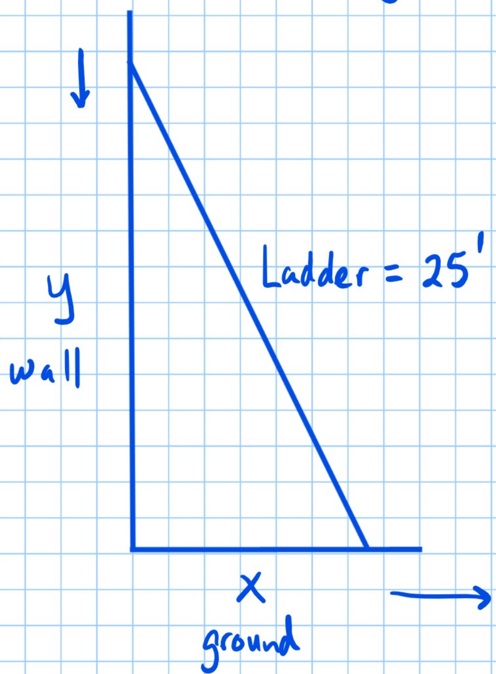
WARMUP

Read pages 153-154 in Larson book

RELATED RATES

When 2 or more variables are related by an equation and their rates vary with time, they are related rates.



x and y are changing as the ladder slides down the wall.

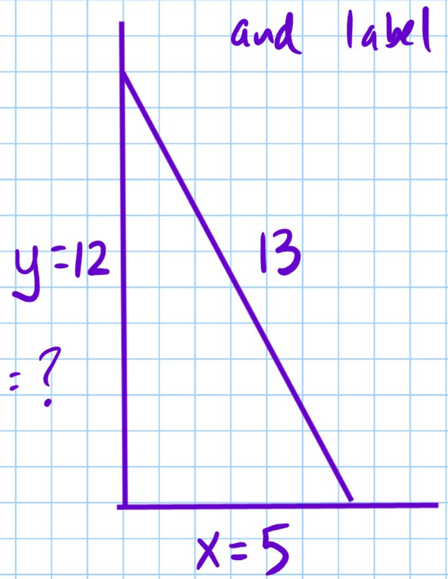
$\frac{dx}{dt}$ = rate of change of x

$\frac{dy}{dt}$ = rate of change of y

ex: A 13-foot ladder is leaning against a wall and sliding down the wall. If the foot of the ladder is pulled away from the wall at 1.5 ft/sec, how fast is the top of the ladder sliding down the wall when the top of the ladder is 12 feet above the ground?

STEP 1: Draw picture (if applicable)

and label it.



$\frac{dy}{dt} = ?$

$\frac{dx}{dt} = 1.5 \text{ ft/sec}$

STEP 2: Write an equation relating the variables.

$$x^2 + y^2 = 13^2$$

STEP 3: Write the related rate equation by using the derivative.

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

have to have these because x and y are functions of time

STEP 4: Sub in what we know and solve for what we do not know.

$y = 12$

$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

$x^2 = 25$

$x = 5$

$$2 \cdot 5 \cdot 1.5 + 2 \cdot 12 \cdot \frac{dy}{dt} = 0$$

$$15 + 24\frac{dy}{dt} = 0$$

$$24\frac{dy}{dt} = -15$$

$$\frac{dy}{dt} = \frac{-15}{24}$$

$$\frac{dy}{dt} = -\frac{5}{8}$$

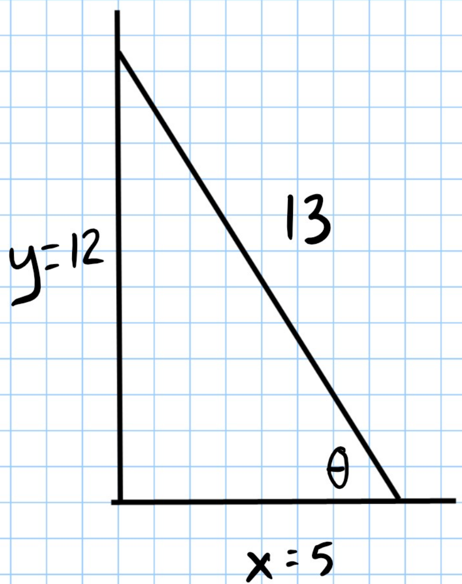
STEP 5: Answer the question

$-\frac{5}{8} \text{ ft/sec}$ ← negative because y is getting smaller ex: Same scenario. How fast is the angle formed by the ladder and the ground changing?

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$



$\tan\theta = \frac{y}{x}$ $\quad \sin\theta = \frac{y}{13} \quad \cos\theta = \frac{x}{13}$

quotient rule so more complicated

either one of these is less complicated.

$$\sin\theta = \frac{1}{13}y$$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{13}\cdot\frac{dy}{dt}$$

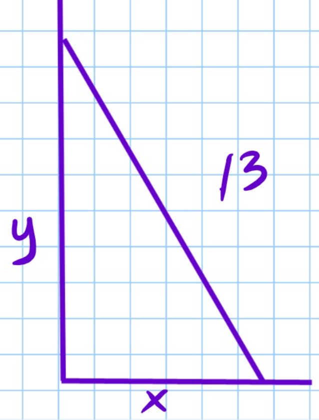
rate of change of angle

$\cos\theta = \frac{5}{13}$

$$\cancel{\frac{13}{5}} \cdot \cancel{\frac{5}{13}} \cdot \frac{d\theta}{dt} = \frac{1}{\cancel{13}}\cdot\left(-\frac{\cancel{5}}{8}\right)\cdot\frac{\cancel{13}}{\cancel{5}}$$

$$\frac{d\theta}{dt} = -\frac{1}{8} \text{ rad/sec}$$

ex: Same scenario. At what rate is the area of the Δ formed by the wall, the ground, and ladder changing?



$$A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}x\cdot\frac{dy}{dt} + y\cdot\frac{1}{2}\cdot\frac{dx}{dt}$$

$$\frac{dA}{dt} = \frac{1}{2}\cdot\frac{5}{1}\cdot\left(\frac{-5}{8}\right) + 12\cdot\frac{1}{2}\cdot 1.5$$

$$\frac{dA}{dt} = \frac{-25}{16} + 9$$

$$\frac{dA}{dt} = \frac{-25}{16} + \frac{144}{16} = \frac{119}{16} \text{ ft}^2/\text{sec}$$

p159 #23 just the 7-ft case