WARMUP

The John Deere Company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p that it charges. If the revenue R is

$$R(p) = -\frac{1}{2}p^2 + 1900p$$

What unit price should be charged to maximize revenue? What is the maximum revenue?

$$R'(p) = -p + 1900 = 0$$

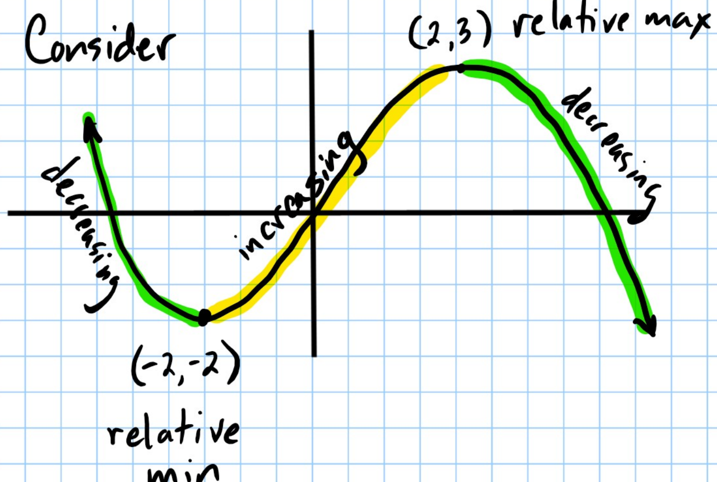
$$p = 1900$$

$$R(1900) = -\frac{1}{2}(1900)^2 + 1900 \cdot 1900$$

$$= \$1{,}805{,}000$$

Section 3.3 The First Derivative Test

Consider



When f' changes from positive to negative, we have a relative max

When f' changes from negative to positive, we have a relative min

The First Derivative Test

Let c be a critical number of f. [$f'(c)=0$ or where $f'(c)$ is undefined]

1) If f' changes from negative to positive at $x=c$ then $(c, f(c))$ is a <u>relative min</u>.

2) If f' changes from positive to negative at $x=c$ then $(c, f(c))$ is a <u>relative max</u>.

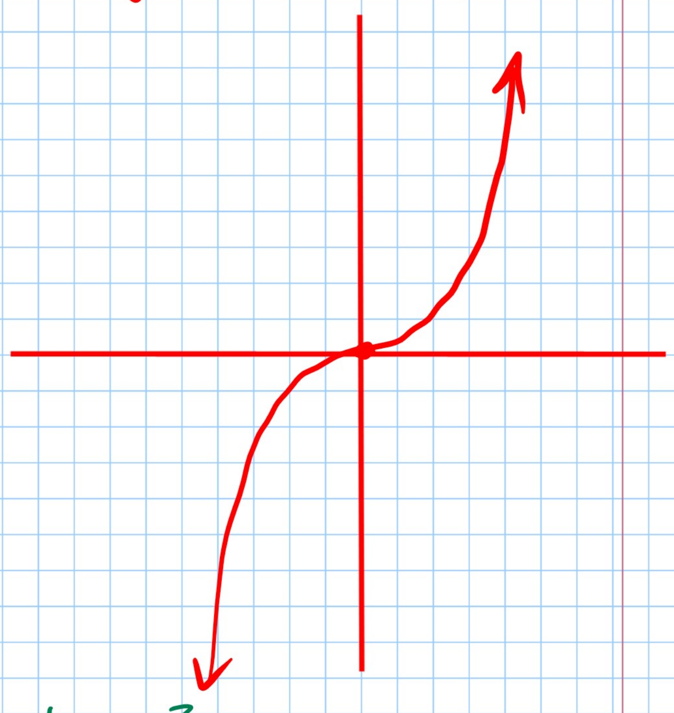
There are cases when a critter does not yield a max or min:

<u>ex</u>: $f(x)=x^3$

$$f'(x)=3x^2=0$$

$$x=0$$

is a critter



<u>ex</u>: Find the relative extrema of $f(x)=x^4-4x^3$

① Find critters:

$$f'(x)=4x^3-12x^2=0$$

$$4x^2(x-3)=0$$

$$4x^2=0 \qquad x-3=0$$

$$x=0 \qquad x=3$$

② Use a number line to determine signs of derivative on the intervals determined by the critters.

	0		3	
-1		2		4

Test Values →

$4(-1)^2(-1-3)$	$4\cdot 2^2(2-3)$	$4\cdot 4^2(4-3)$
$+\cdot+\cdot-$	$+\cdot+\cdot-$	$+\cdot+\cdot+$
$=-$	$=-$	$=+$
↘	↘	↗

Plug test values into the derivative to determine sign of f'

③ Use first derivative to determine maxima and minima

f' does not change sign at $x=0$, so $(0, f(0))$ is not a max or min, but there is a horizontal tangent.

f' changes from $-$ to $+$ at $x=3$ so there is a relative min at $(3, \underline{f(3)}) = (3, -27)$

$$f(3) = 3^4 - 4\cdot 3^3$$
$$= 81 - 108 = -27$$

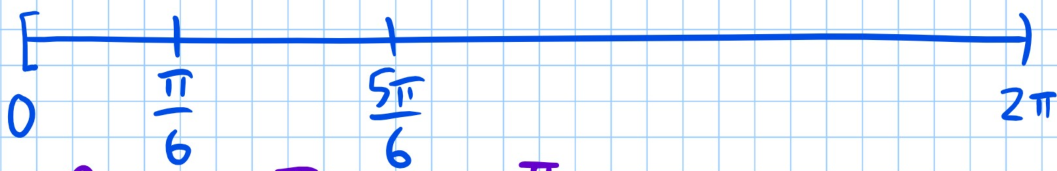
Given ex: $f(x) = \frac{1}{2}x + \cos x$ on $[0, 2\pi)$. find extrema

$$f'(x) = \frac{1}{2} - \sin x = 0$$

$$-\sin x = -\frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$



0	$\frac{\pi}{2}$	π
$\frac{1}{2} - \sin 0$	$\frac{1}{2} - \sin\frac{\pi}{2}$	$\frac{1}{2} - \sin\pi$
$\frac{1}{2}$	$\frac{1}{2} - 1$	$\frac{1}{2} - 0$
$+$	$-$	$+$

relative max at $\left(\frac{\pi}{6}, f\left(\frac{\pi}{6}\right)\right) = \left(\frac{\pi}{6}, 1.13\right)$

relative min at $\left(\frac{5\pi}{6}, f\left(\frac{5\pi}{6}\right)\right) = \left(\frac{5\pi}{6}, 0.44\right)$

p186-187 7, 11, 14, 15, 19, 30, 31, 48, 49, 55