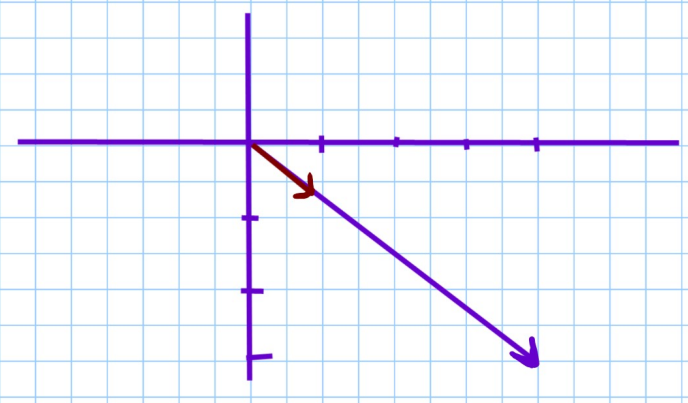
Section 8.4 (Continued and Concluded)

For any nonzero vector $\mathbf{v}$, the vector $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$ is a **unit vector** that has the same direction as $\mathbf{v}$ and a magnitude of 1.

ex: Find the unit vector in the same direction as

$\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$



$$\|\mathbf{v}\| = \sqrt{4^2 + (-3)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25} = 5$$

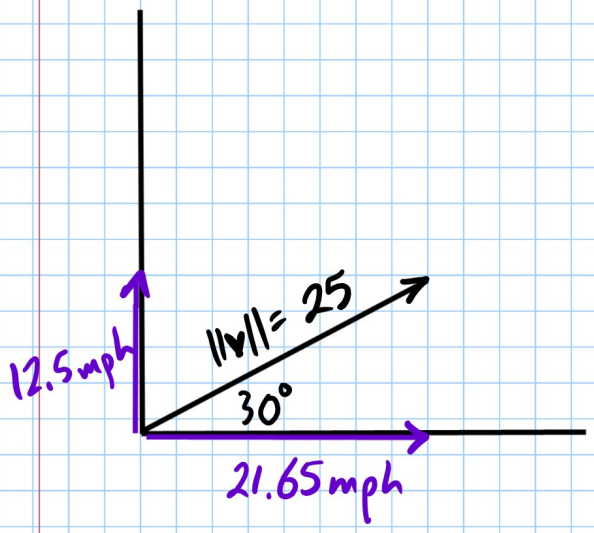
$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{4\mathbf{i} - 3\mathbf{j}}{5}$$

$$= \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$

Often the direction of vector $\mathbf{v}$ is given by the angle $\mathbf{v}$ makes with the positive x-axis. If the angle is α, then $\mathbf{v}$ can be expressed by

$$\mathbf{v} = \|\mathbf{v}\|(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$$

ex: You throw a ball at 25 mph so the angle is 30° with the positive x-axis.

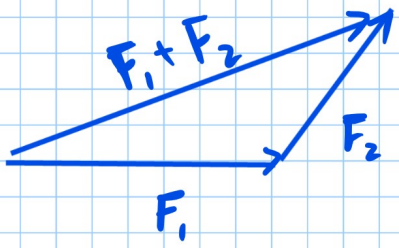


$$\mathbf{v} = 25\left(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j}\right)$$

$$\mathbf{v} = 25\left(0.866\mathbf{i} + 0.5\mathbf{j}\right)$$

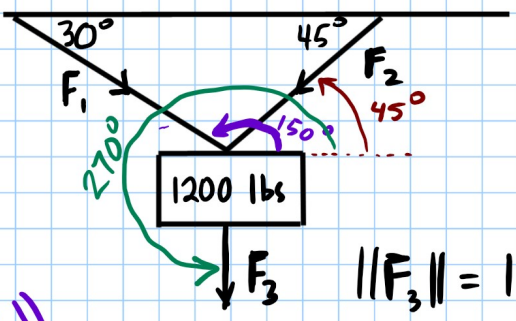
$$\mathbf{v} = 21.65\mathbf{i} + 12.5\mathbf{j}$$

In Physics, force **F** is a vector. If 2 different forces F_1 and F_2 work on an object, the resultant force is $F = F_1 + F_2$



An object is in equilibrium if 1) the sum of all forces is **0**
2) the object is at rest.

ex:



What are the magnitudes of F_1 and F_2?

$$\mathbf{v} = \|\mathbf{v}\|\left(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j}\right)$$

$\|F_3\| = 1200$

$x = \|F_1\|$

$y = \|F_2\|$

$$\mathbf{F_1} = x\left(\cos 150^\circ \mathbf{i} + \sin 150^\circ \mathbf{j}\right) = -0.866x\,\mathbf{i} + 0.5x\,\mathbf{j}$$

$$\mathbf{F_2} = y\left(\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j}\right) = 0.707y\,\mathbf{i} + 0.707y\,\mathbf{j}$$

$$\mathbf{F_3} = 1200\left(\cos 270^\circ \mathbf{i} + \sin 270^\circ \mathbf{j}\right) = -1200\,\mathbf{j}$$

$F_1 + F_2 + F_3 = 0$

$$-0.866x + 0.707y = 0$$
$$(-1)(0.5x + 0.707y - 1200 = 0)$$

$$-0.866x + 0.707y = 0$$
$$-0.5x - 0.707y = -1200$$
$$-1.366x = -1200$$
$$x = 878.5$$

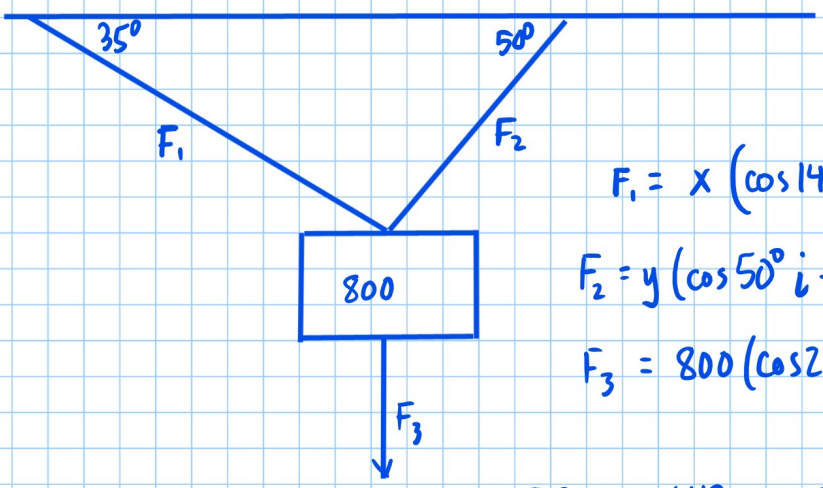
$$-0.866(878.5) + 0.707y = 0$$
$$-760.78 + 0.707y = 0$$
$$0.707y = 760.78$$
$$y = 1076.1$$

$\|F_1\| = 878.5$ lbs

$\|F_2\| = 1076.1$ lbs

p629 39, 41, 42, 49, 50, 59, 60

60 p629



$x = \|F_1\|$

$y = \|F_2\|$

$F_1 = x(\cos 145° i + \sin 145° j) = -.819x\,i + .574x\,j$

$F_2 = y(\cos 50° i + \sin 50° j) = .643y\,i + .766y\,j$

$F_3 = 800(\cos 270° i + \sin 270° j) = -800j$

$$-.819x + .643y = 0$$
$$.574x + .766y = 800$$

$(-819x + 643y = 0)\ 574$

$(574x + 766y = 800000)\ 819$

$\|F_1\| = 516.2$ lbs

$\|F_2\| = 657.2$ lbs

$$-470106x + 369082y = 0$$
$$470106x + 627354y = 655200000$$
$$996436y = 655200000$$
$$y = 657.5$$
$$-.819x + .643(657.5) = 0$$
$$-.819x = -422.8$$
$$x = 516.2$$