Common Instructional Framework for Mathematics

2013-2015

CCS
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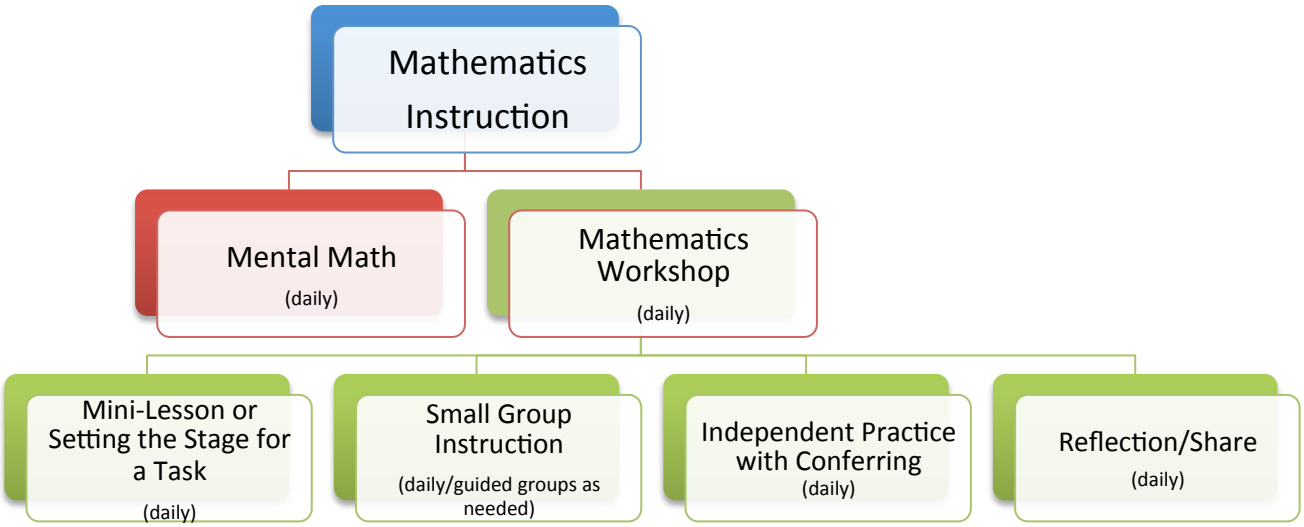
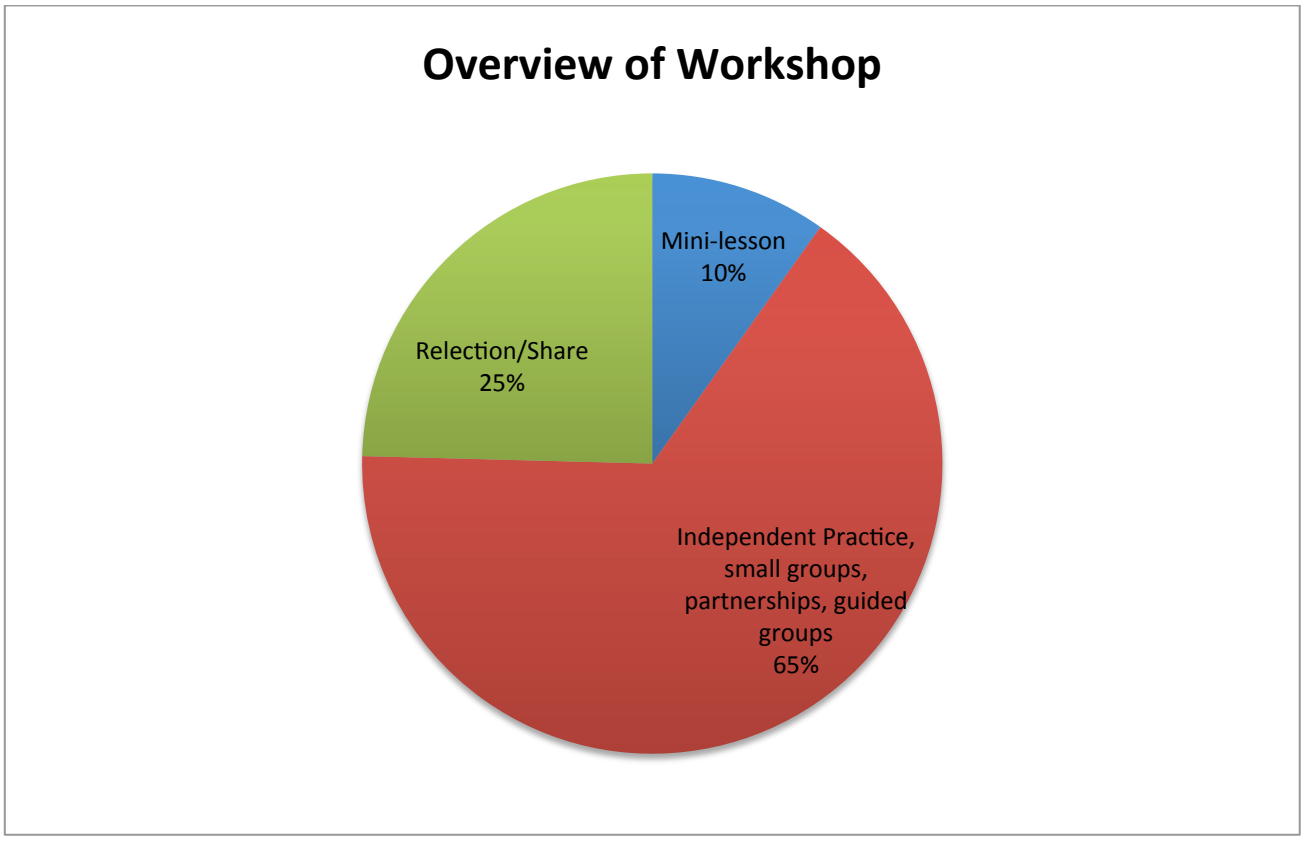
The North Carolina Department of Education adopted the Common Core State Standards for Mathematics. The Common Instructional Framework defines the local expectations for teaching these standards in Cabarrus County elementary schools. Included in the Common Instructional Framework is the following:

- Graphic Representation of the Common Instructional Framework
- Daily Mathematics Instruction and Sample Mathematics Block
- Taking a Closer Look at Mathematics Instruction
- Overview of Workshop: mini-lesson, small group instruction, independent practice, conference, reflection/share
- Overview of Mini-Lesson
- Overview of Small Group Instruction
- Overview of Independent Practice
- Overview of a Conference
- Overview of Reflection/Share
- Overview of Mental Math

Daily Mathematics Instruction

Cabarrus County elementary schools incorporate the structures of workshop into mathematics instruction. Math workshop promotes a culture of engagement and individualization that gives mathematical access to all learners in the classroom community. Math workshop provides students with the opportunity to develop conceptual understanding as they attend explicitly to connections among facts, procedures, and ideas as well as wrestle with important ideas in an intentional and conscious way (NCTM, 2007). When implementing mathematics instruction, the following components are incorporated into the mathematics instructional block.





Sample Mathematics Block

	Kindergarten through Fifth Grade
Mini-Lesson	10-20 minutes
Independent Practice with Conferring	30-60 minutes
Small Group Instruction	Takes place during the 30-60 minute Independent Practice, time with groups will vary according to need of the group
Reflection/Share	10-15 minutes
Mental Math	10 minutes

Taking a Closer Look at Mathematics Instruction

Math workshop is an inquiry-based model of differentiated instruction that fosters a deeper understanding of rigorous and rich mathematics that is attainable by all learners (Legnard & Austin, 2012).

We Believe:

- All students can learn and be successful
- Student learning is best achieved through rigorous, integrated, and culturally responsive lessons
- Teachers facilitate learning by probing student thinking through purposeful, provocative questions that encourage mathematical justifications
- Effective mathematics instruction develops most effectively in a safe learning environment where student ideas are valued and a love of mathematics is fostered
- Research-based instructional techniques that allow for differentiation (product, process, content, etc.) support all learners
- Collaboration and reflection is essential among teachers and students

Therefore, instructional practices should:

- include teaching for conceptual understanding
- balance the building of number sense, computational fluency, and conceptual understanding
- provide for the application of the mathematical concepts students are learning (problem solving)
- include a strong instructional and assessment component for math facts
- focus on number sense development in kindergarten through second grade
- engage students in conversation, interaction, and metacognitive processes
- balance teacher talk and student talk
- integrate the standards for mathematical practice with the content standards
- view student mistakes as opportunities to learn
- include the CPA model/Concrete, Pictorial, Abstract

What should you see…

- Environment:
  - Space dedicated to meeting for a mini-lesson, guided or strategy group, individual work, etc.
  - Additional space for possible small group mathematics work
  - Student seating which allows for interactive learning such as, meeting with partners or listening in on a teacher-led conference
  - Anchor charts which are teacher-student generated or student-generated for reference to strategies and skills
  - Mathematics materials accessible
    - Pencils or pens
    - Index cards, sticky notes, or other organizing pieces
    - Manipulatives
    - Number lines
    - Reference materials
    - Math journals
- Teacher:
  - Meeting with students one-on-one in a conference or in small groups focused on mathematics strategies and making connections between mathematical concepts
  - Modeling with whole class or debriefing different use of strategies with students (note: the teacher should not suggest particular strategies that will lead students to solve problems until after they have had an opportunity to explore these strategies independently or in small groups)
  - Facilitating mental math activities
  - Writing anecdotal notes that reflect students' strengths, weaknesses, and growth
  - Making instructional decisions based on student misconceptions and adjusting instruction to meet the needs of the students
  - Aligning student strategies with the learning goal for the lesson

- Students:
  - Working independently
  - Meeting with a math partner/group discussing ideas about strategies, investigations, etc.
  - Justifying and supporting their approach to the task given
  - Engaging and focusing in thinking about math, with teacher or other students
  - Exhibiting understanding of the procedures and components of mathematics workshop in their classroom
  - Writing notes, generating ideas, or reflecting in a notebook, with sticky notes, graphic organizers, etc.

What should you hear…

- Teacher
  - Referring to a strategy using common phrases; such as,
    - "Yesterday, we…" or "You already know how to… " to connect
    - "Today, we will" or "I am going to show you" to introduce
    - "When I" or "I am thinking I" to model
    - "Let's try it" or "Turn and talk" to actively involve students
  - Conferring with a student during independent work or facilitating guided groups using phrases such as,
    - "How's it going?" or "What are you doing/thinking today?" to start conference
    - What is the problem asking?
    - "Show me where you've done that" to gain knowledge
    - What strategy or path did you take?
    - What generalizations can you make?
    - "I like how you" to compliment
    - "What I want to teach you is" to start teaching point
    - Is there a more effective and efficient way to solve the problem?
    - Is your answer reasonable and how do you know?
    - "When you are problem solving, you can" or "This is something mathematicians do all the time" to close conference
    - * See Appendix A for stems related to the 8 mathematical practices
- Students
  - Talking with teacher
    - responding to questions in conference or small group instruction
    - asking questions or clarifying misunderstandings
  - Talking with classmates
    - During mini-lesson, guided group, etc.
      - Making sure they understand the problem or task
      - Sharing ideas and thinking about strategies
    - During independent work
      - Sharing ideas, notebooks, and thinking about strategies
      - Giving suggestions or praise
      - Critiquing other students' work
    - During Share or Closure time
      - Explaining work completed, thinking behind strategies, etc.
      - Taking notes in their math journals
      - Giving praise or compliment
      - Critiquing other students' work

Overview of a Mini-Lesson

Why do it?

Mini-lessons provide time for instruction as the teacher presents a conceptual problem and related skills. Additionally, an analysis of problem strategies and comparison of related problems may take place. This removes the guesswork out of what students are expected to do independently. Additionally, this supports dependent students who have not intuited or deduced how to think strategically about a concept or strategy, are confused easily, or need all of the cues that can be provided. The shortened time allotted for a mini-lesson allows for increased student engagement time in mathematical processes not only during the mini-lesson but also during independent practice.

What is it?

A mini-lesson is one opportunity for teaching or explorations of new mathematical strategies for students. The mini-lesson may target math content, math process skills, strategies for successful collaboration, or any other understanding that students may need prior to small group instruction and independent practice (Hoffer, 2012). It should be applicable to all or the majority of the students and is not a repetition of the strategies they already know how to do. Student data and the standards are used to determine the skills and strategies on which to focus. It is concise, brief, and purposeful in order to keep students engaged.

What does it look like?

The mini-lesson is around 10-20 minutes, but lasting no longer than 20 minutes. It often occurs at a designated meeting place in the classroom. The teacher's role is to present the problem or task that embodies important mathematical ideas and can be solved in multiply ways. The teacher introduces students to the problem, the tools that are available for working on it, and the nature of the products that the students will be expected to produce (Smith and Stein, 2011). Student talk should be to clarify questions. The teacher should not explicitly suggest particular strategies that will lead students to solve problems during independent practice.

The mini-lesson may address the presentation of a conceptual problem/s, any related skills needed to solve the problem, compare related problems, or analyze problem strategies from previous lessons. The mini-lesson may also include explicit teaching if at other times during the unit students have engaged in task exploration and problem-solving. The important consideration is that of being purposeful and having a balance of setting the stage for tasks and explicit instruction. Instead of, "Here's how to **do** this math," it is about, "Here's how to **think** as a mathematician about this situation" (Hoffer, 2012, p. 106).

Mini-lesson Look–Fors

Look – For	Evident	Working On	Not Evident
Does the teacher connect the work the students have been doing with the new learning they are about to do in the mini-lesson?			
Does the teacher give and gather information and set the stage for the work?			
Does the teacher provide essential information and model the tools for thinking and problem solving that the students will need (modeling the thinking versus showing examples)?			
Does the lesson focus on math content, math process skills, strategies for collaboration, or other specific understandings?			
Does the lesson include opportunities for student engagement and thinking? (turn and talk to a partner, elaborate on their comments, etc.)			
Does the teacher ensure that the students understand the task before sending them off to work?			
Does the task presented align with the mathematics intended for the lesson?			
Does the teacher keep the mini-lesson between 10-20 minutes?			

Teacher: _____ Date: _____

Comments:

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Overview of Independent Practice

Why do it?

Research tells us that complex knowledge and skills are learned through social interaction (Vygotsky, 1978; Lave and Wagner, 1991). Students need to work together on complex tasks or problems to further their mathematical thinking. Independent Practice allows the teacher time to teach to the students' needs while they are working in partnerships or groups. It is the time when the teacher coaches students, asks engaging, refocusing, clarifying, and probing questions to ensure that students' needs are met. It also provides time to practice with peers with teacher coaching as needed. This coaching time allows teachers to assess students' transfer of the modeled skill and strategy (Beers, 2003). The teacher evaluates strategies and selects students to share strategies. During this time the teacher reflects on the direction of the task, reevaluates the lesson for the next day, and determines if the students are engaged and mastering the standards being taught. The teacher needs this time to evaluate all of the students. This may also include opportunities for students to work on their own to demonstrate their understanding of particular standards, concepts, and strategies.

What is it?

Independent practice provides an opportunity for students to engage in mathematics development by focusing on effective strategies for building number sense, computational fluency, and conceptual understanding while providing for application of mathematical concepts. Tasks completed during this time should include those that, "demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas which lead to a different set of opportunities for student thinking" (Stein, Smith, Henningsen, & Silver, 2000, p. 11). Characteristics of tasks may include: challenging the learners to think for themselves, different levels of challenge, opportunities for collaboration and discussion, potential for revealing patterns or leading to generalizations, decision making, accessible to a wide range of learners, something to make sense of, requiring justifications and explanations for answers and methods, making sense of the mathematics involved and thereby increasing understanding (Van de Walle & Lovin, 2004).

What does this look like?

Students move into groups or partnerships after the mini-lesson. During independent practice students are discussing and revising their ideas using materials that are appropriate for the task. The teacher is circulating and questioning students, making observations, evaluating the effectiveness of the lesson and where the class is headed. The teacher is deciding which students will share strategies to further the learning of the class. This may also include opportunities for students to work on their own to demonstrate their understanding of particular standards, concepts, and strategies.

Independent Practice Look-Fors

Look-Fors	Evident	Working On	Not Evident
Does the task align with the standard, content, or strategy presented during the mini-lesson?			
Are students able to explain what they are doing and why they are doing it?			
Are students engaged in complex tasks or problems?			
Are students collaborating with one another in pairs or small groups?			
Is the teacher monitoring the strategies students are using to complete the task and/or coaching students?			
Are students finding multiple pathways to solve the task or problem?			
Are students justifying their answers and explaining their thinking to one another within their small groups?			
Are all students taking ownership of the work being done?			
Does the teacher keep track of the strategies students are using in order to select the most beneficial strategies for the reflections at the end of the lesson?			
If students are working on their own to demonstrate their understanding, does the accurately measure the standard of the lesson?			

Teacher:_____Date:_____

Comments:_____

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Overview of Small Group Instruction

Why do it?

Pulling students into small groups provides teachers with a greater opportunity to scaffold instruction. It allows teachers to work with students who are similar in their math behaviors or who have similar instructional needs. It is designed to help individual students learn how to use strategies and skills to master mathematical practices and mathematical concepts with the goal of helping children learn how to independently use these strategies and skills successfully.

It meets the needs of all students, both struggling and independent, through its varied instruction focusing on constructing meaning and understanding of concepts not previously encountered. It allows for ongoing observation and assessment that informs the teacher's interactions with individuals in the group and helps the teacher determine subsequent teaching points and areas of focus. Small group instruction provides an opportunity for teachers to reteach or provide enrichment to students as needed.

What is it?

Small group instruction is an instructional portion of the math workshop. During this time, the teacher supports students' mathematics development by focusing on effective strategies for building number sense, computational fluency, and conceptual understanding while providing for application of mathematical concepts.

What does this look like?

The teacher works with a small group of students who have similar instructional needs. Decisions can be made through error analysis of student work, anecdotal notes from conferences and observations, formative assessments, etc. The teacher selects tasks or supports the task in progress that offer the students a minimum of new things to learn; that is, they can begin to work through it with the strategies they currently have with an opportunity for a small amount of new learning. Students will take "the known" to work through "the unknown". The teacher provides varying levels of guidance (consider each student's zone of proximal development), takes notes, and provides individual support. One or two teaching points are presented to the group. These groups will occur while other students are working in small groups or individually on a meaningful mathematical task as described in the independent practice section.

Small Group Instruction Look-Fors

Look-Fors	Evident	Working-On	Not Evident
Are the groups flexible and fluid?			
Does the teacher think aloud, talk, question, and teach a specific strategy?			
Is the instruction targeted to meet the needs of the students within the group? (the instruction is different for different groups of students)			
Is there one (or two) teaching point only?			
Do the students use the strategy, skills, or concept to complete the task?			
Are there opportunities for students to share the thinking behind solutions?			
Does the teacher ask questions to elicit deeper explanations?			
Does the group last for 15-20 minutes?			

Teacher:_____Date:_____

Comment:_____

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Overview of a Conference

Why do it?

Conferences provide opportunities for individual, thoughtful, and respectful exchanges about mathematical ideas. They allow us to focus on individual students to support their thinking and understanding. Conferences also give teachers the means to really get to know their students. They are a powerful way to teach them how to be better mathematicians. They are focused on the individual student's or like-group of students' present needs to help them develop their own expertise (Anderson, 2000). They are the teacher's opportunity to provide individualized instruction to each student and assess students' strengths and weaknesses; as well as, determine what the students' needs are collectively as a class or small group to guide further instruction. Finally, conferences provide a key opportunity for teachers to "unveil how a learner comprehends a concept, what gaps may be troubling her, and how to move her to the next level of understanding" (Hoffer, 2012, p. 140).

What is it?

A conference is a conversation about a student's thinking as he or she seeks to improve his or her skills. It is about the work the student is doing as a mathematician and how he or she can become a better mathematician. It has specific characteristics:

- Conferences have a point
- Conferences have a predictable structure
- In conferences, we pursue lines of thinking with students
- Teachers and students have conversational roles in conferences
- In conferences, students are shown that teachers care about them (Anderson, 2000)

Conferences take place during independent practice. There are several different types of conferences: research-decide-teach, coaching, and table. Research-decide-teach conferences are the most common. They include, as the name states, research, teaching decision, and teaching. Coaching conferences "coach" students through the teaching of work the student has already done and been working on during previous conferences. During this type of conference, the teacher is pushing a student toward independence and to do the work he is able to do, but with scaffolded instruction. Table conferences are essentially the same as coaching or research-decide-teach but delivered to a group of students. It may start with one student with others joining in or it may begin by asking a group of students a specific question about a strategy or skill.

What does this look like?

During a conference, the teacher is sitting side by side with the student or talking with the math group for about 5 minutes. The most common type of conference looks as follows:

- Research: Teacher finds out what the student/s are working on
- Decide: Teacher notices what the student/s are able to do and where they are. Then, the teacher guides the student/s on this path.
- Teach: The teacher guides the student/s and gives him the opportunity to try this new strategy or ideas. The teacher ends the conference by telling the student when he should do this type of work or use this strategy.

Conference Look – Fors

Look – For	Evident	Working On	Not Evident
Does the teacher *research* the student? • Begin the conference with an open-ended question that invites the student to talk about his/her work. • Ask the student questions to learn about his/her behavior. • Look at the student's work to gain an understanding of what he/she is doing.			
Does the teacher *decide* what is most useful to guide the student? • The teacher reflects, "Based on what I've learned so far in this conference, what can I teach this student that will help him/her?" • Teaches the student something about a strategy or technique that he is already using. • Guides the student to try a new strategy/technique.			
Does the teacher guide the student toward a new strategy or manipulative by giving an explanation, looking at a problem together with the student, asking for justification for the strategy, or by referring back to previous problems?			
Does the teacher give the student specific feedback about how well he is doing? *(compliment)*			
Does the student talk through (or try out the strategy through writing) what he is going to do in his work after the conference (*active engagement*)?			
Does the teacher end the conference by letting the student know she expects him to follow through with what they have just talked about (*link*)?			
Does the teacher record notes for each conference?			
Is the teacher's conference time with the child five minutes or less?			

Teacher: _____Date: _____

Comments:_____

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Overview of Reflection/Share

Why do it?

In mathematics classrooms, high-quality discussions support student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas (Smith and Stein, 2011). Research tells us that students learn when they are encouraged to become the authors of their own ideas and when they are held accountable for reasoning about and understanding key ideas (Engle and Conant 2002). Reflection or share restates the teaching that students were supposed to glean from the day through students sharing their strategies. Including a reflection or share time provides students with necessary repetition of the teaching and learning. Additionally, as students share, they are, "challenged to articulate their own process as problem solvers; as they listen to peers explain, learners consider how another's thinking matches up with or diverges from their own" (Hoffer, 2012, p. 155).

What is it?

Reflection or share is the wrap-up to workshop. It is a whole-class discussion and summary of various student-generated approaches to solving the problem. It reiterates the teaching and learning of the day. It is the most important part of the lesson pulling all of the pieces together and allowing students to share their thinking and strategies with others. It is a time for students to revise notes in their journals and reflect on the most efficient and effective ways to solve problems.

What does this look like?

The reflection or share may take place at the meeting place in the classroom. The teacher's role is critical during this phase, because he must choose who presents to form a coherent connection between the mathematical ideas and strategies the students will present. Without the teacher guidance, the discussion can turn into a lecture or a show and tell of ideas. Students may bring examples of problem solving, sticky notes, notebooks, chart paper, etc. from which to share their thinking. The teacher concludes the workshop by having partnerships recall and share something learned or highlighting a specific student's work that encompasses the teaching point of the day or creates a trail that other students could follow. Students are recording in their math journals the leaning goals of the day.

Reflection/Share Look-Fors

Look-Fors	Evident	Working On	Not Evident
Does the teacher provide opportunities for students to present thought-provoking strategies that move the mathematical thinking of the classroom toward the learning goal?			
Do students record strategies in their math journals that help form connections to the learning goal for the day?			
Does the teacher encourage students to evaluate the strategies of other students?			
Are there opportunities for students to share the thinking behind solutions?			
Does the teacher ask questions to elicit deeper explanations?			
Does the teacher use language that encourages students to share their thinking in what has been established as a safe and comfortable learning environment?			

Teacher:_____Date:_____

Comments:_____

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Overview of Mental Math

Why do it?

Research indicates that teachers can best support students memorization of basic facts through the varied experiences making 10, breaking numbers apart, and working on metal strategies, rather than repetitive timed tests. Through regular experiences with mental math children come to realize that many calculations are in fact easier to perform mentally. Also, when using mental math children almost always use a method, which they understand (unlike with written computation) and are encouraged to think actively about relationships involving the particular numbers they are dealing with. Mental math works on computational fluency and number sense. It teaches students to use number relationships and the structure of numbers to add, subtract, multiply, and divide. Through focused, frequent experiences children learn the math necessary for computational theory.

What is it?

During mental math students are solving a problem/problems mentally, discussing strategies used, and clarifying their thinking in a safe environment. Students are decomposing and composing numbers to make problems easier and quicker to solve. They are using strategies like doubling/halving to mentally solve problems like 25*12 and turning into 6*50=300. Students are justifying why these strategies work. Students are finding efficient and effective ways to solve problems mentally. Mental math may be a *Number Talk*, *Investigations 10 Minute Math* activity, or any 5-10 minute activity where students compute problems mentally. They share strategies to find efficient and effective ways to solve problems mentally by composing and decomposing numbers.

What does this look like?

In order to be effective Mental Math sessions should:

- occur on a daily basis (5-10 minutes per day)
- encourage all students to attempt the problem
- promote oral discussion
- allow students to see that there are many ways to arrive at a correct answer rather than one correct way
- build up a dense web of connections between numbers and number facts
- emphasize active understanding and use of place value

Mental Math Look-Fors

Look-Fors	Evident	Working On	Not Evident
Are there opportunities for all students to attempt to solve the problem mentally?			
Do students use a signal when they have an answer for the problem?			
Does the teacher provide opportunities for students to share strategies of solving problems?			
Are the strategies that are shared recorded in some way?			
Does the teacher emphasize the understanding of strategies, not just finding an answer?			

Teacher:_____Date:_____

Comments:_____
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APPENDIX A

Common Core State Standards
Standards for Mathematical Practice
Questions for Teachers to Ask

Make sense of problems and persevere in solving them	**Reason abstractly and quantitatively**	**Construct viable arguments and critique the reasoning of others**	**Model with mathematics**
Teachers ask: • What is this problem asking? • How could you start this problem? • How could you make this problem easier to solve? • How is ___'s way of solving the problem like/different from yours? • Does your plan make sense? Why or why not? • What tools/manipulatives might help you? • What are you having trouble with? • How can you check this?	*Teachers ask:* • What does the number ____ represent in the problem? • How can you represent the problem with symbols and numbers? • Create a representation of the problem.	*Teachers ask:* • How is your answer different than _____'s? • How can you prove that your answer is correct? • What math language will help you prove your answer? • What examples could prove or disprove your argument? • What do you think about _____'s argument • What is wrong with ____'s thinking? • What questions do you have for ____? **it is important that the teacher poses tasks that involve arguments or critiques*	*Teachers ask:* • Write a number sentence to describe this situation • What do you already know about solving this problem? • What connections do you see? • Why do the results make sense? • Is this working or do you need to change your model? **It is important that the teacher poses tasks that involve real world situations*
Use appropriate tools strategically	**Attend to precision**	**Look for and make use of structure**	**Look for and express regularity in repeated reasoning**
Teachers ask: • How could you use manipulatives or a drawing to show your thinking? • Which tool/manipulative would be best for this problem? • What other resources could help you solve this problem?	*Teachers ask:* • What does the word ____ mean? • Explain what you did to solve the problem. • Compare your answer to _____'s answer • What labels could you use? • How do you know your answer is accurate? • Did you use the most efficient way to solve the problem?	*Teachers ask:* • Why does this happen? • How is ____ related to ____? • Why is this important to the problem? • What do you know about ____ that you can apply to this situation? • How can you use what you know to explain why this works? • What patterns do you see? **deductive reasoning (moving from general to specific)*	*Teachers ask:* • What generalizations can you make? • Can you find a shortcut to solve the problem? How would your shortcut make the problem easier? • How could this problem help you solve another problem? **inductive reasoning (moving from specific to general)*