Section 6.4 Sum and Difference Formulas

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

ex: Exact value of $\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$$= \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

ex: $\tan\frac{\pi}{12} = \tan 15^\circ = \tan(60^\circ - 45^\circ)$

$$\frac{\cancel{\pi}}{12} \cdot \frac{180}{\cancel{\pi}} = 15^\circ \qquad = \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$$

$$= \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \cdot \frac{(\sqrt{3} - 1)}{\underbrace{(\sqrt{3} - 1)}_{\text{Conjugate of } \sqrt{3}+1}}$$

	$\sqrt{3}$	-1
$\sqrt{3}$	3	$-\sqrt{3}$
-1	$-\sqrt{3}$	1

	$\sqrt{3}$	1
$\sqrt{3}$	3	$\cancel{\sqrt{3}}$
-1	$\cancel{-\sqrt{3}}$	-1

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2 - \sqrt{3})}{\cancel{2}}$$

$$= 2 - \sqrt{3}$$

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4) $\tan\frac{7\pi}{12} = \tan 105^\circ = \tan(60^\circ + 45^\circ)$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1}$$

$$= \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$$

$$= \frac{4 + 2\sqrt{3}}{-2} = \frac{\cancel{-2}(-2-\sqrt{3})}{\cancel{-2}}$$

$$\boxed{= -2-\sqrt{3}}$$

9) $\sin\frac{17\pi}{12} = \sin 255^\circ = \sin(210^\circ + 45^\circ)$

$$= \sin 210^\circ \cos 45^\circ + \cos 210^\circ \sin 45^\circ$$

$$\frac{17\cancel{\pi}}{12} \cdot \frac{180}{\cancel{\pi}} = 255^\circ$$

$$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}$$

$$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$$

46) $\frac{\cos(\alpha-\beta)}{\sin\alpha \cos\beta} = \cot\alpha + \tan\beta$

$$\frac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta} = \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\cos\beta}$$

$$= \frac{\cos\alpha \cancel{\cos\beta}}{\sin\alpha \cancel{\cos\beta}} + \frac{\cancel{\sin\alpha}\sin\beta}{\cancel{\sin\alpha}\cos\beta}$$

$$= \cot\alpha + \tan\beta$$