Section 11.2 Arithmetic Sequences

An example of an arithmetic sequence is

$$3, 7, 11, 15, 19, \ldots$$

The difference between any 2 consecutive terms is the same. This is called the <u>common difference</u>, d.

In the example $d = 4$.

Formulas for arithmetic sequences:

Recursive: $a_1 = a$, $a_n = a_{n-1} + d$

In our example, $a = 3$, $a_n = a_{n-1} + 4$

n^{th} term: $a_n = a + (n-1)d$

In our example, $a = 3$, $d = 4$

$$a_n = 3 + (n-1)4$$

$$a_n = 3 + 4n - 4$$

$$a_n = 4n - 1 \longleftarrow \text{linear}$$

$$a_{4000} = 4 \cdot 4000 - 1 = 15{,}999$$

ex: $3, 6, 10, 13, 17, \ldots$ is <u>not</u> arithmetic

ex: $20, 17, 14, 11, 8, \ldots$ is an arithmetic sequence with $d = -3$

$$a_n = 20 + (n-1)(-3)$$

$$a_n = 20 - 3n + 3$$

$$a_n = -3n + 23$$

d is like the <u>slope</u>

ex: The 8^{th} term of an arithmetic sequence is 75 and the 20^{th} term is 39. Find the first term and the common difference.

$$\left.\begin{aligned} a_8 &= 75 \\ a_{20} &= 39 \end{aligned}\right\} \quad \left.\begin{aligned} &(8, 75) \\ &(20, 39) \end{aligned}\right\} \quad m = \frac{39-75}{20-8} = \frac{-36}{12} = -3$$

$\boxed{d = -3}$

$\boxed{a_1 = 96}$

$$a_n = \underset{=}{a} + (n-1)d$$

$$75 = a + (8-1)(-3)$$

$$75 = a + 7(-3)$$

$$75 = a - 21$$

$$96 = a$$

Sum of first 100 natural numbers:

$$1 + 2 + 3 + \ldots + 98 + 99 + 100$$

$$\left.\begin{aligned} &1 + 100 \\ &2 + 99 \\ &3 + 98 \\ &\;\vdots \qquad \vdots \\ &50 + 51 \end{aligned}\right\} \; 50 \cdot 101 = 5{,}050$$

$$\frac{100}{2}(1 + 100)$$

$$50(101)$$

The sum of the first n terms of an arithmetic sequence is $S_n \ \frac{n}{2}(a+a_n)$

p872-873 3-30 multiples of 3, 47, 49, 53

53) $a_1 = 10$

$d = 4$

$a_n = 10 + (n-1)4$

$a_n = 10 + 4n - 4$

$a_n = 4n + 6$

-1020

-30 34

4

$$S_n = \frac{n}{2}(a + a_n)$$

$$2040 = \frac{n}{2}\left(10 + \overbrace{4n+6}^{a_n}\right)$$

$$4080 = n(16 + 4n)$$

$$4080 = 16n + 4n^2$$

$$4n^2 + 16n - 4080 = 0$$

$$4(n^2 + 4n - 1020) = 0$$

$$4(n-30)(n+34) = 0$$

$n = 30$

so 30 rows