

## WARMUP

Use the table to find:

1)  $h(4)$  if  $h(x) = f(g(x))$

$$h(4) = f(g(4)) = f(3) = 1$$

2)  $h'(4)$  if  $h(x) = f(g(x))$

$$h'(4) = f'(g(4)) \cdot g'(4) = f'(3) \cdot g'(4) = 2 \cdot 1 = 2$$

3)  $h(4)$  if  $h(x) = g(f(x))$

$$h(4) = g(f(4)) = g(4) = 3$$

4)  $h'(4)$  if  $h(x) = g(f(x))$

$$h'(4) = g'(f(4)) \cdot f'(4) = g'(4) \cdot f'(4) = 1 \cdot 3 = 3$$

5)  $h'(4)$  if  $h(x) = f(x) \cdot g(x)$

$$h'(4) = f(4) \cdot g'(4) + g(4) \cdot f'(4) = 4 \cdot 1 + 3 \cdot 3 = 13$$

6)  $h'(4)$  if  $h(x) = \frac{g(x)}{f(x)}$

$$h'(4) = \frac{f(4)g'(4) - g(4) \cdot f'(4)}{(f(4))^2} = \frac{4 \cdot 1 - 3 \cdot 3}{4^2} = \frac{-5}{16}$$

$x$	1	2	3	4
$f(x)$	3	2	1	4
$f'(x)$	1	4	2	3
$g(x)$	2	1	4	3
$g'(x)$	4	2	3	1

20)  $y = \frac{t^{\frac{1}{2}}}{t^2 + 1}$

$$3x^2y^3 + 7x^3y^2$$
$$x^2y^2$$

$$y' = \frac{(t^2 + 1) \cdot \frac{1}{2} t^{-\frac{1}{2}} - t^{\frac{1}{2}} \cdot 2t'}{(t^2 + 1)^2}$$

$$\frac{t^{\frac{1}{2}} \cdot 2t}{\frac{1}{2} t^{-\frac{1}{2}}}$$

$$y' = \frac{\frac{1}{2} t^{-\frac{1}{2}} [t^2 + 1 - 4t^2]}{(t^2 + 1)^2}$$

$$2t' \cdot 2t = 4t^2$$

$$y' = \frac{1 - 3t^2}{2\sqrt{t} (t^2 + 1)^2}$$

$$\frac{2\sqrt{t} \left[ (t^2+1) \cdot \frac{1}{2\sqrt{t}} - 2t^{\frac{3}{2}} \right]}{2\sqrt{t} (t^2+1)^2}$$

$$\frac{t^2+1-4t^2}{2\sqrt{t} (t^2+1)^2} = \frac{1-3t^2}{2\sqrt{t} (t^2+1)^2}$$

3.4  
52)  $f(x) = (2x+1)^{10} (3x-1)^7$

$$f'(x) = \underbrace{(2x+1)^{10}}_{1st} \cdot \underbrace{7(3x-1)^6 \cdot 3}_{der 2nd} + \underbrace{(3x-1)^7}_{2nd} \cdot \underbrace{10(2x+1)^9 \cdot 2}_{der 1st}$$

$$f'(x) = 21(2x+1)^{10} (3x-1)^6 + 20(3x-1)^7 (2x+1)^9$$

$$f'(x) = (2x+1)^9 (3x-1)^6 \left[ 21(2x+1) + 20(3x-1) \right]$$

$$f'(x) = \underbrace{(2x+1)^9 (3x-1)^6}_{1st} \underbrace{(102x+1)}_{2nd}$$

$$f''(x) = (2x+1)^9 (3x-1)^6 \cdot 102 + (102x+1) \left[ (2x+1)^9 \cdot 6(3x-1)^5 \cdot 3 + (3x-1)^6 \cdot 9(2x+1)^8 \cdot 2 \right]$$

$$= (2x+1)^9 (3x-1)^6 \cdot 102 + (102x+1) \cdot 18(2x+1)^8 (3x-1)^5 \left[ \overbrace{2x+1+3x-1}^{5x} \right]$$

$$= 102(2x+1)^9 (3x-1)^6 + 90x(102x+1)(2x+1)^8 (3x-1)^5$$

$$= 2(2x+1)^8 (3x-1)^5 \left[ 51(2x+1)(3x-1) + 45x(102x+1) \right]$$

$$= 2(2x+1)^8 (3x-1)^5 \left( \begin{matrix} 102x^2 + 17x - 17 + 1530x^2 + 15x \\ 306x^2 + 51x - 51 + 4590x^2 + 45x \end{matrix} \right)$$

$$= 6(2x+1)^8 (3x-1)^5 (1632x^2 + 32x - 17)$$



$$54) a) g(x) = \sqrt{f(x)} = (f(x))^{\frac{1}{2}}$$

$$g'(1) = \frac{1}{2}(f(1))^{-\frac{1}{2}} \cdot f'(1)$$

$$= \frac{1}{2} \cdot 4^{-\frac{1}{2}} \cdot 3 = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} \cdot 3 = \frac{3}{4}$$

$$b) h(x) = f(\sqrt{x})$$

$$h'(1) = f'(\sqrt{1}) \cdot \frac{1}{2} \cdot 1^{-\frac{1}{2}} = f'(1) \cdot \frac{1}{2 \cdot \sqrt{1}}$$

$$= 4 \cdot \frac{1}{2} = 2$$