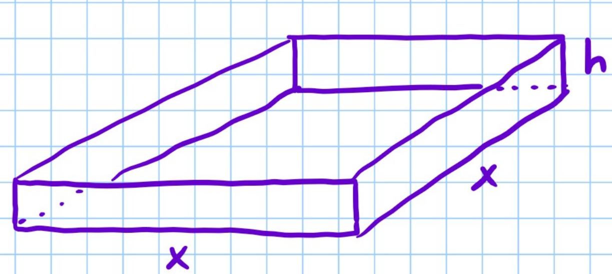
Section 3.7 Optimization Application Problems

ex: A manufacturer wants to design an open box having a square base and a surface area of 108 in. What dimensions will produce a box of maximum volume?

STEP 1: If applicable, draw a picture. Define variables.



STEP 2: Write a primary equation - equation that uses what we are trying to optimize

$$V = x^2 h$$ trying to maximize volume.

STEP 3: Write a constraint equation - equation that relates the quantities that we know with the variables.

$$x^2 + 4xh = 108$$

STEP 4: Solve the constraint equation for one of the variables and substitute for that variable in the primary equation.

Solve for h: $4xh = 108 - x^2$

$$h = \frac{108 - x^2}{4x}$$

$$h = \frac{108}{4x} - \frac{x^2}{4x}$$

$$h = \frac{27}{x} - \frac{x}{4}$$

$$V = \frac{x^2}{1}\left(\frac{27}{x} - \frac{x}{4}\right)$$

STEP 5: Simplify your primary equation and find its derivative.

$$V = \frac{27x^2}{x} - \frac{x^3}{4}$$

$$V = 27x - \frac{1}{4}x^3$$

$$V' = 27 - \frac{3}{4}x^2$$

STEP 6: Set derivative = 0 to find critters

$$27 - \frac{3}{4}x^2 = 0$$

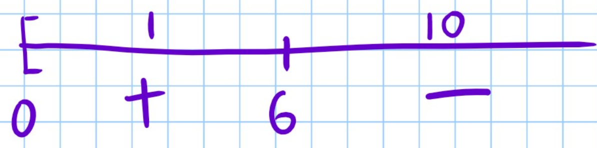
$$\cancel{\frac{4}{3}}\,\cancel{\frac{3}{4}}x^2 = \frac{27}{1}\cdot\frac{4}{3}$$

$$x^2 = 36$$

$$x = \pm\sqrt{36} = \pm 6$$

$$x = 6 \qquad (x \neq -6)$$

STEP 7: Prove the critter yields a max (or min) using the number line.



so $x=6$ yields a max

STEP 8: Answer the question.

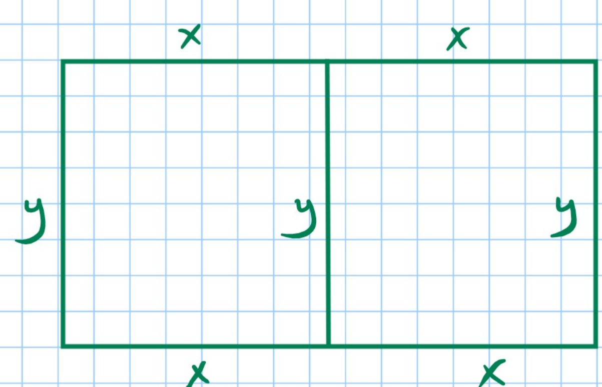
$$x=6 \qquad h=\frac{27}{x}-\frac{1}{4}x=\frac{27}{6}-\frac{1}{4}\cdot 6$$

$$=3$$

so dimensions are $6'' \times 6'' \times 3''$

ex: A rancher has 200ft of fencing with which to enclose 2 congruent adjacent rectangular corrals. What dimensions will maximize the area enclosed?

1)



2) Primary: $A=2xy$

3) Constraint: $4x+3y=200$

4) $\frac{4x}{4}=\frac{200-3y}{4}$

$$x = 50 - \frac{3}{4}y$$

$$A = 2\left(50 - \frac{3}{4}y\right)y$$

5) $A = 2y\left(50 - \frac{3}{4}y\right)$

$$A = 100y - \frac{3}{2}y^2$$

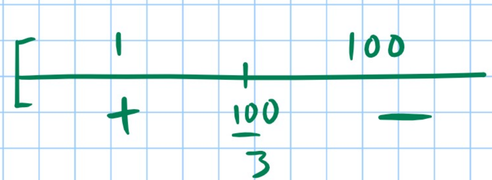
$$A' = 100 - 3y$$

6) $100 - 3y = 0$

$$3y = 100$$

$$y = \frac{100}{3}$$

7)



8) $y = 33\frac{1}{3}'$

$$x = 50 - \frac{3}{4}y$$

$$x = 50 - \frac{\cancel{3}}{4} \cdot \frac{100}{\cancel{3}}$$

$$x = 50 - 25 = 25$$

each corral is $33\frac{1}{3}' \times 25'$