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$f(x) = \underline{a}x^3 + \underline{b}x^2 + cx + d$

min @ $(0,0)$ $\qquad$ $f'(x) = 3ax^2 + 2bx + c$

max @ $(2,2)$ $\qquad$ $f'(0) = c = 0$

$f'(2) = 12a + 4b = 0$

$f(0) = 0 \Rightarrow d = 0$

$$
\begin{array}{rl}
f(2) = 8a + \cancel{4b} & = 2 \\
12a + \cancel{4b} & = 0 \\
\hline
-4a & = 2
\end{array}
$$

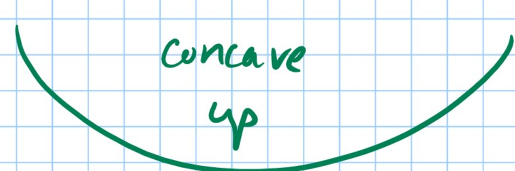
$$a = -\frac{1}{2}$$

$$
\begin{aligned}
8\left(-\tfrac{1}{2}\right) + 4b &= 2 \\
-4 + 4b &= 2 \\
4b &= 6 \\
b &= \frac{3}{2}
\end{aligned}
$$

## Section 3.4 Concavity and Points of Inflection

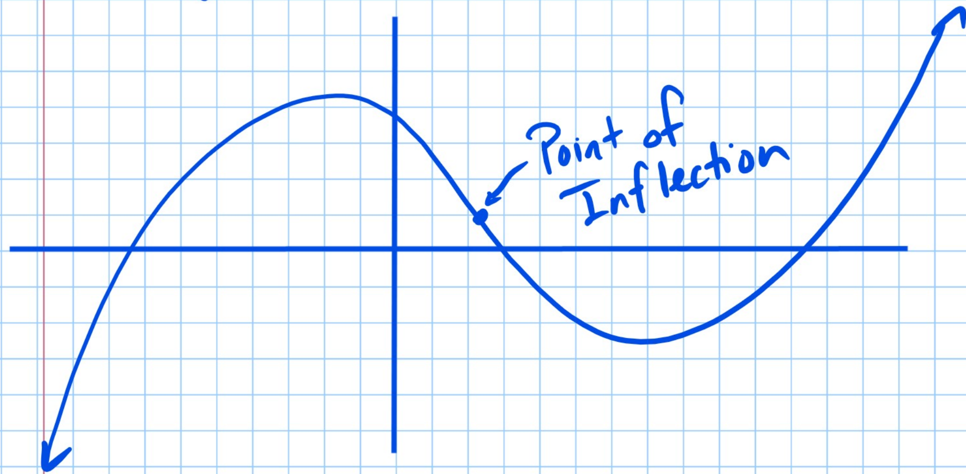
Recall:


$f'' < 0$




$f'' > 0$

A **point of inflection** is a point where the graph changes concavity.




To find points of inflection we start by finding possible inflection points (PIPs) by finding domain values for which $f''(x) = 0$ or $f''(x)$ is undefined. Then we use the same number line test that we used for the first derivative.

ex: Find extrema and points of inflection for

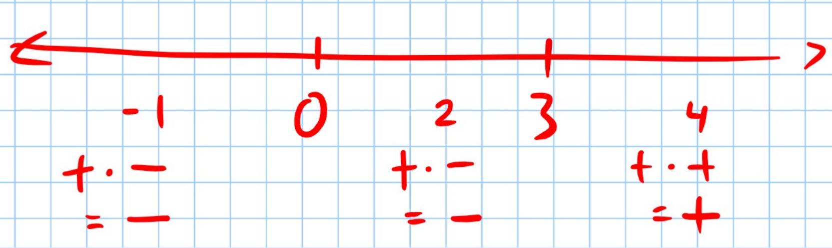
$$f(x) = x^4 - 4x^3$$

Extrema: $f'(x) = 4x^3 - 12x^2 = 0$

$$4x^2(x-3) = 0$$

$4x^2 = 0$ $\qquad$ $x - 3 = 0$

$x = 0$ $\qquad$ $x = 3$



relative min @ $(3, f(3)) = (3, -27)$

**IPs:** $f''(x) = 12x^2 - 24x$

$= 12x(x-2) = 0$

PIPs are $x=0$ $x=2$

0 2

-1 1 3

$- \cdot -$ $+ \cdot -$ $+ \cdot +$

$+$ $-$ $+$

IPs @ $(0, f(0)) = (0,0)$

@ $(2, f(2)) = (2, -16)$ $2^4 - 4 \cdot 2^3$

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23) $f(x) = x^3 - 6x^2 + 12x - 8$

$f'(x) = 3x^2 - 12x + 12$

$= 3(x^2 - 4x + 4)$

$= 3(x-2)(x-2)$

critter: $x = 2$

2

0 3

$- \cdot -$ $+ \cdot +$

$+$ $+$

no max or min

$f''(x) = 6x - 12 = 0$

$6x = 12$

$x = 2$

2

0 4

$-$ $+$

IP @ $(2, f(2))$

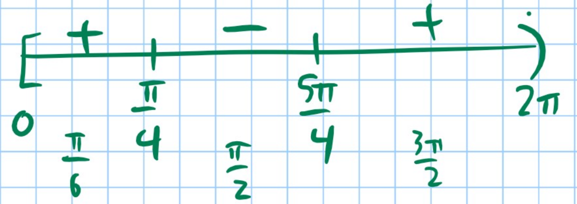
$= (2, 0)$

34) $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



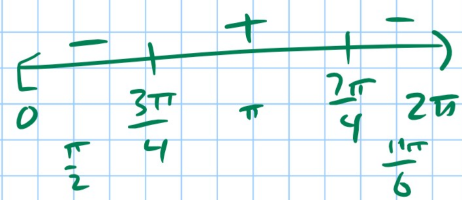

max @ $\left(\frac{\pi}{4}, 1.41\right)$

min @ $\left(\frac{5\pi}{4}, -1.41\right)$

$f''(x) = -\sin x - \cos x = 0$

$\sin x = -\cos x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$




IPs @ $\left(\frac{3\pi}{4}, 0\right)$

$\left(\frac{7\pi}{4}, 0\right)$