WARMUP

find $f'(7)$ if $f(x) = \frac{3}{x}$

Use $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$f'(7) = \lim_{h \to 0} \frac{\frac{3}{7+h} - \frac{3}{7}}{\frac{h}{1}} = \lim_{h \to 0} \frac{21 - 3(7+h)}{(7+h)7} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{21} - \cancel{21} - 3\cancel{h}}{(7+h)7\cancel{h}}$$

$$= \frac{-3}{(7+0)7} = -\frac{3}{49}$$

Section 2.6 The Second Derivative

The derivative of the derivative is the <u>second derivative</u>. It is notated by $f''(x)$

"f double prime"

$$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1} \qquad \frac{d}{dx}\left[cx\right] = c$$

$$\frac{d}{dx}\left[cx^n\right] = cnx^{n-1} \qquad \frac{d}{dx}\left[c\right] = 0$$

ex: $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

ex: $\frac{d}{dx}\left(6x^2 - 5x\right) = 12x - 5$

ex: $f(x) = \frac{7}{x} = 7x^{-1}$

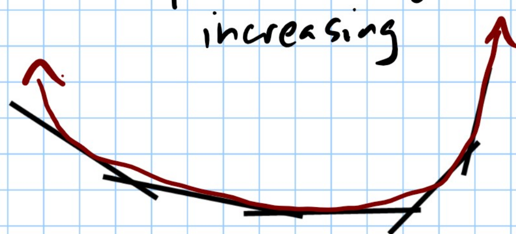
$f'(x) = -7x^{-2} = -\frac{7}{x^2}$

$f''(x) = 14x^{-3} = \frac{14}{x^3}$

When $f'(x) > 0$, $f(x)$ is increasing

so when $f''(x) > 0$, $\underbrace{f'(x)}_{\text{slopes of tangent lines are increasing}}$ is increasing

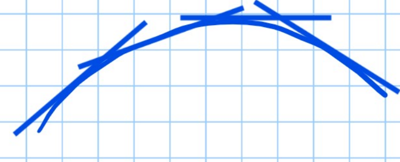
The graph of f is concave up when $f''(x) > 0$



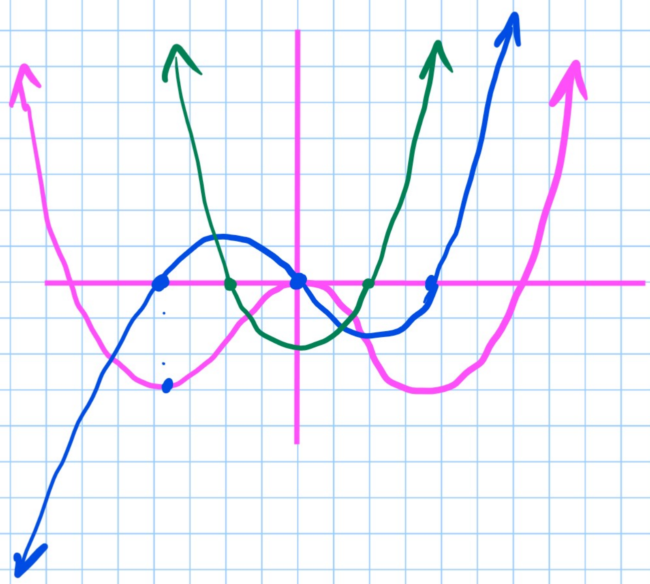
When $f'(x) < 0$, $f(x)$ is decreasing

so when $f''(x) < 0$, $f'(x)$ is decreasing

The graph of f is <u>concave down</u> if $f''(x) < 0$.



ex: Sketch $f'(x)$ and $f''(x)$ for $f(x)$:



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