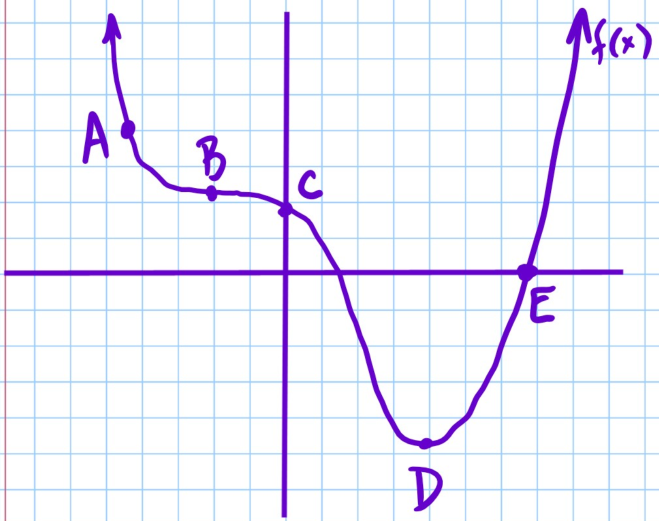
WARMUP

Fill in the table with +, −, or 0

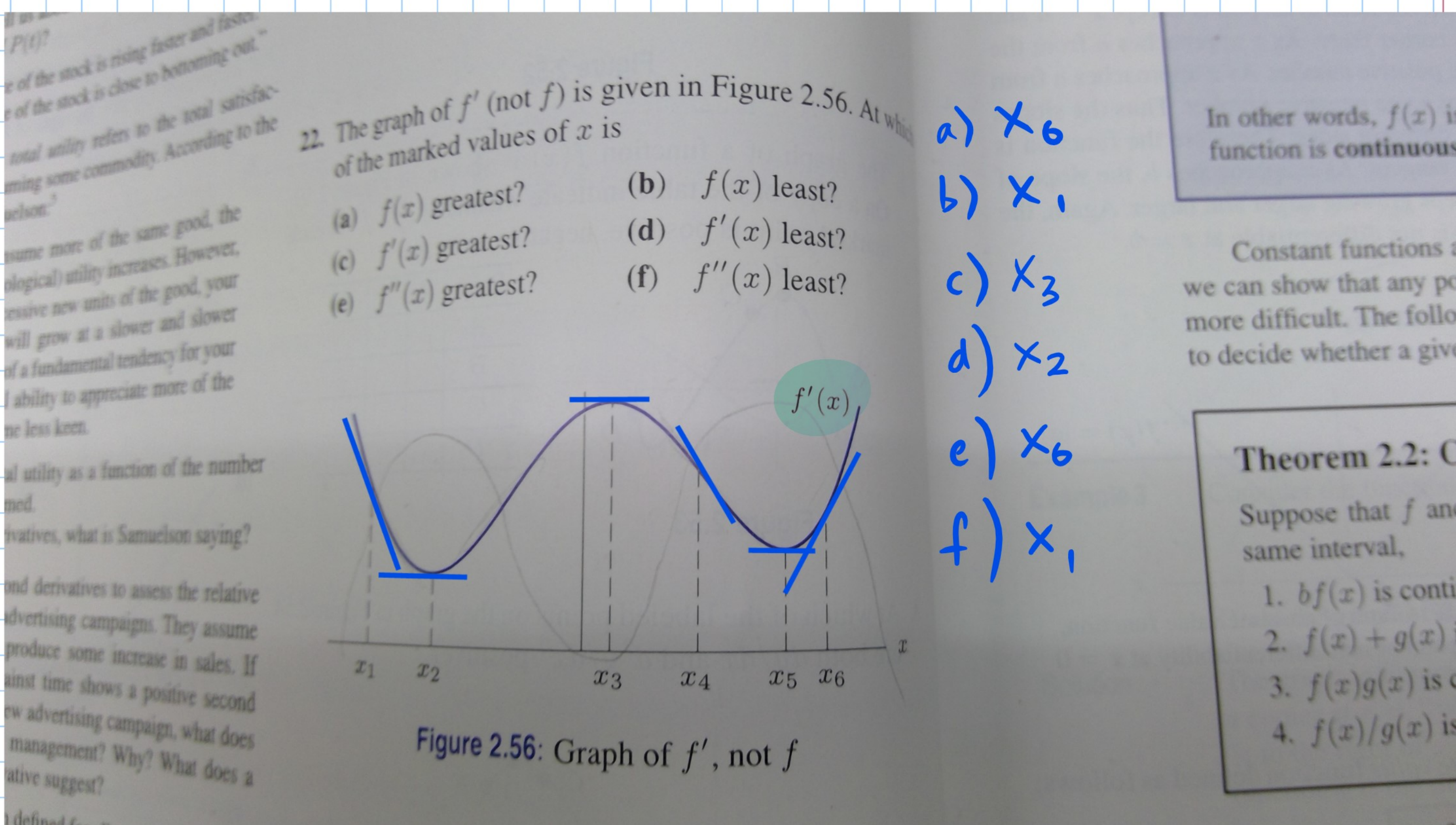


above x-axis +
below x-axis −
on x-axis 0

inc. +
dec. −
horiz tan 0

conc up +
conc. down −
change conc. 0

Point	f	f'	f''
A	+	−	+
B	+	0	0
C	+	−	−
D	−	0	+
E	0	+	+



22. The graph of f' (not f) is given in Figure 2.56. At which of the marked values of x is

(a) $f(x)$ greatest? (b) $f(x)$ least?
(c) $f'(x)$ greatest? (d) $f'(x)$ least?
(e) $f''(x)$ greatest? (f) $f''(x)$ least?

Figure 2.56: Graph of f', not f

In other words, $f(x)$ is ... function is **continuous** ...

Constant functions a... we can show that any pol... more difficult. The follow... to decide whether a give...

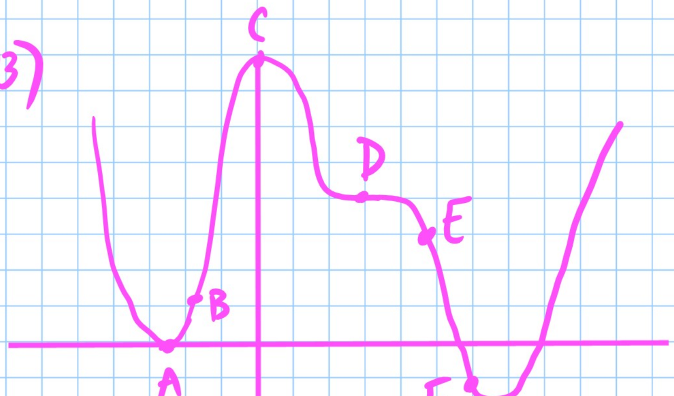
Theorem 2.2: C...

Suppose that f and ... same interval,

1. $bf(x)$ is contin...
2. $f(x) + g(x)$ i...
3. $f(x)g(x)$ is c...
4. $f(x)/g(x)$ is ...

a) x_6
b) x_1
c) x_3
d) x_2
e) x_6
f) x_1

23)



a) f', f'' nonzero same sign

B: $f'>0$, $f''>0$

E: $f'<0$, $f''<0$

b) A: $f=0$, $f'=0$

D: $f'=0$, $f''=0$

Section 2.7 Continuity

A function is continuous at $x = c$ if these conditions hold:

1) $f(c)$ is defined

2) $\lim_{x \to c} f(x)$ exists

3) $\lim_{x \to c} f(x) = f(c)$

ex: $f(x) = \begin{cases} 2x^2 - 3 & x < 1 \\ x - 2 & x > 1 \end{cases}$

is <u>not</u> continuous at $x = 1$ since $f(1)$ does not exist

ex: Is $f(x)$ continuous at $x = 1$?

$f(x) = \begin{cases} 2x^2 - 3 & x < 1 \\ x - 2 & x > 1 \\ 5 & x = 1 \end{cases}$

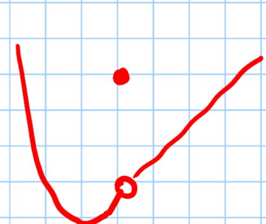
1) $f(1) = 5$ ✓

2) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$-1 = -1$

so $\lim_{x \to 1} f(x)$ exists

3) $\lim_{x \to 1} f(x) \neq f(1)$



ex: $f(x) = \begin{cases} 2x^2 - 3 & x < 1 \\ x - 2 & x \geq 1 \end{cases}$

1) $f(1) = 1 - 2 = -1$ ✓

2) $\lim_{x \to 1^-} f(x) = 2(1)^2 - 3 = -1$

$\lim_{x \to 1^+} f(x) = 1 - 2 = -1$

so $\lim_{x \to 1} f(x)$ exist ✓

3) $\lim_{x \to 1} f(x) = f(1)$

$-1 = -1$ ✓

$f(x)$ is continuous at $x = 1$

ex: $\lim\limits_{x\to 0} \dfrac{(7x+3)^2 - 9}{x}$

$= \lim\limits_{x\to 0} \dfrac{49x^2+42x\cancel{+9}\cancel{-9}}{x}$

$$\begin{array}{c|c|c} & 7x & 3 \\ \hline 7x & 49x^2 & 21x \\ \hline 3 & 21x & 9 \end{array}$$

$= \lim\limits_{x\to 0} \dfrac{\cancel{x}(49x+42)}{\cancel{x}}$

$= 49 \cdot 0 + 42$

$\boxed{= 42}$

ex: $\lim\limits_{x\to 7} \dfrac{x^2-8x+7}{x^2-49}$

$$\begin{array}{ccc} & 7 & \\ -1 & \times & -7 \\ & -8 & \end{array}$$

$= \lim\limits_{x\to 7} \dfrac{(x-1)\cancel{(x-7)}}{(x+7)\cancel{(x-7)}}$

$= \dfrac{7-1}{7+7} = \dfrac{6}{14} = \boxed{\dfrac{3}{7}}$

ex: $\lim\limits_{h\to 0} \dfrac{(\sqrt{7+h}-\sqrt{7})}{h} \dfrac{(\sqrt{7+h}+\sqrt{7})}{(\sqrt{7+h}+\sqrt{7})}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{7}+\cancel{h}^{\,1}-\cancel{7}}{\cancel{h}(\sqrt{7+h}+\sqrt{7})}$

$= \lim\limits_{h\to 0} \dfrac{1}{\sqrt{7+h}+\sqrt{7}}$

$= \dfrac{1}{\sqrt{7+0}+\sqrt{7}}$

$\boxed{= \dfrac{1}{2\sqrt{7}}}$ OR $\dfrac{\sqrt{7}}{14}$

ex: $\lim\limits_{h\to 0} \dfrac{\frac{6}{5+h} - \frac{6}{5}}{\frac{h}{1}}$

$= \lim\limits_{h\to 0} \dfrac{30-6(5+h)}{(5+h)5} \cdot \dfrac{1}{h}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{30}-\cancel{30}-6\cancel{h}}{(5+h)\cdot 5\cancel{h}}$

$= \lim\limits_{h\to 0} \dfrac{-6}{(5+h)5}$

$= \dfrac{-6}{(5+0)5} = \boxed{-\dfrac{6}{25}}$

ex: Calculate $f'(x)$ using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

if $f(x) = 7x^2 - x + 13$

$$f'(x) = \lim_{h \to 0} \frac{(7(x+h)^2 - (x+h) + 13) - (7x^2 - x + 13)}{h}$$

$$= \lim_{h \to 0} \frac{7(x^2 + 2xh + h^2) - x - h + 13 - 7x^2 + x - 13}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{7x^2} + 14xh + 7h^2 - \cancel{x} - h + \cancel{13} - \cancel{7x^2} + \cancel{x} - \cancel{13}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(14x + 7h - 1)}{\cancel{h}}$$

$$= 14x + 7 \cdot 0 - 1 = \boxed{14x - 1}$$

ex: Find the equation of the tangent line to the graph of $f(x) = 3x^4 - 7x^3 + 3$ when $x = -1$. You may use the shortcut.

SLOPE:

$f'(x) = 12x^3 - 21x^2$

$m = f'(-1) = 12(-1)^3 - 21(-1)^2$

$= 12(-1) - 21 \cdot 1$

$= -12 - 21$

$m = -33$

POINT:

$(-1, f(-1))$

$f(-1) = 3(-1)^4 - 7(-1)^3 + 3$

$= 3 \cdot 1 + 7 + 3$

$= 13$

$(-1, 13)$

EQ:

$y = mx + b$

$13 = -33(-1) + b$

$13 = 33 + b$

$-20 = b$

$y = -33x - 20$