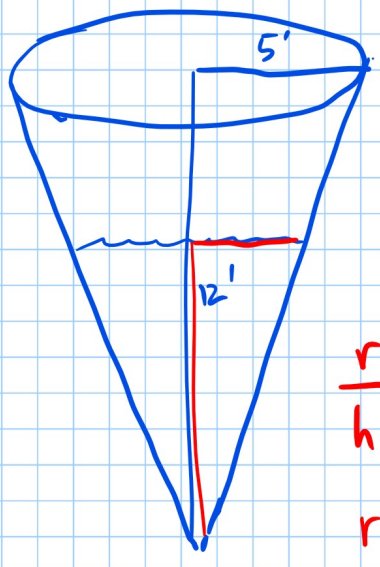
ex: A conical tank (with vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at the rate of $10\ ft^3/min$, find the rate of change of the depth of the water the instant it is 8 feet deep.

$\frac{dh}{dt}$ = rate of change of depth.



$\frac{r}{h} = \frac{5}{12}$

$r = \frac{5}{12}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$

$V = \frac{1}{3}\pi\frac{25}{144}h^2 \cdot h$

$V = \frac{25}{432}\pi h^3$

$\frac{dV}{dt} = \frac{25}{432}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{25}{144}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 10$

$h = 8$

$\frac{dh}{dt} = ?$

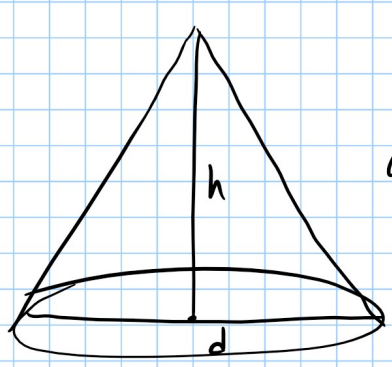
$10 = \frac{25}{144}\cdot\pi\cdot 8^2\frac{dh}{dt}$

$\frac{10}{1}\cdot\frac{144}{25}\cdot\frac{1}{\pi}\cdot\frac{1}{64} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{10\pi}\ ft/min \approx 0.29\ ft/min$

p158-159
11, 15, 19, 27, 28, 29

19) $\frac{dV}{dt} = 10\ ft^3/min$



$d = 3h$

$2r = 3h$

$r = \frac{3}{2}h$

$h = 15\ ft$

$\frac{dV}{dt} = 10\ ft^3/min$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \frac{3}{4} h^2 \cdot h$

$V = \frac{3}{4}\pi h^3$

$\frac{dV}{dt} = \frac{3}{4}\pi \cdot 3h^2 \frac{dh}{dt}$

$10 = \frac{3}{4}\pi \cdot 3 \cdot 15^2 \frac{dh}{dt}$

$\frac{2}{10} \cdot \frac{4}{3} \cdot \frac{1}{\pi} \cdot \frac{1}{3} \cdot \frac{1}{225_{45}} = \frac{dh}{dt}$

$10 = 1590.4 \frac{dh}{dt}$

$\frac{8}{405\pi} = \frac{dh}{dt}$

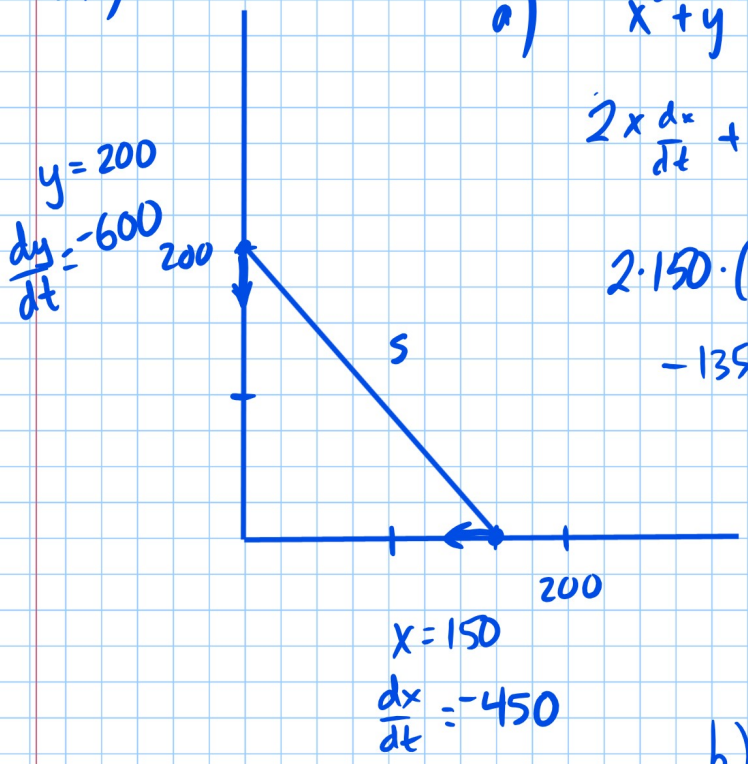
$\frac{dh}{dt} = .0063\ ft/min$

$\frac{dh}{dt} = 0.0063\ ft/min$

15) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

27)



a) $x^2 + y^2 = s^2 \Rightarrow 150^2 + 200^2 = s^2$

$s = 250$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$2 \cdot 150 \cdot (-450) + 2 \cdot 200(-600) = 2 \cdot 250\frac{ds}{dt}$

$-135000 - 240000 = 500\frac{ds}{dt}$

$-375000 = 500\frac{ds}{dt}$

$\frac{ds}{dt} = -750$ mph

b) $d = rt$

$250 = 750t$

$\frac{1}{3} = t$ → so 20 minutes