WARMUP

Calculate $f'(x)$ and $f''(x)$ for $f(x) = \dfrac{x^2}{e^x}$

$$f'(x) = \frac{e^x \cdot 2x - x^2 \cdot e^x}{(e^x)^2}$$

$$f'(x) = \frac{x\cancel{e^x}(2-x)}{(e^x)^{\cancel{2}}}$$

$$f'(x) = \frac{x(2-x)}{e^x} \quad \text{or} \quad \frac{2x-x^2}{e^x}$$

$$f''(x) = \frac{e^x(2-2x)-(2x-x^2)e^x}{(e^x)^2}$$

$$f''(x) = \frac{\cancel{e^x}(2-2x-2x+x^2)}{(e^x)^{\cancel{2}}}$$

$$f''(x) = \frac{x^2-4x+2}{e^x}$$

STEPS FOR GRAPHING

STEP 1: Determine intercepts using calculator

STEP 2: Determine asymptotes if the function is a fraction

V.A.s set denominator $= 0$

H.A.s find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

STEP 3: Determine extrema

- critters – domain values for which $f'(x)=0$ or $f'(x)$ is undefined.
- number line: include critters and VAs
- determine where max or min

STEP 4: Determine IPs:

- PIPs: domain values for which $f''(x)=0$ or $f''(x)$ is undefined.
- number line: include PIPs and VAs
- determine IPs.

STEP 5 : Draw a beautiful graph!

ex: $f(x) = \dfrac{x^2}{e^x}$

STEP 1 : intercepts $(0,0)$

STEP 2 : H.A. $y = 0$

STEP 3 : $f'(x) = \dfrac{x(2-x)}{e^x}$ (see warmup)

0   2

-1 1   3

$-$ $+$ $-$

relative min @ $(0, f(0)) = (0,0)$

relative max @ $(2, f(2)) = (2, 0.54)$

STEP 4 : $f''(x) = \dfrac{x^2 - 4x + 2}{e^x}$

PIPs: $x^2 - 4x + 2 = 0$

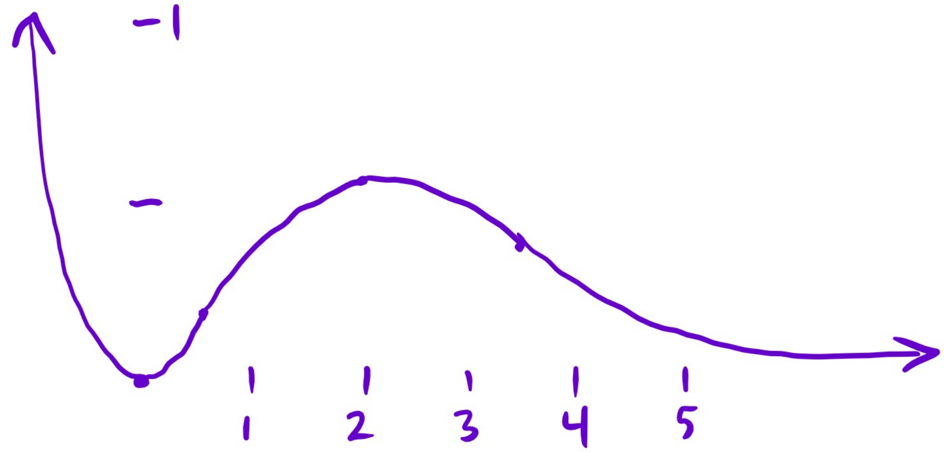
0   1   4

$+$ 0.59 $-$ 3.41 $+$

IPs @ $(0.59, f(0.59)) = (0.59, 0.19)$

@ $(3.41, f(3.41)) = (3.41, 0.38)$

STEP 5:



56 p195 $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$\left.\begin{array}{l} \text{Max: } (2,4) \\ \text{Min: } (4,2) \end{array}\right\} \begin{array}{ll} f(2) = 4 & f'(2) = 0 \\ f(4) = 2 & f'(4) = 0 \end{array}$

IP: $(3,3)$ $f(3) = 3$ $f''(3) = 0$

$\begin{cases} f'(2) = 12a + 4b + c = 0 \\ f'(4) = 48a + 8b + c = 0 \end{cases}$

$\left.\begin{array}{l} 8a + 4b + 2c + d = 4 \\ 64a + 16b + 4c + d = 2 \end{array}\right\}$ subtract $56a + 12b + 2c = -2$

$27a + 9b + 3c + d = 3$

$f''(3) = 18a + 2b = 0$

$2b = -18a$

$b = -9a$

$c = -12a - 4b$

$c = -12a + 36a = 24a$

$56a + 12(-9a) + 2(24a) = -2$

$56a - 108a + 48a = -2$

$-4a = -2$

$a = \frac{1}{2}$

$b = -9\left(\frac{1}{2}\right) = -\frac{9}{2}$

$c = 24\left(\frac{1}{2}\right) = 12$

$8\left(\frac{1}{2}\right) + 4\left(-\frac{9}{2}\right) + 2 \cdot 12 + d = 4$

$4 - 18 + 24 + d = 4$

$10 + d = 4$

$d = -6$

$f(x) = \frac{1}{2}x^3 - \frac{9}{2}x^2 + 12x - 6$

$f'(x) = \frac{3}{2}x^2 - 9x + 12 = 0 \Rightarrow x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2, \ x = 4$

$f''(x) = 3x - 9 = 0$

$x = 3$

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$f(x) = (x-a)(x-b)(x-c)$ a, b, c are zeros

$f(x) = (x-a)(x^2 - bx - cx + bc)$

$f(x) = x^3 - bx^2 - cx^2 + bcx - ax^2 + abx + acx - abc$

$f(x) = x^3 + (-a-b-c)x^2 + (ab+bc+ac)x - abc$

$f'(x) = 3x^2 + 2(-a-b-c)x + (ab+bc+ac)$

$f''(x) = 6x + 2(-a-b-c) = 0$

$6x = -2(-a-b-c)$

$x = \frac{-2(-a-b-c)}{6} = \frac{a+b+c}{3}$

PIP: $x = \frac{a+b+c}{3}$ $\Rightarrow$ avg of zeros