Solutions to Paper 1 MAY 2015

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Tuesday 12 May 2015 (morning)

1 hour 30 minutes

Candidate session number

0	0	0	4	4	0	0	0	2	5

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **Mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Given to Students 11-20-15

2215 – 7403

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. $T = \dfrac{(\tan(2z)+1)(2\cos(z)-1)}{y^2 - x^2}$, where $x = 9$, $y = 41$ and $z = 30^\circ$.

(a) Calculate the **exact** value of T. [2]

(b) Give your answer to T correct to

(i) two significant figures;

(ii) three decimal places. [2]

Pyotr estimates the value of T to be 0.002.

(c) Calculate the percentage error in Pyotr's estimate. [2]

Working:

$T = \dfrac{[(\tan(2\cdot 30)+1)(2\cos(30)-1)]}{41^2 - 9^2} = .00125$

(c) Approx = .002
Exact = .00125

$\%\ \text{error} = \left|\dfrac{.002 - .00125}{.00125}\right| \times 100$

$=$

Answers:

(a) 0.00125 M1 A1

(b) (i) .0013 A1 (ft)

(ii) .001 A1 (ft)

(c) 60% M1 A1 (ft)

2. The IB grades attained by a group of students are listed as follows.

6 4 5 3 7 3 5 4 2 5

(a) Find the median grade. [2]

(b) Calculate the interquartile range. [2]

(c) Find the probability that a student chosen at random from the group scored at least a grade 4. [2]

Working:

(b) IQR = $5 - 3 = 2$

Q_3 Q_1

ON (a) and (b) just use your GDC to quickly get the 5 number summary.

(c) $\frac{7}{10}$

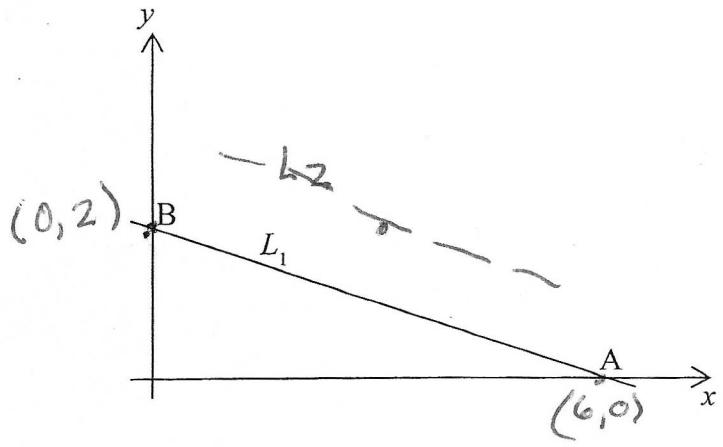
Answers:

(a) 4.5 M1A1

(b) 2 M1A1

(c) $\frac{7}{10}$ A2

3. The diagram shows the straight line L_1, which intersects the x-axis at $A(6, 0)$ and the y-axis at $B(0, 2)$.



(a) Write down the coordinates of M, the midpoint of line segment AB. [2]

(b) Calculate the gradient of L_1. [2]

The line L_2 is parallel to L_1 and passes through the point $(3, 2)$.

(c) Find the equation of L_2. Give your answer in the form $y = mx + c$. [2]

Working:

(a) midpoint $\left(\frac{0+6}{2}, \frac{2+0}{2}\right) \Rightarrow (3, 1)$

(b) gradient (slope) $= \frac{2-0}{0-6} = \frac{2}{-6} = -\frac{1}{3}$

(c) Use gradient $-\frac{1}{3}$ and point $(3, 2)$

I'll start with Point-slope, then convert to $y = mx + c$

$y - 2 = -\frac{1}{3}(x - 3)$

$y - 2 = -\frac{1}{3}x + 1$

$y = -\frac{1}{3}x + 3$

Answers:

(a) $(3, 1)$ A1A1

(b) $-\frac{1}{3}$ M1A1

(c) $y = -\frac{1}{3}x + 3$ ..... M1A1 (ft)

4. Identical mosquito traps are placed at different distances from a lake. On one day the number of mosquitoes caught in 10 of the traps is recorded.

Distance, m (x)	8	15	22	30	34	45	50	60	74	82
Number of mosquitoes (y)	78	75	72	67	66	59	59	53	48	43

It is believed the number of mosquitoes caught varies linearly with the distance, in metres, of the trap from the lake.

(a) Find

(i) Pearson's product–moment correlation coefficient, r;

(ii) the equation of the regression line y on x. [4]

enter data, Get Answers directly from GDC

(b) Use the equation of the regression line y on x to estimate the number of mosquitoes caught in a trap that is 28 m from the lake. [2]

Working:

(b) $y = -.470(28) + 81.7$

$= 68.6$ mosquitos

Answers:

(a) (i) $r = -0.998$ A2

(ii) $y = -.470x + 81.7$ A1 A1

(b) 68.6 (mosquitos) ..... M1 A1 (ft)

If I write the unit in parentheses, than the question was not penalized on the exam

5. Assume the Earth is a perfect sphere with radius $6371\,\text{km}$.

(a) Calculate the volume of the Earth in km^3. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [3]

The volume of the Moon is $2.1958 \times 10^{10}\,\text{km}^3$.

(b) Calculate how many times greater in volume the Earth is compared to the Moon. Give your answer correct to the nearest **integer**. [3]

Working:

a) $V_{\text{earth}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6371)^3 = 1.0833206\ldots \times 10^{12}$

b)

$$\frac{V_{\text{earth}}}{V_{\text{moon}}} = \frac{1.0833206\ldots \times 10^{12}}{2.1958 \times 10^{10}} = 49.330855\ldots$$

A1 A1 (ft)

M1 A2

M1 A1(ft) A1

Answers:

(a) 1.08×10^{12} (km^3)

(b) 49

6. Pietro arrives in Singapore and, at the airport, changes 800 euros (EUR) to Singapore dollars (SGD).

The bank rates quoted at the airport for exchanging EUR with SGD are given in the following table. Also given are the rates for exchanging SGD with British pounds (GBP) and US dollars (USD). There is no commission charged on exchanges.

Bank Buys	Bank Sells
1 EUR = 1.55 SGD	1 EUR = 1.75 SGD
1 GBP = 1.92 SGD	1 GBP = 2.05 SGD
1 USD = 1.15 SGD	1 USD = 1.28 SGD

(a) Calculate the number of SGD Pietro receives. [2]

Pietro also has 100 GBP that he wishes to change to USD for a trip to Cambodia.
To perform this transaction, the GBP must first be converted to SGD and then to USD.

(b) Calculate the number of USD Pietro receives. [4]

Working: The bank buys Pietro's euros

(a) $800 \text{ euros} \times \frac{1.55 \text{ SGD}}{1 \text{ EUR}} = 1240 \text{ SGD}$

The bank buys Pietro's euros

(b) $100 \text{ euro} \times \frac{1.92 \text{ SGD}}{1 \text{ euro}} = 192 \text{ SGD}$

Then bank sells his SGD

$192 \text{ SGD} \times \frac{1 \text{ USD}}{1.28 \text{ SGD}} = 150 \text{ USD}$

AFO accepted ☆

Answers:

(a) 1240 (SGD) M1A1

(b) 150 (USD) A1M1 M1A1

7. The second term of an arithmetic sequence is 30. The fifth term is 90.

(a) Calculate

(i) the common difference of the sequence;

(ii) the first term of the sequence. [3]

The first, second and fifth terms of this arithmetic sequence are the first three terms of a geometric sequence.

(b) Calculate the seventh term of the **geometric** sequence. [3]

$U_n = u_1 + d(n-1)$

Working:

a) (i) $u_2 = 30$ $u_5 = 90$

$30 = u_1 + d(2-1)$ $90 = u_1 + d(5-1)$

$30 = u_1 + d$ $90 = u_1 + 4d$

$d = 30 - u_1$

$90 = u_1 + 4(30 - u_1)$

$90 = u_1 + 120 - 4u_1$

$-30 = -3u_1$

$u_1 = 10$

(ii) $30 = u_1 + 10$

$u_1 = 20$

Geometric

b) 10, 30, 90, ...

$U_n = u_1 \cdot r^{n-1}$

$u_7 = 10\,(3)^{7-1}$

$=$

Answers:

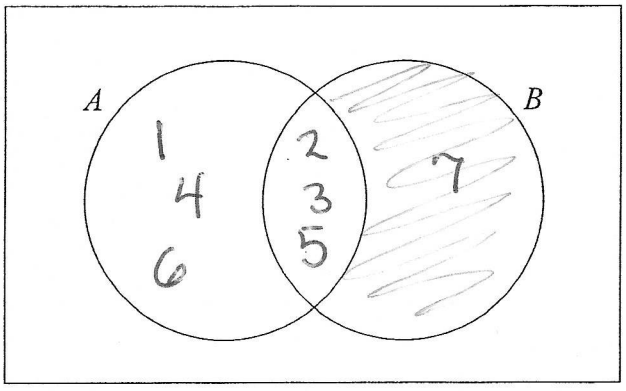
(a) (i) 10 M1A1

(ii) 20 A1

(b) 7290 A1M1A1

8. Aleph has an unbiased cubical (six faced) die on which are written the numbers 1, 2, 3, 4, 5 and 6.
Beth has an unbiased tetrahedral (four faced) die on which are written the numbers 2, 3, 5 and 7.

(a) Complete the Venn diagram with the numbers written on Aleph's die (A) and Beth's die (B). A1 A1



[2]

(b) Find $n(B \cap A')$. [2]

Aleph and Beth are each going to roll their die once only. Shin says the probability that each die will show the same number is $\frac{1}{8}$.

(c) Determine whether Shin is correct. Give a reason. [2]

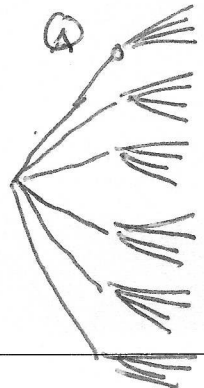
Working:

(c)

(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
1	2	4	2	6	2	2	2	3	3	5	5
1	3	4	3	6	3	2	7	3	7	5	7
1	5	4	5	6	5	2	3	3	2	5	3
1	7	4	7	6	7	2	5	3	5	5	2

$\frac{3}{24} = \frac{1}{8}$

Or you could use a tree diagram



Answers:

(b) 1 C2

(c) Correct (see above) R2

9. A right pyramid has apex V and rectangular base ABCD, with AB = 8 cm, BC = 6 cm and VA = 13 cm. The vertical height of the pyramid is VM.

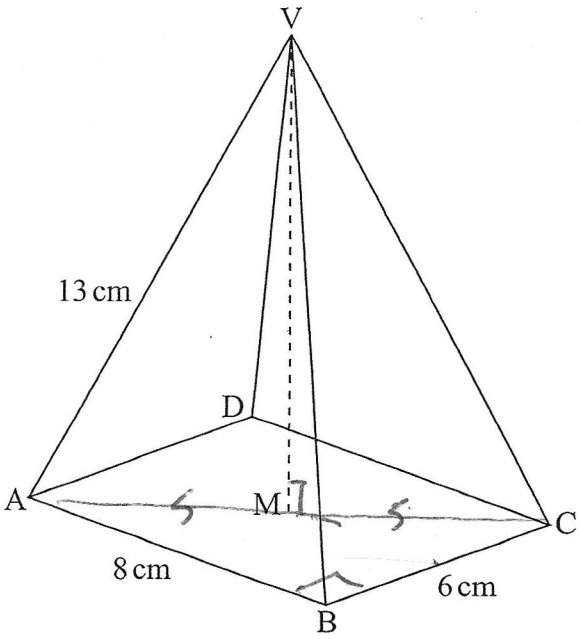


diagram not to scale

(a) Calculate VM. [4]

(b) Calculate the volume of the pyramid. [2]

Working:

(a)

$AC^2 = 8^2 + 6^2$

$AC = \sqrt{8^2+6^2}$

$= 10$

$VA^2 = AM^2 + VM^2$

$VM^2 = VA^2 - AM^2$

$VM^2 = 13^2 - 5^2$

$VM = \sqrt{13^2-5^2} = 12$

(b)

$V = \frac{1}{3}$ (area of base) × (height)

$= \frac{1}{3}(48)(12)$

$= 192 \text{ cm}^3$

Answers:

(a) 12 (cm) M1A1 M1A1

(b) 192 (cm^3)

10. Pierre invests 5000 euros in a fixed deposit that pays a nominal annual interest rate of 4.5 %, compounded **monthly**, for seven years.

(a) Calculate the value of Pierre's investment at the end of this time. Give your answer correct to two decimal places. [3]

Carla has 7000 dollars to invest in a fixed deposit which is compounded **annually**. She aims to double her money after 10 years.

(b) Calculate the minimum annual interest rate needed for Carla to achieve her aim. [3]

Working:

(a) $FV = PV\left(1 + \frac{r}{100k}\right)^{kn}$

$= 5000\left(1 + \frac{4.5}{1200}\right)^{(12 \cdot 7)} = 6847.26$ euros

(b) $14000 = 7000\left(1 + \frac{r}{100}\right)^{10}$ $k=1$ for annually

$\left(1 + \frac{r}{100}\right)^{10} = 2$

$\sqrt[10]{\left(1 + \frac{r}{100}\right)} = \sqrt[10]{2}$

$1 + \frac{r}{100} = \sqrt[10]{2}$

$\frac{r}{100} = \sqrt[10]{2} - 1$

$r = 100\left(\sqrt[10]{2} - 1\right)$

$= 7.17734....$ %

Answers:

(a) 6847.26 euros

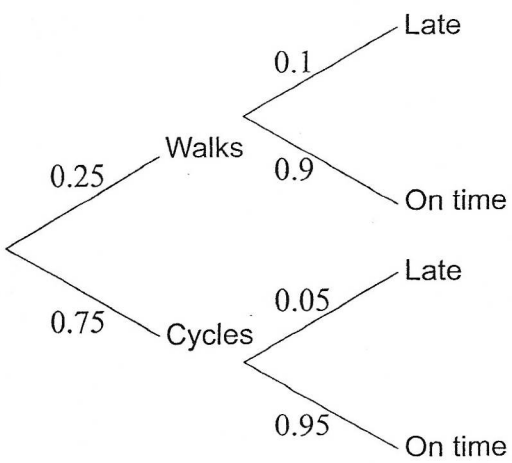
(b) 7.18 %

.0718 % is NOT correct

M1A1 A1

M1A1 A1

11. Peter either walks or cycles to work. The probability that he walks is 0.25. If Peter walks to work, the probability that he is late is 0.1. If he cycles to work, the probability that he is late is 0.05. The tree diagram for this information is shown.



(.25)(.1) = .025

(.75)(.05) = .0375

On a day chosen at random, Peter walked to work.

(a) Write down the probability that he was on time. [1]

For a different day, also chosen at random,

(b) find the probability that Peter cycled to work and was late; [2]

(c) find the probability that, given Peter was late, he cycled to work. [3]

Working:

(b) P(cycled and was late) = (.75)(.05) = .0375

(c) P(cycled given that he was late) = $\frac{.0375}{.025 + .0375}$ = 0.6

or 3.75%

or 3/80

Answers:

(a) 0.9

(b) .0375 M1A1

(c) 0.6 M1A1(ft)

or 60%

or 3/5

12. An iron bar is heated. Its length, L, in millimetres can be modelled by a linear function, $L = mT + c$, where T is the temperature measured in degrees Celsius (°C).

At $150\,°C$ the length of the iron bar is $180\,\text{mm}$.

(a) Write down an equation that shows this information. [1]

At $210\,°C$ the length of the iron bar is $181.5\,\text{mm}$.

(b) Write down an equation that shows this second piece of information. [1]

(c) Hence, find the length of the iron bar at $40\,°C$. [4]

Working:

(b) $181.5 = 210m + C$

(c) $180 = 150m + C$

$C = 180 - 150m$

$181.5 = 210m + 180 - 150m$

$1.5 = 60m$

$m = .025$

$C = 180 - 150(.025)$

$= 176.25$

So the linear equation is

$L = .025T + 176.25$

$L = .025(40) + 176.25$

$= 177.25$ exact

Answers:

(a) $180 = 150m + C$ ... A1

(b) $181.5 = 210m + C$ ...

(c) 177.25 (mm) ... A1A1 M1A1

or 177

13. The weight, W, of bags of rice follows a normal distribution with mean $1000\,\text{g}$ and standard deviation $4\,\text{g}$.

(a) Find the probability that a bag of rice chosen at random weighs between $990\,\text{g}$ and $1004\,\text{g}$. [2]

$95\,\%$ of the bags of rice weigh less than k grams.

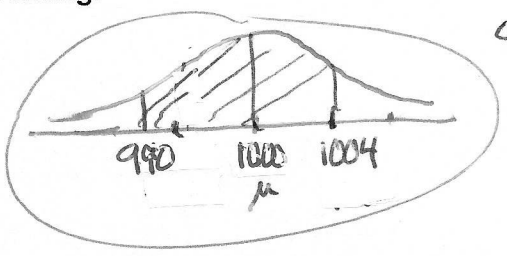
(b) Find the value of k. [2]

For a bag of rice chosen at random, $\text{P}(1000 - a < W < 1000 + a) = 0.9$.

(c) Find the value of a. [2]

Working:

a) A1

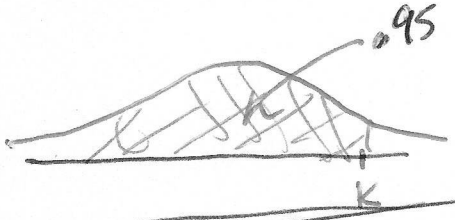


a) $P(990 < W < 1004)$ ← Use GDC

$= .835135...$

normalcdf (990, 1004, 1000, 4)

b)



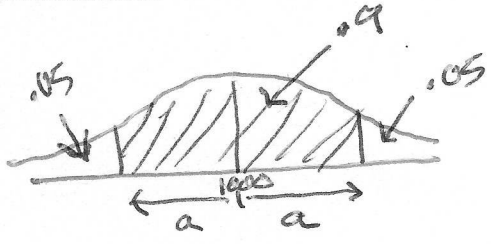
$P(W < k) = .95$

$K = 1006.5794$

$= 1010$ ← 3 sf

invnorm (0.95, 1000, 4)

c)



$P(W < 1000 - a) = .05$

so $W = 6.5794...$

6.58

invnorm (.05, 1000, 4)

sketch

Answers:

(a) .835 M1 A1

(b) 1010 M1 A1

(c) 6.58 M1 A1

14. Consider the quadratic function, $f(x) = px(q - x)$, where p and q are positive integers. The graph of $y = f(x)$ passes through the point $(6, 0)$.

(a) Calculate the value of q. [2]

The vertex of the function is $(3, 27)$.

(b) Find the value of p. [2]

(c) Write down the range of f. [2]

Working:

(a) Substitute (6,0) for x and y

$0 = p(6)(q-6)$

$0 = 6p(q-6)$

Use zero product property → $q-6=0$, $q=6$

(b) Vertex $(3, 27)$ (x, y)

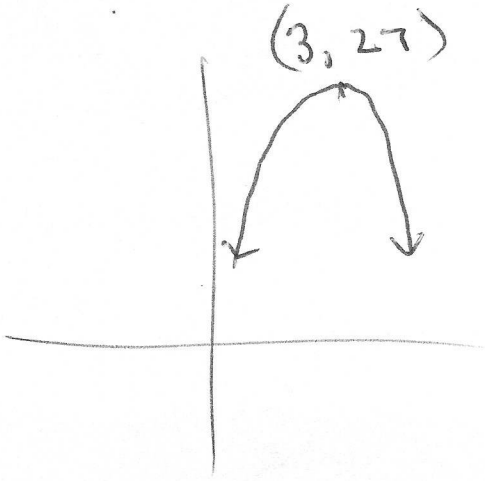
$27 = p(3)(6-3)$

$27 = 9p$

$p = 3$

(c) $f(x) = 3x(6-x) = 18x - 3x^2 = -3x^2 + 18$

negative orientation so parabola has a maximum



Answers:

(a) 6 M1A1

(b) 3 M1A1

(c) $y \leq 27$ or $-\infty < y \leq 27$

15. A cuboid has a rectangular base of width x cm and length $2x$ cm. The height of the cuboid is h cm. The total length of the edges of the cuboid is 72 cm.

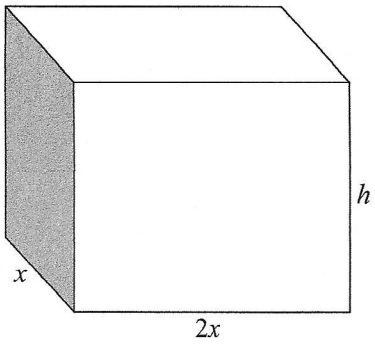


diagram not to scale

Volume of any cuboid = length × width × height

so $V = (x)(2x)(h)$

$V = 2x^2h$

4 of each edge

so $4x + 4(2x) + 4h = 72$

$12x + 4h = 72$ (A1)

$4h = 72 - 12x$

$h = 18 - 3x$

The volume, V, of the cuboid can be expressed as $V = ax^2 - 6x^3$.

(a) Find the value of a. [3]

(b) Find the value of x that makes the volume a maximum. [3]

Working:

a) $V = 2x^2h$

$V = 2x^2(18 - 3x)$

$V = 36x^2 - 6x$

so $a = 36$

b) TO MAXIMIZE find derivative and set it equal to zero (where gradients are flat

$V = 36x^2 - 6x^3$

$\frac{dV}{dx} = 72x - 18x^2$

$72x - 18x^2 = 0$

$18x(4 - x) = 0$

$4 - x = 0$

$x = 4$

so 4 is the value that will maximize the volume

Answers:

(a) 36 M1 A1 A1

(b) 4 M1 A1 (ft) A1

Name _____

Details about taking **Paper 1**
IB Math Studies Exam

Paper 1 and Paper 2 have different formats. This "Mock" exam you are doing now is a Paper 1 style.

Things to know about the format of Paper 1:

- ✓ 15 questions, 6 marks each.
- ✓ 90 minutes total allowed, a mark a minute
- ✓ All Responses written on question paper provided.
- ✓ GDC allowed
- ✓ Use your formula packet.
- ✓ Full marks awarded for the correct answer in the correct place.
- ✓ Method marks possible (showing work may get you points on some if answer is incorrect).
- ✓ Answers to be given to 3 significant figures unless it is otherwise stated or financial.
- ✓ Unit penalty applies at specific points in the mark scheme, but you won't know where. (therefore, write units on answers where appropriate)
- ✓ You are given 5 minutes reading time at the start of the exam during which time they may not write, but may read the questions. this is not part of the 90 minutes and should be used well.