WARMUP

Evaluate

$\log_2 32 = 5$ $(4 \cdot 8)$

1) $\log_2 4 + \log_2 8 = 2 + 3 = 5$

2) $\log_2 32 = 5$

3) $\log_3 81 - \log_3 27 = 4 - 3 = 1$

4) $\log_3 3 = 1$

1

$\frac{81}{27}$

Section 4.4 Properties of Logarithms

1) $\log_a 1 = 0$ $\qquad \ln 1 = 0$

2) $\log_a a = 1$ $\qquad \ln e = 1$ $\qquad \ln e^x = 5$, $x = 5$

3) $\log_a a^r = r$ $\qquad \ln e^r = r$

4) $a^{\log_a M} = M$ $\qquad e^{\ln M} = M$

5) $\log_a M + \log_a N = \log_a MN$ $\qquad \ln M + \ln N = \ln MN$

6) $\log_a M - \log_a N = \log_a \frac{M}{N}$ $\qquad \ln M - \ln N = \ln \frac{M}{N}$

7) $\log_a M^r = r \log_a M$ $\qquad \ln M^r = r \ln M$

ex: Write $\ln \frac{x^2}{(x-1)^3}$ as a sum/difference of logs.

Express all powers as factors.

$$\ln x^2 - \ln (x-1)^3 \quad \text{(PROP 6)}$$

$$2\ln x - 3\ln(x-1) \quad \text{(PROP 7)}$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

ex: $\log_a \left(x\sqrt{x^2+1}\right)$

$$= \log_a x + \log_a \sqrt{x^2+1} \quad \text{(PROP 5)}$$

$$= \log_a x + \log_a (x^2+1)^{\frac{1}{2}}$$

$$= \log_a x + \tfrac{1}{2}\log_a (x^2+1) \quad \text{(PROP 7)}$$

ex: Write as a single logarithm

$$\log_a 7 + 4\log_a 3$$

$$\log_a 7 + \log_a 3^4 \quad \text{(PROP 7)}$$

$$\log_a 7 + \log_a 81 \quad \text{arithmetic}$$

$$\log_a 7 \cdot 81 \quad \text{(PROP 5)}$$

$$\log_a 567 \quad \text{arithmetic}$$

ex: $\ln(x-3) - \ln x - \ln(5+2x)$

$$= \ln \frac{x-3}{x} - \ln(5+2x) \quad \text{(PROP 6)}$$

$$= \ln \frac{\frac{x-3}{x}}{\frac{5+2x}{1}} \quad \text{(PROP 6)}$$

$$= \ln \frac{x-3}{x} \cdot \frac{1}{5+2x}$$

$$= \ln \frac{x-3}{x(5+2x)}$$

ex: $\ln(x+3) - \ln x + \ln y - \ln(y+1)$

$$= \ln \frac{(x+3)y}{x(y+1)}$$

$$= \ln \frac{y(x+3)}{x(y+1)}$$

Change-of-Base Formula

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

ex: $\log_5 32 = \frac{\log 32}{\log 5} \approx 2.153$

$$= \frac{\ln 32}{\ln 5} \approx 2.153$$

p321-322 5-55 by 5's, 61, 62, 75, 77

18) $\ln \frac{2}{3} = \underbrace{\ln 2}_{a} - \underbrace{\ln 3}_{b}$

$\ln 2 = a$

$\ln 3 = b$

20) $\ln 0.5$

$= \ln \frac{1}{2}$

15) $e^{\log_{e^2} 16} = x$

$\ln x = \log_{e^2} 16$

$\ln x = \log_{e^2} e^{2x}$

$\ln x = \log_{e^2} (e^2)^x$

$\ln x = x \log_{e^2} (e^2)$

$\ln x = x$

$\log_{e^2} 16 = x$

$e^{2x} = 16$

$e^{\log_{e^2} 4^2}$

$a^{\log_a x} = x$

$e^{2\log_{e^2} 4}$

$(e^2)^{\log_{e^2} 4}$

$= 4$

$e^{\log_{e^2} 16} = e^{\frac{\ln 16}{\ln e^2}}$

$= e^{\frac{\ln 16}{2}}$

$= e^{\frac{1}{2}\ln 16}$

$= e^{\ln 16^{\frac{1}{2}}}$

$= e^{\ln 4}$

$= 4$