Section 6.7 Solving Trig Equations

An equation like $\sin\theta = \frac{1}{2}$ is asking for what angles does the $\sin\theta = \frac{1}{2}$?

ex: Solve $\sin\theta = \frac{1}{2}$ for $\underbrace{0 \le \theta < 2\pi}_{\text{radians}}$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

If we were asked to give general solution (or solve for all real numbers)

$$\theta = \frac{\pi}{6} + 2\pi k \quad (k \text{ is any integer})$$

$$\theta = \frac{5\pi}{6} + 2\pi k$$

ex: Solve $-2\cos\theta = \sqrt{3}$. Give general solutions.

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{5\pi}{6} + 2\pi k$$

$$\theta = \frac{7\pi}{6} + 2\pi k$$

ex: Solve $\tan\theta = -1$. Give general solutions

Period of tangent is π. So we add πk

$$\theta = \frac{3\pi}{4} + \pi k$$

$$\theta = \frac{7\pi}{4} + \pi k$$

} stating both answers is redundant since $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$ are π apart.

So answer is $\frac{3\pi}{4} + \pi k$

ex: Solve $\cos(2\theta) = \frac{1}{2}$ on $0 \le \theta < 2\pi$

period $= \frac{2\pi}{2} = \pi$

$\frac{1}{2} \cdot 2\theta = \frac{\pi}{3} \cdot \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} + \pi = \frac{\pi}{6} + \frac{6\pi}{6} = \frac{7\pi}{6}$

$\frac{1}{2} \cdot 2\theta = \frac{5\pi}{3} \cdot \frac{1}{2} \Rightarrow \theta = \frac{5\pi}{6} + \pi = \frac{5\pi}{6} + \frac{6\pi}{6} = \frac{11\pi}{6}$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Assignment

For 1-4, give general formula

1) $\tan\theta = \frac{\sqrt{3}}{3}$

2) $3\sin\theta + 3 = 0$

3) $2\sec\theta = 4$

4) $2\sin\theta + 1 = 0$

In 5-8 solve on $0 \le \theta < 2\pi$

5) $2\cos\theta + 3 = 2$

6) $\sin(2\theta) = \frac{\sqrt{3}}{2}$

7) $\cos^2\theta = \frac{1}{4}$

8) $\tan\theta = \sqrt{3}$