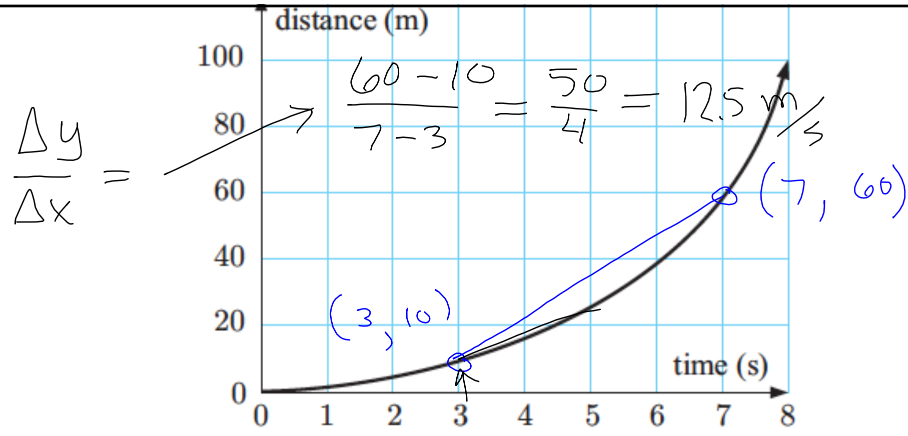
2.

The Graph below shows how a cyclist accelerates away from an intersection.

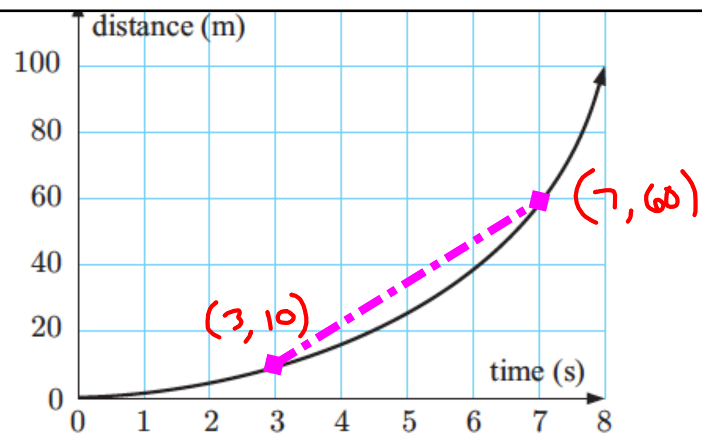
The average speed over the first 8 seconds is

$$\frac{100 \text{ m}}{8 \text{ sec}} = 12.5 \text{ m s}^{-1}. \qquad \frac{m}{s'}$$

Notice that the cyclist's early speed is quite small, but it increases as time goes by.



Estimate the average speed of the cyclist between 3 and 7 seconds

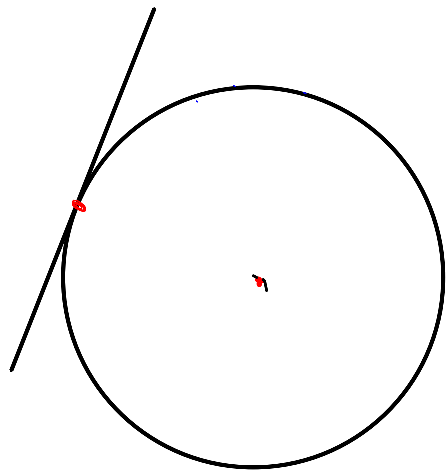


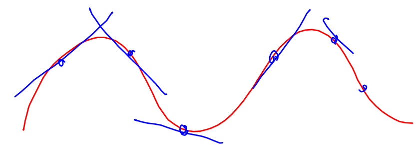
Estimate the average speed of the cyclist between 3 and 7 seconds

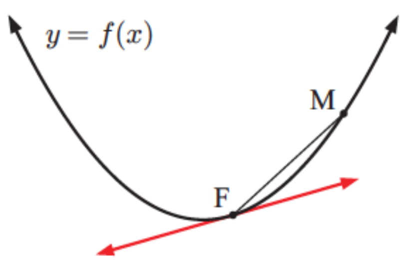
Avg speed $= \frac{60-10}{7-3} = \frac{50}{4} = 12.5$ m/sec

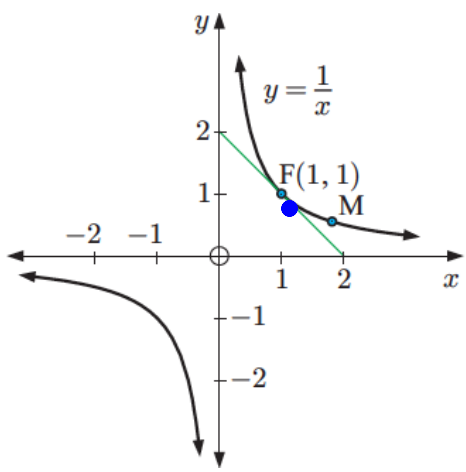
UNIT

From Geometry : A tangent is a line (or segment) that touches a circle at exactly one point.

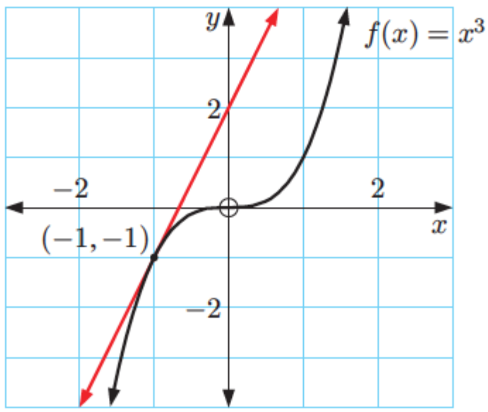


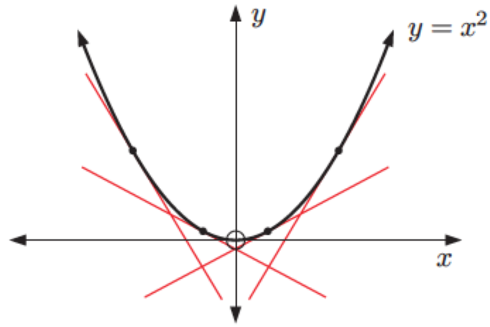




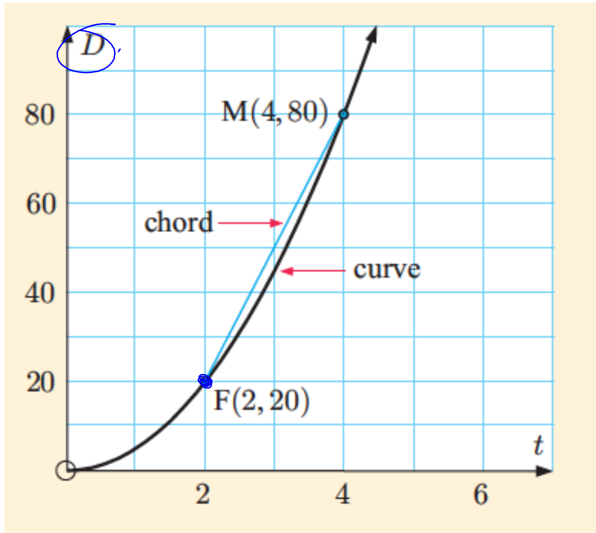


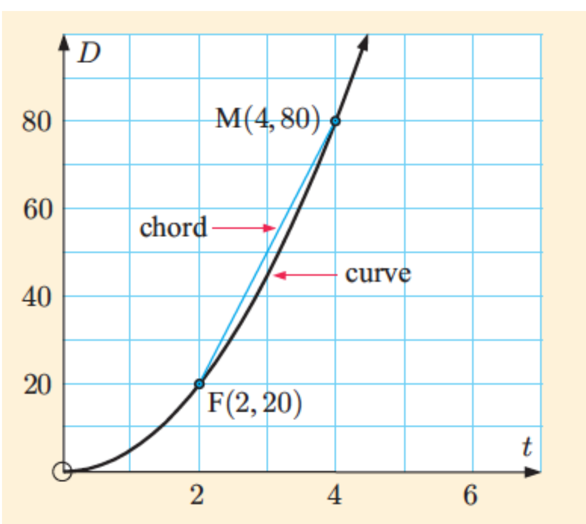
$\frac{1}{x}$


y
f(x) = x^3
2
−2
2
x
(−1, −1)
−2


y
y = x^2
x

Instantaneous Rates of Change





The average speed in the interval from 2 to 4 seconds is shown

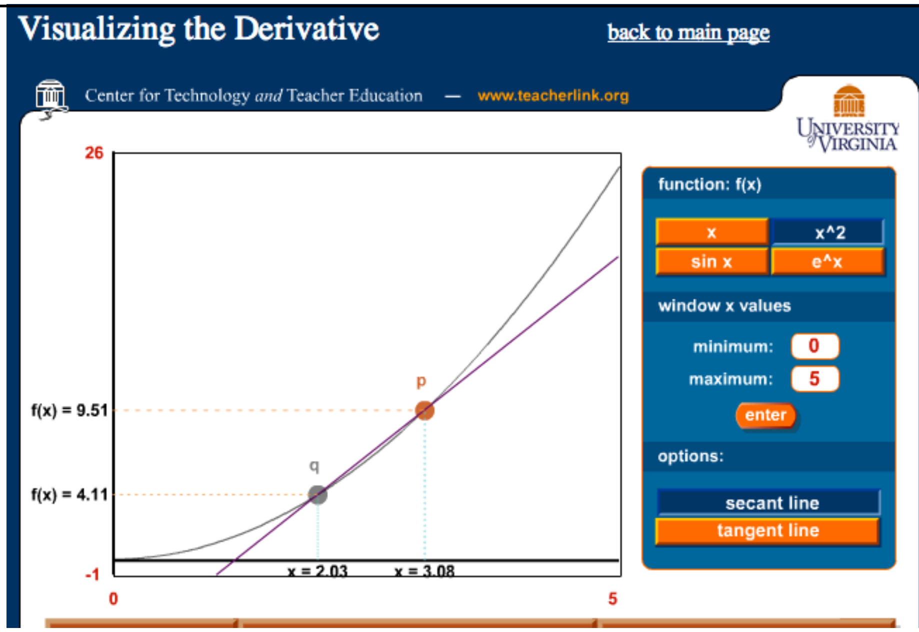
$$= \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{(80 - 20)\text{ m}}{(4 - 2)\text{ s}}$$

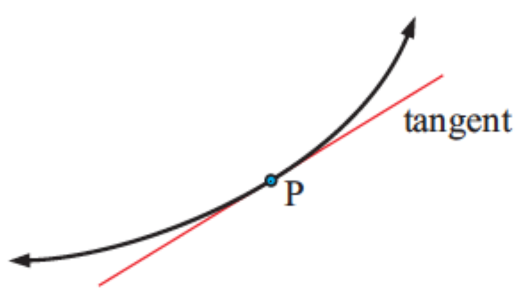
$$= \frac{60}{2}\text{ m s}^{-1}$$

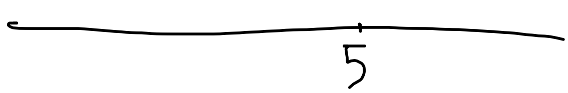
$$= 30\text{ m s}^{-1}$$

But that does not tell us the instantaneous speed at any particular time



The <u>instantaneous</u> rate of change of a variable at a particular instant is given by the gradient of the tangent line to the graph <u>at</u> that point.





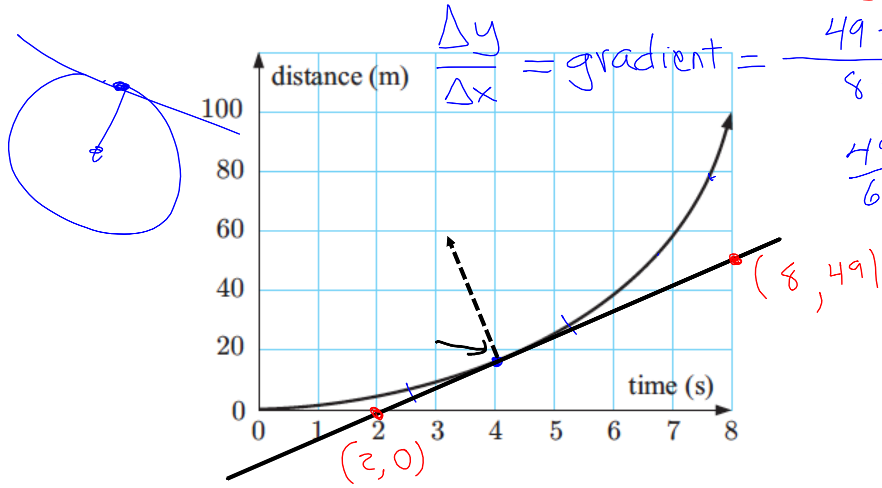
Now back to the

Cyclist problem

Find the instaneous speed at $t = 4$ seconds

$\frac{\Delta y}{\Delta x}$ = gradient = $\frac{49 - 0}{8 - 2}$

$\frac{49}{6} = 8.17$ m/s



C

The tangent passes through (2, 0) and (7, 40).

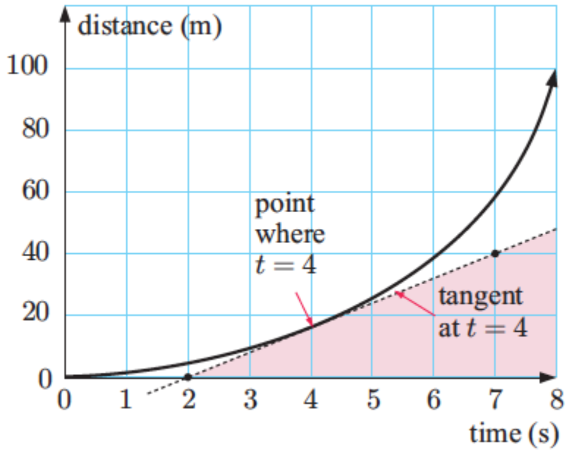
$\therefore$ the instantaneous speed at $t = 4$

$= $ the gradient of the tangent

$$= \frac{(40-0)\text{ m}}{(7-2)\text{ s}}$$

$$= \frac{40}{5}\text{ m s}^{-1}$$

$$= 8\text{ m s}^{-1}$$

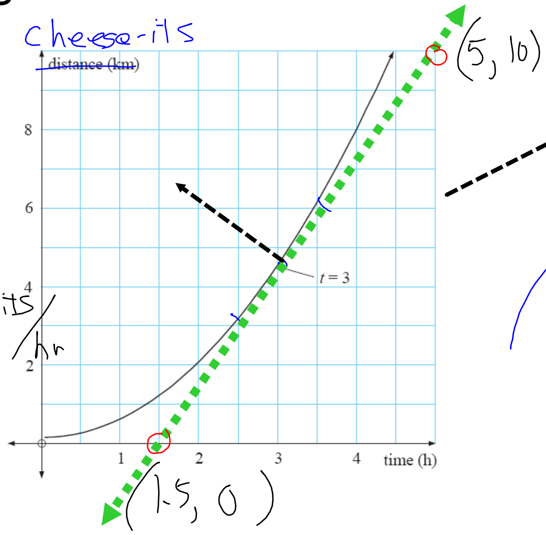


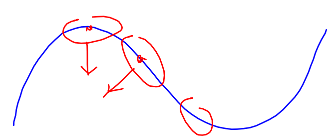
D

Find the instantaneous rate of change at t = 3 seconds

$$\frac{\Delta y}{\Delta x} = \frac{10-6}{5-1.5}$$

$= 2.86$ cheese-its / hr





Finding the Gradient Algebraically

Investigation #3

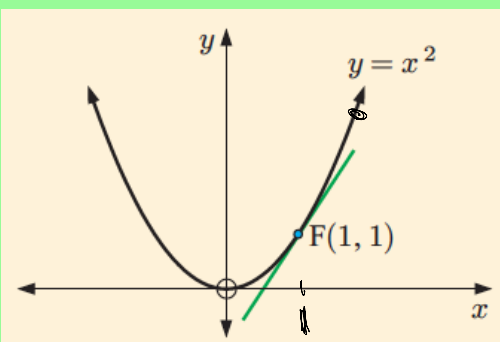
✓

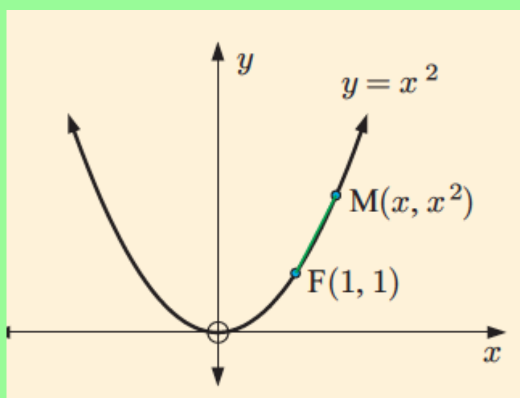
✓

E

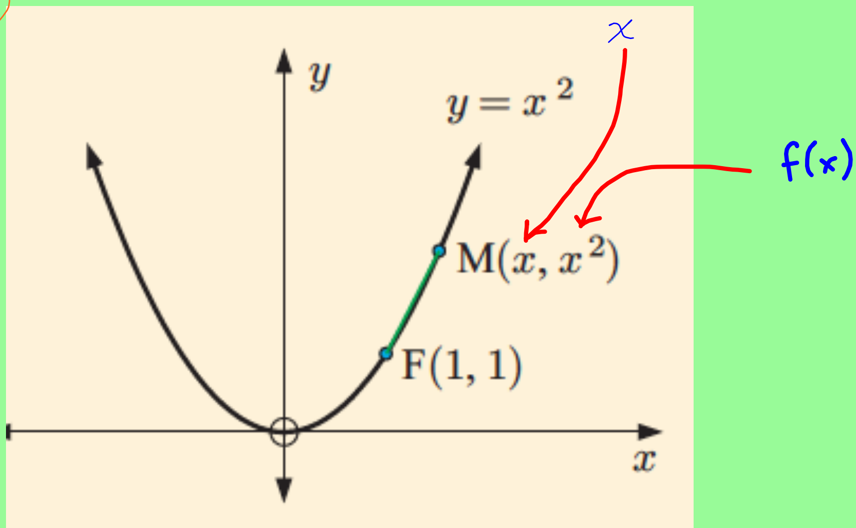
E

Given the curve $y = x^2$ *we will find the gradient of the tangent at the point* **F (1, 1)**





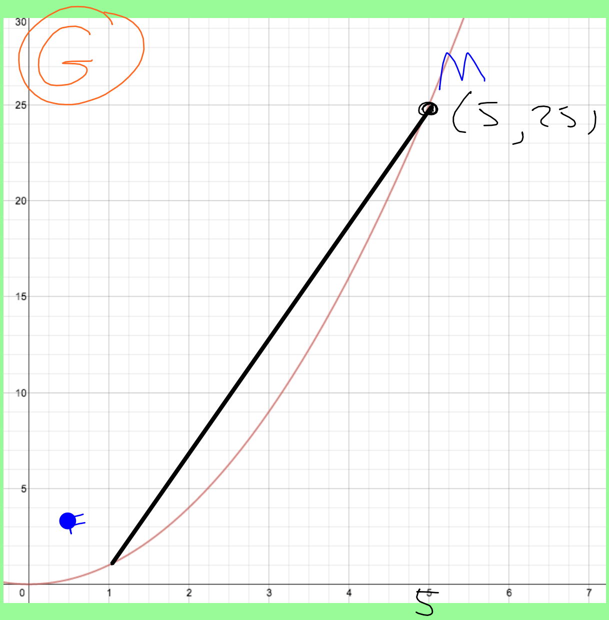
E



F

$f(x) = x^2$

x	Point M	gradient of FM
5	(5, 25)	$\frac{25-1}{5-1} = 6$
3		4 5
2		
1.5		
1.1		
1.01		
1.001		

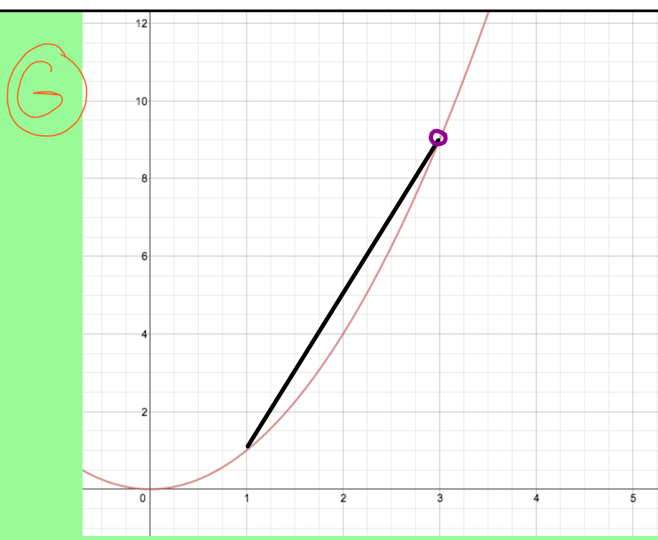


(5,25)

gradient

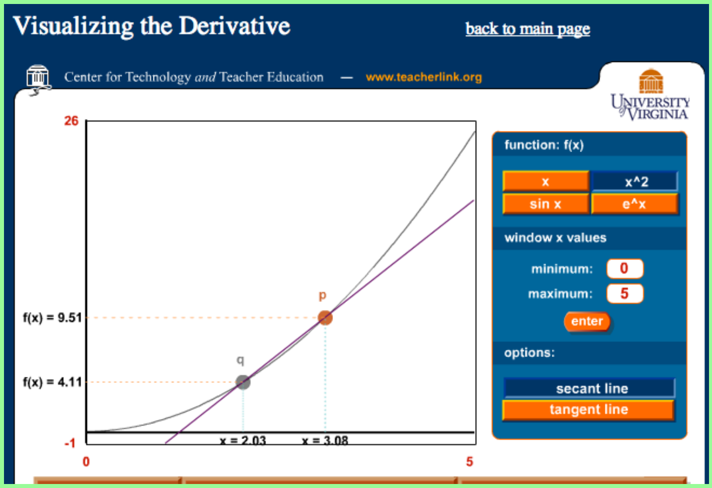
$$= \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

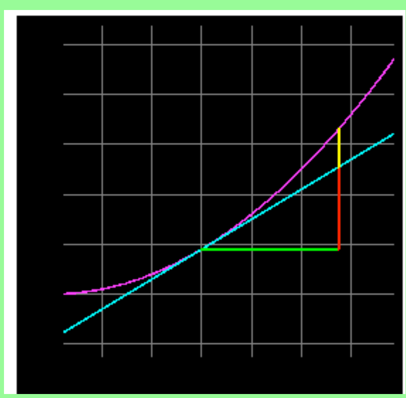
=



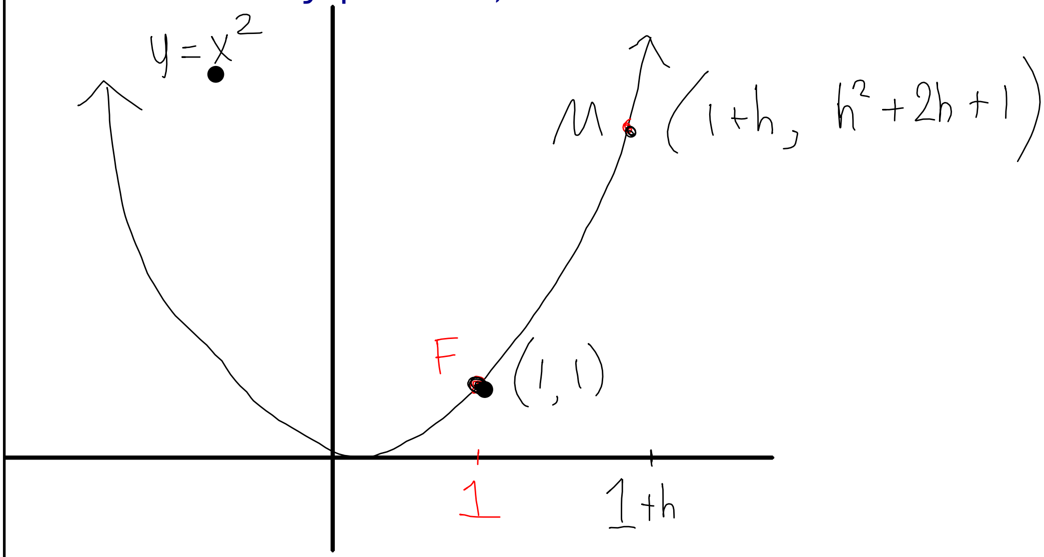
(3,)

for the rest


Visualizing the Derivative
back to main page
Center for Technology and Teacher Education — www.teacherlink.org
University of Virginia
26
f(x) = 9.51
f(x) = 4.11
-1
0
5
p
q
x = 2.03
x = 3.08
function: f(x)
x
x^2
sin x
e^x
window x values
minimum:
0
maximum:
5
enter
options:
secant line
tangent line



Add an arbitrary point **M**, h seconds later



$$y = (1+h)^2$$

$$y = (1+h)(1+h)$$

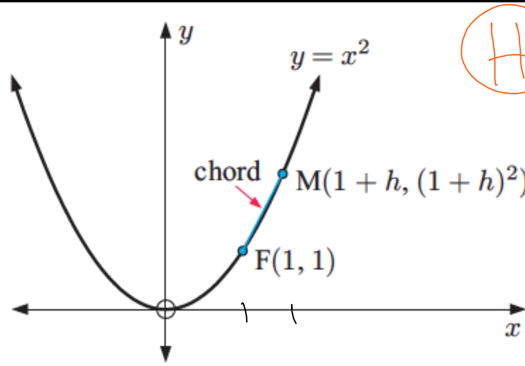
$$y = 1 + 2h + h^2$$

$$y = h^2 + 2h + 1$$

~~$1 + h^2$~~

H

the algebraic method:



$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{y - y}{x - x}$$

$$= \frac{(1+h)^2 - 1}{(1+h) - 1}$$

$$= \frac{h^2 + 2h + 1 - 1}{1 + h - 1} = \frac{h^2 + 2h}{h} = \frac{h(h+2)}{h} = h + 2$$

As $h \to 0$ gradient $= 2 \frac{m}{s}$

I

As **M** *approaches* **F**, h *approaches 0*

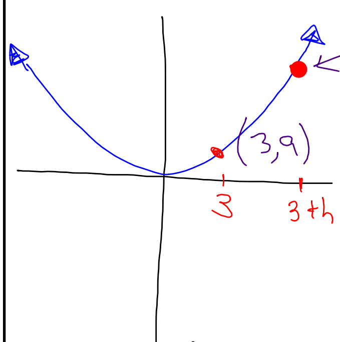
so 2 + h *approaches* **2**

so the gradient of the tangent at (1,1) is 2.

examples for your notes

example

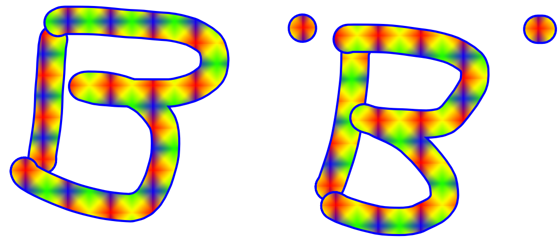
Use the algebraic method to find the gradient of the tangent to **y = x²** at the point where **x = 3**



$(3+h,\ h^2+6h+9)$

$$\frac{\Delta y}{\Delta x} = \frac{h^2+6h+9 - 9}{3+h - 3} = \frac{h^2+6h}{h} = \frac{h(h+6)}{h} = h+6$$

As $h \to 0$ $\frac{\Delta y}{\Delta x} \to 6$



example 2

Use the algebraic method to find the gradient of the tangent to
$\mathbf{y = x^2 + 3x}$ at the point where $\mathbf{x = 2}$

How to Draw CHET GECKO

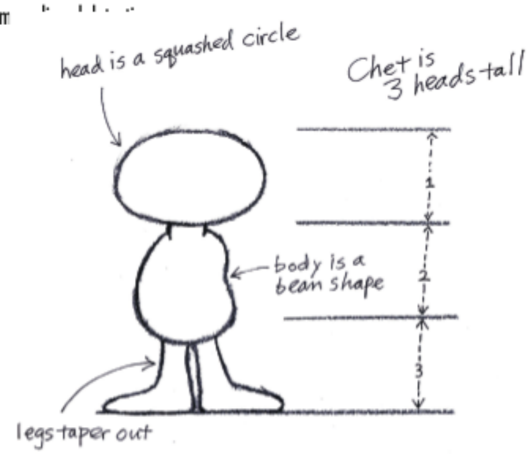
Feeling artistic? Here's how you draw Emerson Hicky Elementary's fam

Get a pencil, paper, and an eraser (yes, even genius artists use erasers). Put on your fancy artist cap (or whatever cap you can find). Then follow these simple instructions.

Have fun!

STEP 1

Start with the basic shapes of a circle head and a bean body.

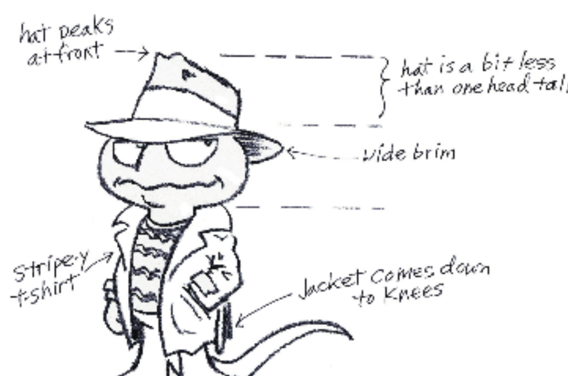


STEP 2

During this step, you erase some of the bean shape—at the neck, arms, and legs. (I told you that eraser would come in handy!)

STEP 3

As you add Chet's clothes, you'll erase even more of the original shapes. But they're still the basis for your drawing. Notice how the brim of the hat comes right down over Chet's eyes. The brim looks almost like eyebrows. Also notice how his front hand rests in a pocket. This is a great trick when you're still learning how to draw hands—hide them!



Why do gorillas have such big nostrils?

Because they have such big fingers.

Why don't seagulls fly over the bay?

Because then they'd be bagels.

What's made of plastic and hangs around French cathedrals?

The lunchpack of Notre Dame.

J

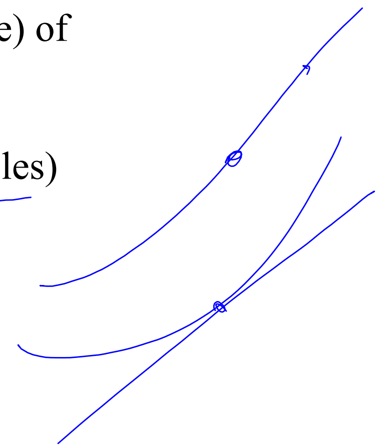
This **instantaneous rate of change** *at specific point on a curve* can be calculated

a) **visually**, by estimating the gradient (slope) of the line that is tangent *at that point*

b) Using the algebraic method (First principles)

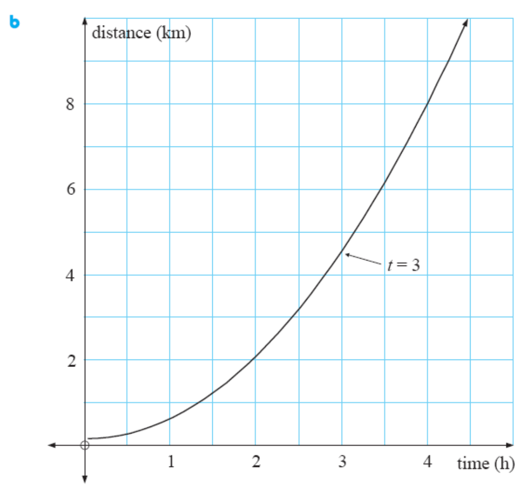
c) Finding the derivative (tomorrow)

d) with your GDC



K

Find the instaneous rate of change at t=3 hours



$f(x) = 0.5(2)^x$

✓ Graph (zoom 6)

✓ 2nd Calc

✓ $\frac{dy}{dx}$

t=3 seconds

at t=1 ?

Assignment:

Calculus packet: p. 565..... 1, 2
p. 568..... 1abe, 3

Next test: Friday, November 3rd

Save Mr. C !!!