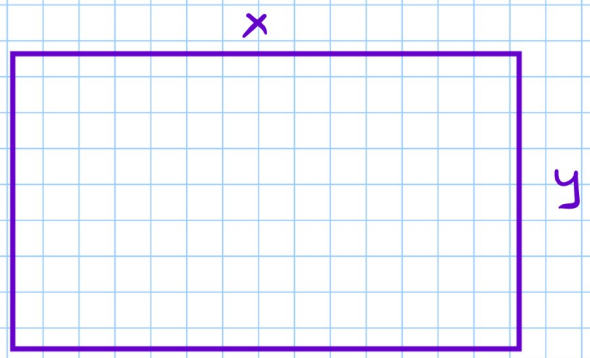
3)



Primary: Minimize $P = 2x + 2y$

Constraint: $xy = 64$

$$y = \frac{64}{x}$$

$$P = 2x + 2 \cdot \frac{64}{x}$$

$$P = 2x + 128x^{-1}$$

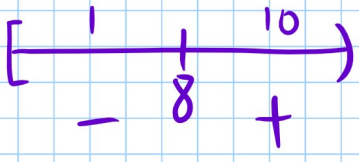
$$P' = 2 - 128x^{-2} = 0$$

$$\frac{2}{1} = \frac{128}{x^2}$$

$$2x^2 = 128$$

$$x^2 = 64$$

$$x = 8$$

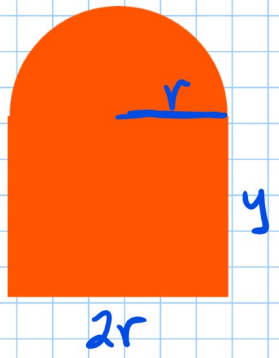


min when $x = 8$

$$y = \frac{64}{x} = \frac{64}{8} = 8$$

Perimeter is minimized when rectangle is 8 by 8

5)



Primary: $A = \frac{\pi r^2}{2} + 2ry$

Constraint: $2r + 2y + \pi r = 16$

$2y = 16 - 2r - \pi r$

$y = 8 - r - \frac{\pi r}{2}$

$A = \frac{\pi r^2}{2} + 2r\left(8 - r - \frac{\pi r}{2}\right)$

$A = \frac{\pi r^2}{2} + 16r - 2r^2 - \pi r^2$

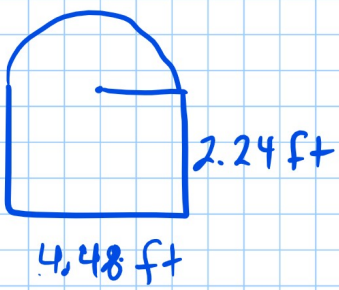
$A = 16r - 2r^2 - \frac{1}{2}\pi r^2$

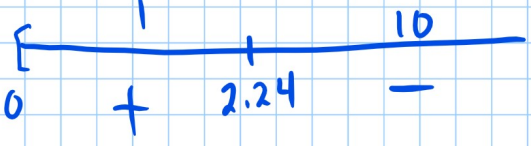
$A' = 16 - 4r - \pi r = 0$

$16 = 4r + \pi r$

$16 = r(4 + \pi)$

$r = \frac{16}{4+\pi} \approx 2.24$

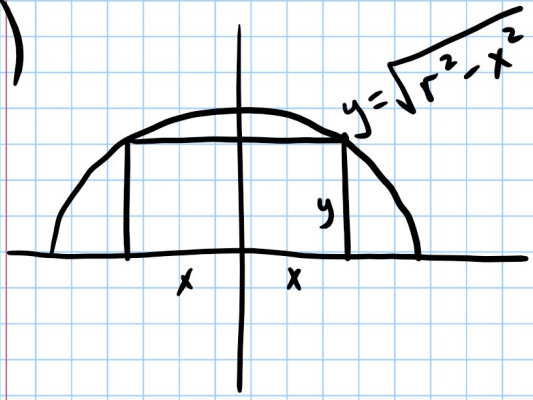




max when $r = 2.24$

$y = 8 - 2.24 - \frac{\pi(2.24)}{2} = 2.24$

6)



$A = 2x\sqrt{r^2 - x^2} = 2x(r^2 - x^2)^{1/2}$

$A' = 2x \cdot \frac{1}{2}(r^2 - x^2)^{-1/2}(-2x) + (r^2 - x^2)^{1/2} \cdot 2$

Primary: $A = 2xy$

Constraint: $y = \sqrt{r^2 - x^2}$

max when $x = \frac{r\sqrt{2}}{2}$

$$y = \sqrt{r^2 - \frac{r^2 \cdot 2}{4}}$$

$$y = \sqrt{\frac{1}{2}r^2} = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$$

So rectangle is $r\sqrt{2}$ by $\frac{r\sqrt{2}}{2}$

$$A' = \frac{-2x^2}{\sqrt{r^2 - x^2}} + \frac{2\sqrt{r^2 - x^2}}{1} \cdot \frac{\sqrt{r^2 - x^2}}{\sqrt{r^2 - x^2}}$$

$$A' = \frac{-2x^2 + 2(r^2 - x^2)}{\sqrt{r^2 - x^2}} = \frac{2r^2 - 4x^2}{\sqrt{r^2 - x^2}}$$

Critters set $2r^2 - 4x^2 = 0$

$$-4x^2 = -2r^2$$

$$x^2 = \frac{1}{2}r^2$$

$$x = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$$

$+$ | $-$

$\frac{r\sqrt{2}}{2}$