

Section 6.5 Continued

Half-Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

Determine what quadrant $\frac{\theta}{2}$ is in and apply the appropriate sign

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$$

ex: $\sin 112.5^\circ = \sin \frac{225^\circ}{2} = + \sqrt{\frac{1 - \cos 225^\circ}{2}} = \sqrt{\frac{(1 - (-\frac{\sqrt{2}}{2})) \cdot \frac{2}{2}}{2}}$

QII
so sine > 0
112.5° is $\frac{1}{2}$ of 225°

$$= \sqrt{\frac{2 + \sqrt{2}}{4}}$$

$$\sin 112.5^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

ex: $\cos 195^\circ = \cos \frac{390^\circ}{2} = - \sqrt{\frac{1 + \cos 390^\circ}{2}}$

QIII
so cosine < 0

$\cos 30^\circ = \cos 390^\circ$

$$= - \sqrt{\frac{(1 + \frac{\sqrt{3}}{2}) \cdot \frac{2}{2}}{2}}$$

$$= - \sqrt{\frac{2 + \sqrt{3}}{4}} = - \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\underline{\text{ex:}} \quad \tan \frac{\pi}{8} = \tan 22.5^\circ = \tan \frac{45^\circ}{2} = \frac{1 - \cos 45^\circ}{\sin 45^\circ}$$

$$\frac{\pi}{8} \cdot \frac{180}{\pi} = 22.5^\circ$$

$$= \frac{\left(1 - \frac{\sqrt{2}}{2}\right) \cdot 2}{\frac{\sqrt{2}}{2} \cdot 2}$$

$$= \frac{(2 - \sqrt{2}) \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$$= \frac{2\sqrt{2} - 2}{2}$$

$$= \frac{\cancel{2}(\sqrt{2} - 1)}{\cancel{2}}$$

$$= \sqrt{2} - 1$$

p501

13, 14, 16, 17, 18