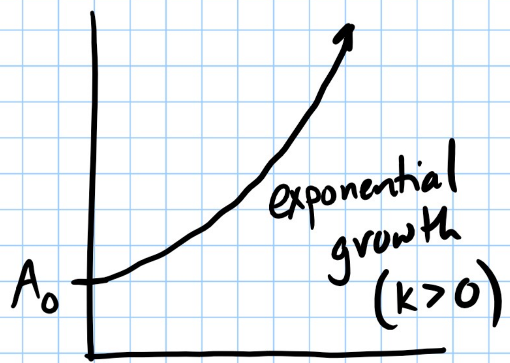
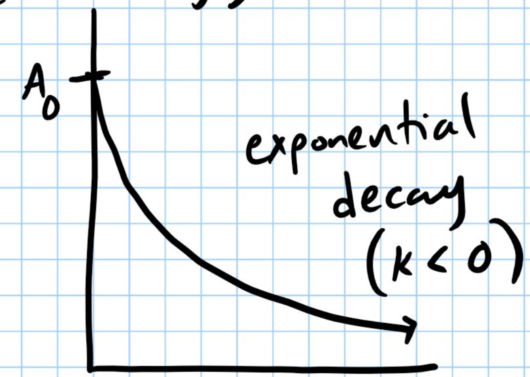
Section 4.7 Growth and Decay

Growth and Decay models follow the formula $A = A_0 e^{kt}$ where A_0 is the initial amount (when $t = 0$) and k is a constant ($k \neq 0$) and k is the growth (or decay) rate.





ex: $N(t) = 1000e^{0.07t}$

Bacteria after t hours

$A_0 = 1000$

$k = .07$ growth rate $= 7\%$

a) How many bacteria are present in 12 hours?

$N(12) = 1000e^{.07 \cdot 12} = 2{,}316$ bacteria

on calculator 1000*e^(.07*12) enter

b) How long will it take for the bacteria population to reach 5000?

$$\frac{5000}{1000} = \frac{\cancel{1000}e^{.07t}}{\cancel{1000}}$$

$$5 = e^{.07t}$$

$$\ln 5 = \cancel{\ln} e^{.07t}$$

$$\frac{\ln 5}{.07} = \frac{.07t}{.07}$$

$$t = 22.99 \text{ hrs}$$

ex: A population of snakes is living under the school. At the beginning of the school year there were 112 snakes. 2 months later there were 275 snakes. How many will there be on the last day of school? (in 9 months)

$$A = A_0 e^{kt}$$

$$A = 112e^{kt}$$

$(2, 275)$
t, A

$$\frac{275}{112} = \frac{\cancel{112}e^{k \cdot 2}}{\cancel{112}}$$

$$2.455 = e^{2k}$$

$$\ln 2.455 = 2k$$

$$k = 0.449$$

$$A = 112e^{0.449t}$$

$$A = 112e^{(0.449 \cdot 9)}$$

$$A = 6{,}371 \text{ snakes}$$

The half-life of something is the time it takes for half of the initial amount to be left.

ex: The half-life of a hostess ding dong is 6 years. If a ding dong weighs 3.5oz, how much will be left in 21 years.

$A = A_0 e^{kt}$

$A = 3.5e^{kt}$

$(\underset{t}{6}, \underset{A}{1.75})$ In 6 years the ding dong weighs $\frac{1}{2}$ of 3.5, or 1.75 oz.

$\frac{1.75}{3.5} = \frac{\cancel{3.5}e^{k \cdot 6}}{\cancel{3.5}}$

$0.5 = e^{6k}$

$\ln 0.5 = 6k$

$k = \frac{\ln 0.5}{6} = -0.1155$

In half-life problems, we can find k with $k = \frac{\ln 0.5}{\text{half-life}}$

$A = 3.5e^{-0.1155t}$

$A = 3.5e^{(-0.1155 \cdot 21)}$

$A = 0.31$ oz

$\boxed{\text{p347 1 (skip c), 5, 7, 9, 11a}}$