WARMUP

Given $\sec\alpha = \frac{41}{9} = \frac{r}{x}$ with $\frac{3\pi}{2} < \alpha < 2\pi$,

calculate $\sin\alpha$ and $\cos\alpha$.

$x = 9$

$y = -40$

$r = 41$

$\sin\alpha = -\frac{40}{41}$

$\cos\alpha = \frac{9}{41}$

Section 6.4 Continued

Given $\tan\alpha = \frac{3}{4}$ ($3 \leftarrow y$, $4 \leftarrow x$) α in QI

For α $x = 4, y = 3, r = 5$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\sin\beta = \frac{7}{25}$ ($7 \leftarrow y$, $25 \leftarrow r$) QII $90^\circ < \beta < 180^\circ$

For β $x = -24, y = 7, r = 25$

$\sin\beta = \frac{7}{25}$

$\cos\beta = \frac{-24}{25}$

Find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{3}{5}\cdot\left(\frac{-24}{25}\right) + \frac{4}{5}\cdot\frac{7}{25} = \frac{-72}{125} + \frac{28}{125} = -\frac{44}{125} = \frac{y}{r}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \frac{4}{5}\left(\frac{-24}{25}\right) - \frac{3}{5}\cdot\frac{7}{25} = \frac{-96}{125} - \frac{21}{125} = \frac{-117}{125} = \frac{x}{r}$$

$x = -117, y = -44, r = 125$

$$\tan(\alpha+\beta) = \frac{y}{x} = \frac{-44}{-117} = \frac{44}{117}$$

What quadrant is $\alpha+\beta$ in? QIII

44, 117, 125 is a Pythagorean Triple

$44^2 + 117^2 = 125^2$

$1936 + 13689 = 15625$

Assignment

1) Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ and what quadrant $\alpha-\beta$ is in if: $\sin\alpha = \frac{3}{5}$ $0<\alpha<\frac{\pi}{2}$; $\cos\beta = \frac{40}{41}$ $\frac{3\pi}{2}<\beta<2\pi$

2) Find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$ and what quadrant $\alpha+\beta$ is in if:

$\sec\alpha = -\frac{25}{7}$ $\frac{\pi}{2}<\alpha<\pi$; $\cot\beta = \frac{12}{5}$ β in QI

3) Establish the identity: $1 - \frac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$