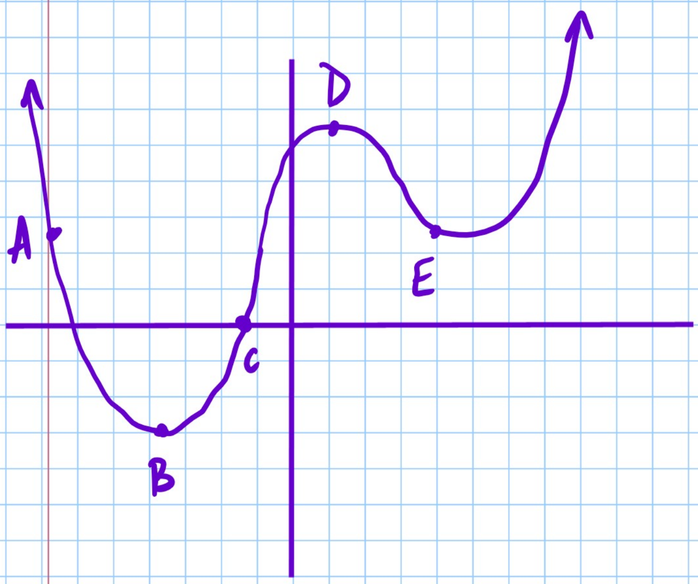
WARMUP (Like Study Guide #6 - you can put it on your notecard instead of warmup sheet)



Point	$f(x)$	$f'(x)$	$f''(x)$
A	+	—	+
B	—	0	+
C	0	+	+ OR 0
D	+	0	—
E	+	—	+

1) $f(x) = \begin{cases} 3x-4 & x < -1 \\ x^2-8 & x > -1 \end{cases}$

a) $\lim\limits_{x \to -1^-} f(x) = 3(-1)-4 = -7$

$\lim\limits_{x \to -1^+} f(x) = (-1)^2-8 = -7$

$\lim\limits_{x \to -1} f(x) = -7$

2) i) $f'(16) = \lim\limits_{h \to 0} \dfrac{f(16+h) - f(16)}{h}$

$= \lim\limits_{h \to 0} \dfrac{(\sqrt{16+h} - \sqrt{16})}{h} \dfrac{(\sqrt{16+h} + \sqrt{16})}{(\sqrt{16+h} + \sqrt{16})}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{16}+\cancel{h}^{\,1} - \cancel{16}}{\cancel{h}(\sqrt{16+h} + 4)}$

$= \dfrac{1}{\sqrt{16+0} + 4} = \dfrac{1}{4+4} = \boxed{\dfrac{1}{8}}$

ii) $$f'(3)=\lim_{h\to 0}\frac{\frac{7}{3+h}-\frac{7}{3}}{h}$$

$$=\lim_{h\to 0}\frac{21-7(3+h)}{(3+h)3}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{21}-\cancel{21}-7\cancel{h}}{(3+h)3\cancel{h}}$$

$$=\lim_{h\to 0}\frac{-7}{(3+h)3}$$

$$=\frac{-7}{(3+0)3}=\boxed{-\frac{7}{9}}$$

iii) $$f'(-2)=\lim_{h\to 0}\frac{\left(3(-2+h)^2-(-2+h)\right)-\left(\overbrace{3(-2)^2-(-2)}^{3\cdot 4+2}\right)}{h}$$

	-2	h
-2	4	$-2h$
h	$-2h$	h^2

$$=\lim_{h\to 0}\frac{3(4-4h+h^2)+2-h-14}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{12}-12h+3h^2+\cancel{2}-h-\cancel{14}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(-12+3h-1)}{\cancel{h}}$$

$$=-12+3\cdot 0-1\ \boxed{=-13}$$

3) i) $$f'(x)=\lim_{h\to 0}\frac{\left(7(x+h)-(x+h)^2\right)-\left(7x-x^2\right)}{h}$$

$$=\lim_{h\to 0}\frac{7x+7h-(x^2+2xh+h^2)-7x+x^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{7x}+7h-\cancel{x^2}-2xh-h^2-\cancel{7x}+\cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(7 - 2x - h)}{\cancel{h}}$$

$$= 7 - 2x - 0$$

$$\boxed{= 7 - 2x}$$

ii) $f(x) = 8\sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\left(8\sqrt{x+h} - 8\sqrt{x}\right)}{h} \frac{\left(8\sqrt{x+h} + 8\sqrt{x}\right)}{\left(8\sqrt{x+h} + 8\sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{64(x+h) - 64x}{h(8\sqrt{x+h} + 8\sqrt{x})}$$

Check $f(x) = 8x^{\frac{1}{2}}$

$f'(x) = 4x^{-\frac{1}{2}}$

$= \frac{4}{x^{\frac{1}{2}}} = \frac{4}{\sqrt{x}}$

$$= \lim_{h \to 0} \frac{\cancel{64x} + 64\cancel{h} - \cancel{64x}}{\cancel{h}(8\sqrt{x+h} + 8\sqrt{x})}$$

$$= \frac{64}{8\sqrt{x+0} + 8\sqrt{x}} = \frac{64}{16\sqrt{x}} = \boxed{\frac{4}{\sqrt{x}}}$$

iii) $f'(x) = \lim_{h \to 0} \dfrac{\frac{5}{7(x+h)} - \frac{5}{7x}}{h}$

$$= \lim_{h \to 0} \frac{35x - 35(x+h)}{7(x+h) \cdot 7x} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{35x} - \cancel{35x} - 35\cancel{h}}{7(x+h) \cdot 7x\cancel{h}}$$

$$= \frac{-35}{7(x+0) \cdot 7x} = \frac{-35}{49x^2} = \boxed{-\frac{5}{7x^2}}$$

4) $f(x) = 8x^3 - 10x^2 + 5$

SLOPE:

$f'(x) = 24x^2 - 20x$

$m = f'(-2) = 24(-2)^2 - 20(-2)$

$= 24 \cdot 4 + 40$

$= 96 + 40$

$m = 136$

POINT:

$(-2, f(-2))$

$f(-2) = 8(-2)^3 - 10(-2)^2 + 5$

$= 8(-8) - 10 \cdot 4 + 5$

$= -64 - 40 + 5$

$= -99$

$(-2, -99)$

EQ:

$-99 = 136(-2) + b$

$-99 = -272 + b$

$173 = b$

$\boxed{y = 136x + 173}$

5) i) $\lim_{h \to 0} \dfrac{\frac{7}{3+h} - \frac{7}{3}}{h} = \lim_{h \to 0} \dfrac{21 - 7(3+h)}{(3+h)3} \cdot \dfrac{1}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{21} - \cancel{21} - 7\cancel{h}}{(3+h)3\cancel{h}}$

$= \dfrac{-7}{(3+0)3} = \boxed{-\dfrac{7}{9}}$

ii) $\lim_{x \to 8} \dfrac{x-8}{x^2 - 12x + 32} = \lim_{x \to 8} \dfrac{\cancel{x-8}^{\,1}}{(x-4)\cancel{(x-8)}} = \dfrac{1}{8-4} = \boxed{\dfrac{1}{4}}$