Section 3.6 Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln f(x)) = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$$

$\frac{d}{dx}(\ln(f(x)))$

ex: $\frac{d}{dx}\left(\ln(\cos x)\right) = \frac{1}{\cos x} \cdot \frac{-\sin x}{1} = \frac{-\sin x}{\cos x} = -\tan x$

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1. **POWER:** $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$ $\quad \frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1} \cdot f'(x)$

2. **e:** $\frac{d}{dx}\left[e^x\right] = e^x$ $\quad \frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$

3. **Exponential:** $\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$ $\quad \frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)} \cdot \ln a \cdot f'(x)$

4. **TRIGS:**
   A. $\frac{d}{dx}\left[\sin x\right] = \cos x$ $\quad \frac{d}{dx}\left[\sin(f(x))\right] = \cos(f(x)) \cdot f'(x)$
   B. $\frac{d}{dx}\left[\cos x\right] = -\sin x$ $\quad \frac{d}{dx}\left[\cos(f(x))\right] = -\sin(f(x)) \cdot f'(x)$
   C. $\frac{d}{dx}\left[\tan x\right] = \sec^2 x$ $\quad \frac{d}{dx}\left[\tan(f(x))\right] = \sec^2(f(x)) \cdot f'(x)$

5. **Natural Log:** $\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$ $\quad \frac{d}{dx}\left[\ln(f(x))\right] = \frac{1}{f(x)} \cdot f'(x)$

6. **Product:** $\frac{d}{dx}\left[f(x) \cdot g(x)\right] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

   1st · deriv of 2nd + 2nd · deriv of 1st

7. __Quotient__: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\text{BOT} \cdot \text{der TOP} - \text{TOP} \cdot \text{der BOT}}{\text{BOT}^2}$

8. __Constants__: $\frac{d}{dx}\left[c\,f(x)\right] = c\,f'(x)$ $\quad \frac{d}{dx}\left[c + f(x)\right] = f'(x)$

__ex__: $h(x) = x^3 \ln(10x)$ RULES used: 6, 1, 5

$$h'(x) = \underbrace{x^3}_{\text{1st}} \cdot \underbrace{\frac{1}{\cancel{10}x} \cdot \cancel{10}}_{\text{der 2nd}} + \underbrace{\ln(10x)}_{\text{2nd}} \cdot \underbrace{3x^2}_{\text{der 1st}}$$

$$h'(x) = x^2 + 3x^2 \ln(10x)$$

$$h'(x) = x^2\left(1 + 3\ln(10x)\right)$$

__ex__: $f(x) = e^{\overbrace{\ln\left(e^{2x^2+3}\right)}^{2x^2+3}} = e^{2x^2+3}$ Rule 2, 1, 8

$e^{\ln N} = N$

$\ln e^M = M$

$$f'(x) = e^{2x^2+3} \cdot 4x$$

$$f'(x) = 4x e^{2x^2+3}$$

__ex__: $f(x) = \frac{\ln x}{\sin x}$ RULES: 7, 5, 4A

$$f'(x) = \frac{\left(\sin x \cdot \frac{1}{x} - \ln x \cdot \cos x\right)}{(\sin x)^2} \quad \frac{\frac{x}{1}}{\frac{x}{1}}$$

$$f'(x) = \frac{\sin x \cdot \frac{1}{\cancel{x}} \cdot \frac{\cancel{x}}{1} - \ln x \cdot \cos x \cdot \frac{x}{1}}{\sin^2 x \cdot \frac{x}{1}}$$

$$f'(x) = \frac{\sin x - x \ln x \cos x}{x \sin^2 x}$$

p136 1-15 odd, 21, 29-32

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$$f(\alpha) = \sin^5 \alpha \cos^3 \alpha \qquad 6, 1, 4A, 4B$$

$$(\sin\alpha)^5 (\cos\alpha)^3$$

$$f'(\alpha) = (\sin\alpha)^5 \cdot 3(\cos\alpha)^2(-\sin\alpha) + (\cos\alpha)^3 \cdot 5(\sin\alpha)^4 \cos\alpha$$

$$= -3\sin^6\alpha \cos^2\alpha + 5\cos^4\alpha \sin^4\alpha$$

$$= \sin^4\alpha \cos^2\alpha \left(-3\sin^2\alpha + 5\cos^2\alpha\right)$$

11) $j(x) = \ln(e^{ax} + b)$

$$j'(x) = \frac{1}{e^{ax}+b} \cdot ae^{ax} = \frac{ae^{ax}}{e^{ax}+b}$$

30) $y = 2x(\ln x + \ln 2) - 2x + e$

$$y' = 2\cancel{x}\left(\frac{1}{\cancel{x}}\right) + (\ln x + \ln 2) \cdot 2 - 2 + 0$$

$$y' = \cancel{2} + 2(\ln x + \ln 2) \cancel{-2} = 2(\ln x + \ln 2)$$

5) $f(z) = (\ln z)^{-1}$

$$f'(z) = -1(\ln z)^{-2} \cdot \frac{1}{z}$$

$$= -\frac{1}{z(\ln z)^2}$$

15) $f(w) = \ln(\cos(w-1))$

$$f'(w) = \frac{1}{\cos(w-1)} \cdot -\sin(w-1) \cdot 1 = \frac{-\sin(w-1)}{\cos(w-1)}$$

$$= -\tan(w-1)$$

21) $g(t) = \cos(\ln t)$

$$g'(t) = -\sin(\ln t) \cdot \frac{1}{t} = -\frac{\sin(\ln t)}{t}$$

31) $f(x) = \ln(\sin x + \cos x)$

$$f'(x) = \frac{1}{\sin x + \cos x} \cdot \frac{\cos x + (-\sin x)}{1}$$

$$= \frac{\cos x - \sin x}{\sin x + \cos x}$$