Section 2.2 Properties of Limits

Theorem 1.2 Properties of Limits

1) If b is a constant, $\lim\limits_{x \to c} (b f(x)) = b\left(\lim\limits_{x \to c} f(x)\right)$

2) $\lim\limits_{x \to c} (f(x) \pm g(x)) = \lim\limits_{x \to c} f(x) \pm \lim\limits_{x \to c} g(x)$

3) $\lim\limits_{x \to c} (f(x) \cdot g(x)) = \lim\limits_{x \to c} f(x) \cdot \lim\limits_{x \to c} g(x)$

4) $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)}$

5) $\lim\limits_{x \to c} k = k$

6) $\lim\limits_{x \to c} x = c$

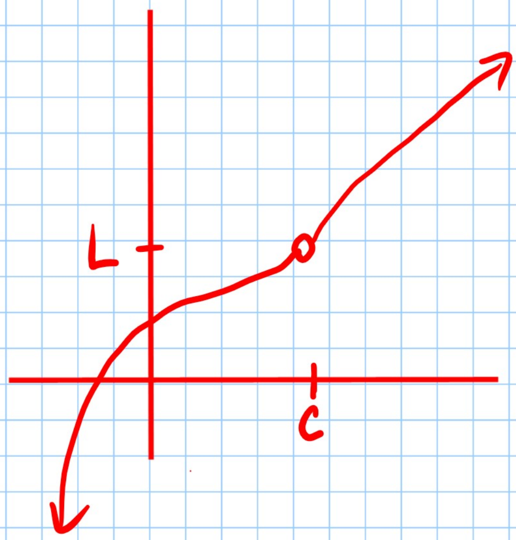
ex: $\lim\limits_{x \to 4} \dfrac{x^2 - 3}{2x+1} = \dfrac{\lim\limits_{x \to 4} (x^2 - 3)}{\lim\limits_{x \to 4} (2x+1)} = \dfrac{\lim\limits_{x \to 4} x^2 - \lim\limits_{x \to 4} 3}{\lim\limits_{x \to 4} 2x + \lim\limits_{x \to 4} 1}$

$$= \frac{\lim\limits_{x \to 4} x \cdot \lim\limits_{x \to 4} x - 3}{2\lim\limits_{x \to 4} x + 1}$$

$$= \frac{4 \cdot 4 - 3}{2 \cdot 4 + 1}$$

$$= \frac{13}{9}$$

Consider the graph of $f(x)$:



We say $\lim_{x \to c} f(x) = L$

limit as x approaches c of $f(x)$ is L.

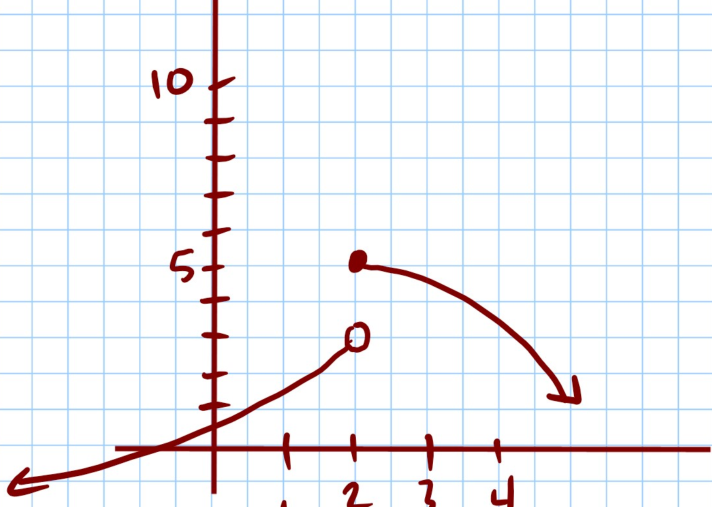
As x gets close to c, y gets close to L.

Note that $f(c) \neq L$ since there's a hole in the graph

We can use a table to evaluate a limit.

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Left-hand and Right-hand Limits



Left-hand limit

$\lim_{x \to 2^-} f(x) = 3$

limit as x approaches 2 from the left.

Right-hand limit

$\lim_{x \to 2^+} f(x) = 5$

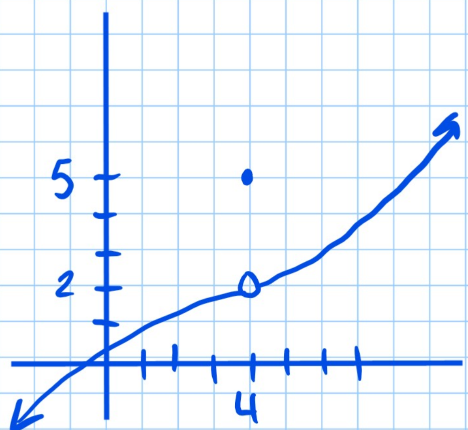
limit as x approaches 2 from the right.

In this case $\lim\limits_{x\to 2^-} f(x) \neq \lim\limits_{x\to 2^+} f(x)$

we say $\underbrace{\lim\limits_{x\to 2} f(x)}$ does not exist

2-sided limit $\Rightarrow$ to exist, left-hand and right-hand limit must exist and be equal.

ex:



$\lim\limits_{x\to 4^-} f(x) = 2$

$\lim\limits_{x\to 4^+} f(x) = 2$

$\lim\limits_{x\to 4} f(x) = 2$

$f(4) = 5$

ex: $\lim\limits_{x\to 5} \dfrac{x^2-16}{x+4} = \dfrac{5^2-16}{5+4} = \dfrac{25-16}{9} = \dfrac{9}{9} = 1$

ex: $\lim\limits_{x\to -4} \dfrac{x^2-16}{x+4} = \lim\limits_{x\to -4} \dfrac{(x-4)\cancel{(x+4)}}{\cancel{x+4}} = \lim\limits_{x\to -4}(x-4)$

$= -4-4 = -8$

$$\frac{5}{8} - \frac{2}{7} = \frac{5\cdot 7 - 2\cdot 8}{8\cdot 7} = \frac{35-16}{56} = \frac{19}{56}$$

ex: $$\lim_{h\to 0} \frac{\frac{5}{4+h} - \frac{5}{4}}{h} = \lim_{h\to 0} \frac{\frac{20-5(4+h)}{(4+h)4}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{20-20-5h}{(4+h)4} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-5\cancel{h}}{(4+h)4\cancel{h}}$$

$$= \frac{-5}{(4+0)4}$$

$$= \frac{-5}{16}$$

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1, 2, 15-20, 22, 37, 38