

Section 3.4 Rational Functions

A rational function can be written as a fraction of polynomials, $p(x) = \frac{f(x)}{g(x)}$ where f and g are polynomials. To find the domain, figure out what makes $g(x) = 0$. The domain is all real numbers except where $g(x) = 0$.

Many rational functions have vertical asymptotes, which are vertical lines that the graph never crosses. To find V.A.s, reduce $\frac{f(x)}{g(x)}$ to lowest terms and then set denominator = 0.

Ex: Find domain and V.A.s of

$$f(x) = \frac{x^2 - 16}{2x^2 + 7x + 6} = \frac{(x+4)(x-4)}{(2x+3)(x+2)}$$

$$\begin{array}{c} 12 \\ 3 \quad 4 \\ \cancel{3} \quad \cancel{4} \\ 7 \end{array}$$

$$\begin{array}{r} 2x \quad 3 \\ \times \quad | \quad | \\ \hline 2x^2 \quad 3x \\ \hline 4x \quad 6 \end{array}$$

$$2x+3=0 \quad x+2=0$$

$$2x=-3 \quad x=-2$$

$$x = -\frac{3}{2}$$

$$\leftarrow \textcolor{pink}{\overbrace{\quad 0 \quad 0 \quad}} \rightarrow$$

$$-2 \quad -\frac{3}{2}$$

$$D = (-\infty, -2) \cup (-2, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$$

V.A.s simplified denominator = 0

$x = -\frac{3}{2}$, $x = -2$ are V.A.s.

Ex: Find domain and V.A.s. for $f(x) = \frac{x+3}{x^2 - 9} = \frac{x+3}{(x+3)(x-3)}$

$$x = -3 \quad x = 3$$

$$D = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

V.A.s $f(x) = \frac{1}{x-3}$

so V.A. $x = 3$

A horizontal asymptote (H.A.) is a horizontal line the graph approaches as $x \rightarrow \infty$ or $x \rightarrow -\infty$

To find the H.A., we compare the degrees of the numerator and denominator.

1) If degree of top < degree of bottom

then $y = 0$ is the H.A.
↳ the x-axis

2) If degree of top = degree of bottom

then $y = \text{ratio of leading coefficients}$

3) If degree of top > degree of bottom

then there is no H.A.

However, if the degree of top = 1 + degree of bottom,
then we have an oblique asymptote, O.A. (slant asymptote)

Ex: Find H.A. of

a) $f(x) = \frac{3x^4 + 7x - 5}{8x^4 - 13x^2 + 7}$ $y = \frac{3}{8}$ is H.A.

degree of top = degree of bottom
 $4 = 4$

$$b) f(x) = \frac{x}{x^2 + 7x - 6}$$

deg of top < deg of bot

1 2

$y=0$ is H.A.

To find the oblique asymptote (O.A.) which is a line in $y=mx+b$ form, we long divide the numerator by the denominator. Then $y=\text{quotient}$ is the O.A.

$$\begin{array}{r} 350 \div 8 \\ \hline 8) 350 \\ \quad 43 \text{ quotient} \\ \quad 32 \downarrow \\ \quad 30 \\ \quad 24 \\ \hline \quad 6 \end{array}$$

ex: $f(x) = \frac{2x^2 - 5x + 4}{x + 7}$

$\deg \text{ of top} = 1 + \deg \text{ of bot}$

$$\begin{array}{r} 2x - 19 \\ x + 7 \sqrt{2x^2 - 5x + 4} \\ \underline{- (2x^2 + 14x)} \\ \quad -19x + 4 \\ \underline{- (-19x - 133)} \\ \quad 137 \end{array}$$

So $y = 2x - 19$ is O.A.

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