WARMUP - Copy Into Your Notes

Section 6.3 Establishing Identities

QUOTIENT: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

RECIPROCAL: $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

PYTHAGOREAN: $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta + 1 = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1 \Rightarrow \sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta \Rightarrow \cot^2\theta = \csc^2\theta - 1 \Rightarrow \csc^2\theta - \cot^2\theta = 1$

An identity is an equation that is true for any number.

ex: $(x+1)^2 = x^2 + 2x + 1$

Establish the identity:

ex: $\sec\theta \cdot \sin\theta = \tan\theta$

To do these problems, rewrite the more complicated side and use our trig identities to transform the expression to what's on the other side.

$$\sec\theta \cdot \sin\theta = \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

ex: $\sin\theta \cdot \csc\theta - \cos^2\theta = \sin^2\theta$

$$\sin\theta \cdot \csc\theta - \cos^2\theta = \sin\theta \cdot \frac{1}{\sin\theta} - \cos^2\theta$$
$$= 1 - \cos^2\theta$$
$$= \sin^2\theta$$

p480 1, 5, 6, 9, 13, 15

Establish each identity

1) $\csc\theta \cdot \cos\theta = \cot\theta$

5) $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$

6) $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

9) $(\sec\theta - 1)(\sec\theta + 1) = \tan^2\theta$

13) $\cos^2\theta(\tan^2\theta + 1) = 1$

15) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

$$\frac{1}{\cos^2\theta} = \sec^2\theta$$

$$\frac{5}{9} \cdot \frac{4}{7} = \frac{20}{63} \qquad \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \frac{\sin\theta}{\cos\theta}$$

5) $\cos\theta(\tan\theta + \cot\theta) = \csc\theta$

$$\cos\theta(\tan\theta + \cot\theta) = \frac{\cos\theta}{1}\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$\frac{\cancel{\cos\theta}}{1} \cdot \frac{\sin\theta}{\cancel{\cos\theta}} + \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{1} + \frac{\cos^2\theta}{\sin\theta}$$

LCD $=\sin\theta$

$$= \frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta}$$

$$= \csc\theta$$

15) $(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2 = 2$

$$(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2$$

$$= (\sin\theta+\cos\theta)(\sin\theta+\cos\theta) + (\sin\theta-\cos\theta)(\sin\theta-\cos\theta)$$

$$= \sin^2\theta + \cancel{2\sin\theta}\cos\theta + \cos^2\theta + \sin^2\theta - \cancel{2\sin\theta}\cos\theta + \cos^2\theta$$

$$= \underbrace{\sin^2\theta + \cos^2\theta} + \underbrace{\sin^2\theta + \cos^2\theta}$$

$$= 1 + 1$$

$$= 2$$

	$\sin\theta$	$\cos\theta$
$\sin\theta$	$\sin^2\theta$	$\sin\theta\cos\theta$
$\cos\theta$	$\sin\theta\cos\theta$	$\cos^2\theta$

13) $\cos^2\theta(\tan^2\theta+1) = 1$

$$\cos^2\theta(\tan^2\theta+1) = \cos^2\theta\cdot\sec^2\theta$$

$$= \cos^2\theta\cdot\frac{1}{\cos^2\theta}$$

$$= 1$$