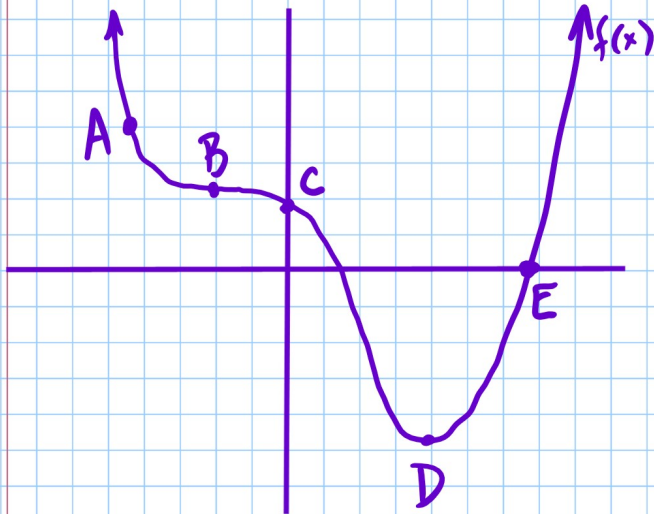


# WARMUP

Fill in the table with +, -, or 0



above x-axis +  
below x-axis -  
on x-axis 0

inc. +  
dec. -  
horiz. turn 0

conc. up +  
conc. down -  
change conc. 0

Point	$f$	$f'$	$f''$
A	+	-	+
B	+	0	0
C	+	-	-
D	-	0	+
E	0	+	+

total utility refers to the total satisfaction from some commodity. According to the Wilson?

assume more of the same good, the (logical) utility increases. However, successive new units of the good, your will grow at a slower and slower of a fundamental tendency for your ability to appreciate more of the or less keen.

al utility as a function of the number med.

derivatives, what is Samuelson saying?

and derivatives to assess the relative advertising campaigns. They assume produce some increase in sales. If ainst time shows a positive second ew advertising campaign, what does management? Why? What does a ative suggest?

defined for all

22. The graph of  $f'$  (not  $f$ ) is given in Figure 2.56. At which of the marked values of  $x$  is

- (a)  $f(x)$  greatest?
- (b)  $f(x)$  least?
- (c)  $f'(x)$  greatest?
- (d)  $f'(x)$  least?
- (e)  $f''(x)$  greatest?
- (f)  $f''(x)$  least?

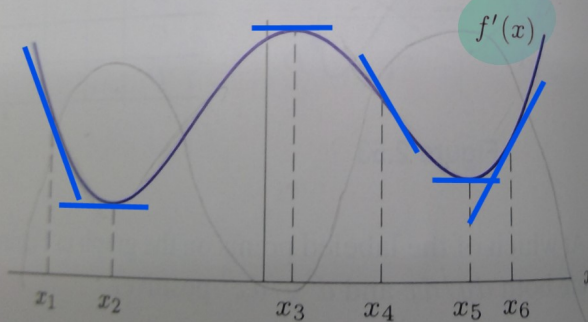


Figure 2.56: Graph of  $f'$ , not  $f$

- a)  $x_6$
- b)  $x_1$
- c)  $x_3$
- d)  $x_2$
- e)  $x_6$
- f)  $x_1$

In other words,  $f(x)$  is function is continuous

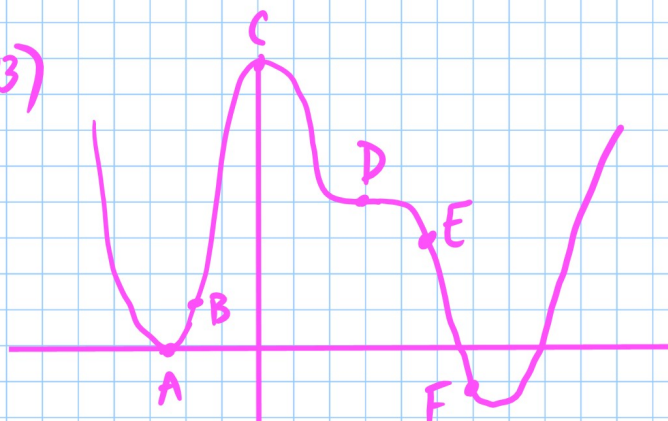
Constant functions a we can show that any pol more difficult. The follo to decide whether a give

## Theorem 2.2: C

Suppose that  $f$  and same interval,

1.  $bf(x)$  is contin
2.  $f(x) + g(x)$  i
3.  $f(x)g(x)$  is c
4.  $f(x)/g(x)$  is

23)



a)  $f', f''$  nonzero same sign

B:  $f' > 0, f'' > 0$

E:  $f' < 0, f'' < 0$

b) A:  $f = 0, f' = 0$

D:  $f' = 0, f'' = 0$

## Section 2.7 Continuity

A function is continuous at  $x = c$  if these conditions hold:

- 1)  $f(c)$  is defined
- 2)  $\lim_{x \rightarrow c} f(x)$  exists
- 3)  $\lim_{x \rightarrow c} f(x) = f(c)$

$$\text{ex: } f(x) = \begin{cases} 2x^2 - 3 & x < 1 \\ x - 2 & x > 1 \end{cases}$$

is not continuous at  $x = 1$   
since  $f(1)$  does not exist

ex: Is  $f(x)$  continuous at  $x = 1$ ?

$$f(x) = \begin{cases} 2x^2 - 3 & x < 1 \\ x - 2 & x > 1 \\ 5 & x = 1 \end{cases}$$

$$1) f(1) = 5 \quad \checkmark$$

$$2) \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$
$$-1 = -1$$

so  $\lim_{x \rightarrow 1} f(x)$  exists

$$3) \lim_{x \rightarrow 1} f(x) \neq f(1)$$

$$\text{ex: } f(x) = \begin{cases} 2x^2 - 3 & x < 1 \\ x - 2 & x \geq 1 \end{cases}$$

$$1) f(1) = 1 - 2 = -1 \quad \checkmark$$

$$2) \lim_{x \rightarrow 1^-} f(x) = 2(1)^2 - 3 = -1$$

$$\lim_{x \rightarrow 1^+} f(x) = 1 - 2 = -1$$

so  $\lim_{x \rightarrow 1} f(x)$  exist  $\checkmark$

$$3) \lim_{x \rightarrow 1} f(x) = f(1)$$

$$-1 = -1 \quad \checkmark$$

$f(x)$  is continuous at  
 $x = 1$





$$\text{ex: } \lim_{x \rightarrow 0} \frac{(7x+3)^2 - 9}{x}$$

$$= \lim_{x \rightarrow 0} \frac{49x^2 + 42x + 9 - 9}{x}$$

$7x$	$3$
$49x^2$	$21x$
$21x$	$9$

$$= \lim_{x \rightarrow 0} \frac{x(49x+42)}{x}$$

$$= 49 \cdot 0 + 42$$

$$\boxed{= 42}$$

$$\text{ex: } \lim_{x \rightarrow 7} \frac{x^2 - 8x + 7}{x^2 - 49}$$

$$= \lim_{x \rightarrow 7} \frac{(x-1)(x-7)}{(x+7)(x-7)}$$

$$\begin{array}{c} 7 \\ -1 \quad -7 \\ -8 \end{array}$$

$$= \frac{7-1}{7+7} = \frac{6}{14} = \boxed{\frac{3}{7}}$$

$$\text{ex: } \lim_{h \rightarrow 0} \frac{(\sqrt{7+h} - \sqrt{7})(\sqrt{7+h} + \sqrt{7})}{h(\sqrt{7+h} + \sqrt{7})}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{7+h} - 7}{\cancel{1}(\sqrt{7+h} + \sqrt{7})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{7+h} + \sqrt{7}}$$

$$= \frac{1}{\sqrt{7+0} + \sqrt{7}}$$

$$\boxed{= \frac{1}{2\sqrt{7}}} \text{ OR } \frac{\sqrt{7}}{14}$$

$$\text{ex: } \lim_{h \rightarrow 0} \frac{\frac{6}{5+h} - \frac{6}{5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{30 - 6(5+h)}{(5+h)5} \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{30} - \cancel{30} - 6h}{(5+h)5h}$$

$$= \lim_{h \rightarrow 0} \frac{-6}{(5+h)5}$$

$$= \frac{-6}{(5+0)5} = \boxed{-\frac{6}{25}}$$

ex: Calculate  $f'(x)$  using  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

if  $f(x) = 7x^2 - x + 13$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(7(x+h)^2 - (x+h) + 13) - (7x^2 - x + 13)}{h} \\ &= \lim_{h \rightarrow 0} \frac{7(x^2 + 2xh + h^2) - x - h + 13 - 7x^2 + x - 13}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{7x^2} + 14xh + \cancel{7h^2} - \cancel{x} - h + \cancel{13} - \cancel{7x^2} + \cancel{x} - \cancel{13}}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(14x + 7h - 1)}{h} \\ &= 14x + 7 \cdot 0 - 1 = \boxed{14x - 1} \end{aligned}$$

ex: Find the equation of the tangent line to the graph of  $f(x) = 3x^4 - 7x^3 + 3$  when  $x = -1$ . You may use the shortcut.

SLOPE:

$$\begin{aligned} f'(x) &= 12x^3 - 21x^2 \\ m = f'(-1) &= 12(-1)^3 - 21(-1)^2 \\ &= 12(-1) - 21 \cdot 1 \\ &= -12 - 21 \\ m &= -33 \end{aligned}$$

POINT:

$$\begin{aligned} &(-1, f(-1)) \\ f(-1) &= 3(-1)^4 - 7(-1)^3 + 3 \\ &= 3 \cdot 1 + 7 + 3 \\ &= 13 \\ &(-1, 13) \end{aligned}$$

EQ:

$$\begin{aligned} y &= mx + b \\ 13 &= -33(-1) + b \\ 13 &= 33 + b \\ -20 &= b \\ y &= -33x - 20 \end{aligned}$$