WARMUP

1) $\lim_{h\to 0} \frac{(\sqrt{100+h}-10)}{h} \cdot \frac{(\sqrt{100+h}+10)}{(\sqrt{100+h}+10)}$

$\lim_{h\to 0} \frac{\cancel{(100}+h) + \cancel{10\sqrt{100+h}} - \cancel{10\sqrt{100+h}} - \cancel{100}}{h(\sqrt{100+h}+10)}$

$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{100+h}+10)} = \frac{1}{\sqrt{100}+10} = \frac{1}{10+10} = \boxed{\frac{1}{20}}$

2) $\lim_{h\to 0} \frac{\frac{3}{\sqrt{16+h}} - \frac{3}{4}}{h} = \lim_{h\to 0} \frac{12-3\sqrt{16+h}}{\sqrt{16+h}\cdot 4} \cdot \frac{1}{h}$

$= \lim_{h\to 0} \frac{(12-3\sqrt{16+h})}{4h\sqrt{16+h}} \frac{(12+3\sqrt{16+h})}{(12+3\sqrt{16+h})}$

$= \lim_{h\to 0} \frac{144-9(16+h)}{4h\sqrt{16+h}\,(12+3\sqrt{16+h})} = \lim_{h\to 0} \frac{\cancel{144}-\cancel{144}-9\cancel{h}}{4\cancel{h}\sqrt{16+\underset{0}{h}}\,(12+3\sqrt{16+\underset{0}{h}})}$

$= \frac{-9}{4\sqrt{16}\,(12+3\sqrt{16})} = \frac{-9}{4\cdot 4(12+3\cdot 4)} = \frac{\overset{-3}{\cancel{-9}}}{4\cdot 4\cdot \underset{8}{\cancel{24}}}$

$= \frac{-3}{128}$

Section 2.3 The Derivative at a Point

Avg. rate of change on $[a, a+h]$ $= \dfrac{f(a+h)-f(a)}{h}$

The <u>derivative of f at a</u>, written $f'(a)$ "f prime of a" is defined as

Rate of change of f at a = Slope of the tangent line at $(a, f(a))$ $= f'(a) = \lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

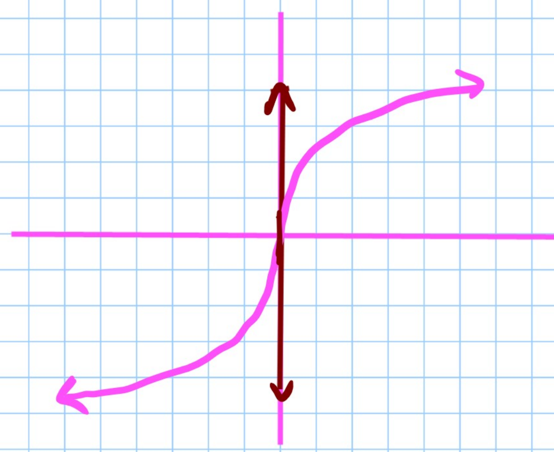
★ Rate of Change $\Longleftrightarrow$ Slope of Tangent Line $\Longleftrightarrow$ Derivative

When you are asked to calculate $f'(a)$ using <u>definition of derivative</u> you must use

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

If $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ exists, we say f is <u>differentiable</u> at $x=a$. The process of calculating the derivative is called <u>differentiation</u>.

ex: $f(x) = x^{\frac{1}{3}} = \sqrt[3]{x}$



Not differentiable at $x=0$ since there's a vertical tangent so slope is undefined.

ex: Calculate $f'(-2)$ if $f(x) = 2x^2+3x-4$ using definition of derivative.

$$f'(-2) = \lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h}$$

$$= \lim_{h\to 0} \frac{\overbrace{\left(2(-2+h)^2+3(-2+h)-4\right)}^{f(-2+h)} - \overbrace{\left(2(-2)^2+3(-2)-4\right)}^{f(-2)}}{h}$$

$$= \lim_{h\to 0} \frac{2(4-4h+h^2)-6+3h-4-(\overbrace{2\cdot 4-6-4}^{-2})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{8}-8h+2h^2\cancel{-6}+3h\cancel{-4}\cancel{-(-2)}}{h}$$

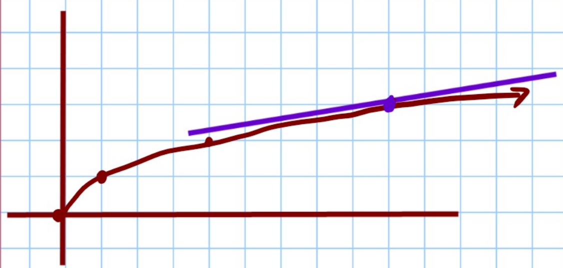
$$= \lim_{h\to 0} \frac{2h^2-5h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2h-5)}{\cancel{h}}$$

$$= 2\cdot 0-5 = \boxed{-5}$$

ex: Find equation of the tangent line to the graph of $f(x) = \sqrt{x}$ when $x = 9$

1) SLOPE using $f'(a)$
2) POINT using $(a, f(a))$
3) EQ using $y = mx + b$



1) $m = f'(9) = \lim_{h \to 0} \dfrac{f(9+h) - f(9)}{h}$

$= \lim_{h \to 0} \dfrac{(\sqrt{9+h} - 3)(\sqrt{9+h} + 3)}{h\,(\sqrt{9+h} + 3)}$

$= \lim_{h \to 0} \dfrac{\cancel{9} + \cancel{h} - \cancel{9}}{\cancel{h}(\sqrt{9+h} + 3)}$

$= \dfrac{1}{\sqrt{9+0} + 3} = \dfrac{1}{3+3} = \dfrac{1}{6}$

2) Point: $(9, f(9)) = (9, 3)$

3) $3 = \dfrac{1}{\cancel{6}_2} \cdot \cancel{9}^{3} + b$

$3 = \dfrac{3}{2} + b$

$\dfrac{6}{2} - \dfrac{3}{2} = b$

$b = \dfrac{3}{2}$

$\boxed{y = \dfrac{1}{6}x + \dfrac{3}{2}}$

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7, 9-14, 16, 17,
20, 21