WARMUP Find $f'(x)$ if $f(x) = (3x^2+2)^2$

$$
\begin{aligned}
f(x) &= (3x^2+2)^2 \\
&= (3x^2+2)(3x^2+2) \\
&= 9x^4 + 12x^2 + 4 \\
f'(x) &= 36x^3 + 24x \\
&= 12x(3x^2+2)
\end{aligned}
$$

Section 3.4 The Chain Rule

Let's do the derivative of $f(x) = (3x^2+2)^2$ using the chain rule.

$$f(x) = (3x^2+2)^2$$

$$f'(x) = \underbrace{2(3x^2+2)^1}_{\text{POWER RULE}} \cdot \underbrace{6x}_{\text{deriv. of what's in the parentheses}}$$

$$f'(x) = 12x(3x^2+2)$$

ex: $f(x) = (3x^2+2)^{19}$

$$f'(x) = 19(3x^2+2)^{18} \cdot 6x$$

$$f'(x) = 114x(3x^2+2)^{18}$$

CHAIN RULE: $\dfrac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

BASIC

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[a^x\right] = a^x \ln a$$

CHAIN RULE

$$\frac{d}{dx}\left[(f(x))^n\right] = n\,(f(x))^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

$$\frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)} \cdot \ln a \cdot f'(x)$$

ex: $f(x) = \sqrt{x^4 + 1} = (x^4+1)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(x^4+1)^{-\frac{1}{2}} \cdot 4x^3$$

$$f'(x) = \frac{2x^3}{(x^4+1)^{1/2}} = \frac{2x^3}{\sqrt{x^4+1}}$$

ex: $f(x) = e^{2x}$

$$f'(x) = e^{2x} \cdot 2$$

$$f'(x) = 2e^{2x}$$

In general

$$\frac{d}{dx}\left[e^{kx}\right] = ke^{kx}$$

ex: $g(x) = 3^{7x^3 + 2x}$

$$g'(x) = 3^{7x^3+2x} \cdot \ln 3 \cdot (21x^2 + 2)$$

ex: $f(x) = \underline{x}\,\underline{e^{5-2x}}$

$$f'(x) = \underset{\text{1st}}{x} \cdot \underbrace{e^{5-2x} \cdot (-2)}_{\text{deriv of 2nd}} + \underset{\text{2nd}}{e^{5-2x}} \cdot \underset{\text{deriv of 1st}}{1}$$

$$f'(x) = e^{5-2x}(-2x+1)$$

$\boxed{\text{p126 1-45 eoo}}$

9) $f(x) = e^{\pi x} \Rightarrow f'(x) = e^{\pi x} \cdot \pi = \pi e^{\pi x}$

$$f'(x) = \pi e^{\pi x}$$

17) $g(t) = e^{(1+3t)^2}$

$$e^{1+6t+9t^2}$$

$$e^{1+6t+9t^2}(6+18t)$$

$$g'(x) = e^{(1+3t)^2} \cdot 2(1+3t)^1 \cdot 3$$

$$g'(x) = 6(1+3t)e^{(1+3t)^2}$$

21) $f(w) = e^{\frac{3w}{2}} = e^{\frac{3}{2}w}$ $\quad f'(w) = \frac{3}{2}e^{\frac{3}{2}w}$

25) $y = \underline{t}\,\underline{e^{-t^2}}$

$$\frac{d}{dt}\Big[\underbrace{e^{-t^2}}_{e^{f(t)}}\Big]$$

$$y' = t \cdot e^{-t^2}(-2t) + e^{-t^2} \cdot 1$$

$$\underbrace{e^{-t^2}}_{e^{f(t)}} \cdot \underbrace{(-2t)}_{f'(t)}$$

$$y' = e^{-t^2}\left(-2t^2 + 1\right)$$

33) $y = \dfrac{1}{e^{3x} + x^2}$

$$y' = \frac{\left(e^{3x} + x^2\right) \cdot 0 - 1\left(3e^{3x} + 2x\right)}{\left(e^{3x} + x^2\right)^2}$$

$$y' = \frac{-3e^{3x} - 2x}{\left(e^{3x} + x^2\right)^2}$$

41) $f(y) = \sqrt{10^{(5-y)}} = \left(10^{(5-y)}\right)^{\frac{1}{2}} = 10^{\frac{5}{2} - \frac{1}{2}y}$

$$f'(y) = 10^{\frac{5}{2} - \frac{1}{2}y} \cdot \ln 10 \cdot \left(-\tfrac{1}{2}\right)$$

$$f'(y) = \frac{-\sqrt{10^{(5-y)}} \cdot \ln 10}{2}$$