Section 2.1 How Do We Measure Speed?

Consider the table which shows the height of a grapefruit at t seconds.

t (sec)	0	1	2	3	4	5	6
$s(t)=y$ (feet)	6	90	142	162	150	106	30

If $s(t)$ is the position of an object at time t, then the <u>average velocity</u> from $t=a$ to $t=b$ is

$$\frac{s(b)-s(a)}{b-a} = \frac{\text{change in position}}{\text{change in time}}$$

ex: Find the avg. velocity for $4 \le t \le 5$

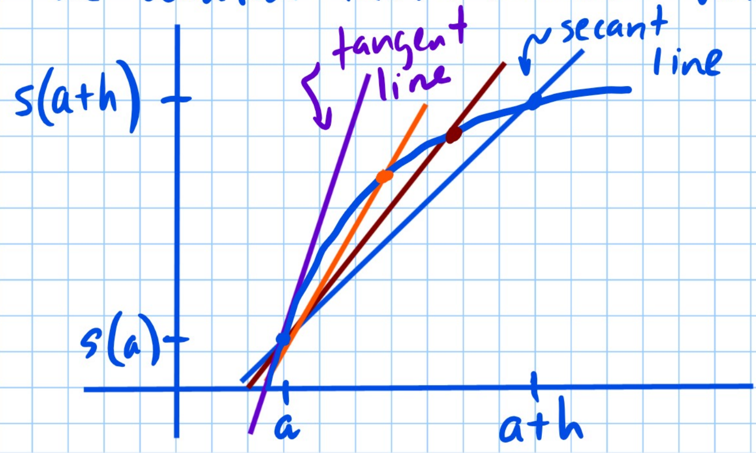
$$\frac{s(5)-s(4)}{5-4} = \frac{106-150}{1} = -44 \text{ ft/sec}$$

going down

ex: Find the avg. velocity for $1 \le t \le 3$

$$\frac{s(3)-s(1)}{3-1} = \frac{162-90}{2} = \frac{72}{2} = 36 \text{ ft/sec}$$

What if we wanted instantaneous velocity at $t=a$?



avg. velocity from $t=a$ to $t=a+h$

$$\frac{s(a+h)-s(a)}{a+h-a}$$

$$= \frac{s(a+h)-s(a)}{h}$$

slope of the secant line

If we let a+h approach a, the average velocity gets closer to becoming the instantaneous velocity

We can say therefore that

$$\text{instantaneous velocity at } t=a = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h}$$

As $h \to 0$, the secant line joining $(a, s(a))$ and $(a+h, s(a+h))$ approaches the tangent line at $(a, s(a))$. So instantaneous velocity is the <u>slope of the tangent line</u>.

Slope of tan line at $x=a$ means the same as the slope of the curve at $x=a$.

<u>ex 2 p61</u> $s = 3t^2$

a) $s=1$ to $s=1+h$ when $h=0.1$

$$\frac{s(\underbrace{1.1}_{\substack{1+h\\1+0.1}}) - s(1)}{1.1 - 1} = \frac{3\cdot 1.1^2 - 3\cdot 1^2}{0.1} = \frac{3.63-3}{0.1} = 6.3 \text{ m/sec}$$

b) $s=1$ to $s=1+h$ when $h=0.01$

$$\frac{s(1.01)-s(1)}{1.01-1} = \frac{3\cdot 1.01^2 - 3\cdot 1^2}{0.01} = \frac{3.0603-3}{0.01} = 6.03 \text{ m/sec}$$

We predict inst. velocity is 6 m/sec when $t = 1$ sec.

$$\text{Inst. velocity at } t=1 = \lim_{h \to 0} \frac{s(1+h) - s(1)}{h}$$

$$= \lim_{h \to 0} \frac{\overbrace{3(1+h)^2}^{s(1+h)} - \overbrace{3 \cdot 1^2}^{s(1)}}{h}$$

$$= \lim_{h \to 0} \frac{3(1+2h+h^2) - 3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3} + 6h + 3h^2 - \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6+3h)}{\cancel{h}}$$

$$= 6 + 3 \cdot 0$$

$$= 6$$

$\boxed{\text{p61-62} \quad 1, 3, 4, 5, 7, 13\text{-}16}$