

WARMUP

Solve

$$x^2 = 16$$

$x = 4, -4$ since $4^2 = 16$ and $(-4)^2 = 16$

$$x^2 = -25 \Rightarrow x = \pm \sqrt{-25} = \pm i\sqrt{25} = \pm 5i$$

$$\frac{4}{5}x^2 = \frac{85}{5}$$

$$x^2 = 17$$

$$x = \pm \sqrt{17}$$

Section 11.1 The Square Root Property and Completing the Square

If $u^2 = d$ then $u = \sqrt{d}$ or $u = -\sqrt{d}$
or $u = \pm \sqrt{d}$

ex: $\frac{4}{4}x^2 = \frac{28}{4}$

$$x^2 = 7$$

$$x = \pm \sqrt{7}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

ex: $4x^2 + 9 = 0$

$$\frac{4}{4}x^2 = -\frac{9}{4}$$

$$x^2 = -\frac{9}{4}$$

$$x = \pm \sqrt{-\frac{9}{4}}$$

$$x = \pm i\sqrt{\frac{9}{4}} = \pm i\frac{3}{2}$$

$$x = \pm \frac{3}{2}i$$

$$\text{ex: } (x-3)^2 = 10$$

$$\begin{array}{r} \cancel{x-3} = \pm \sqrt{10} \\ +\cancel{3} \quad +3 \\ \hline x = 3 \pm \sqrt{10} \end{array}$$

Completing the Square:

means to make a perfect square trinomial

$$(x+4)^2 = x^2 + 8x + 16$$

x	x	4
x	x^2	$4x$
4	$4x$	16

perfect square trinomial (p.s.t)

since it can be written
as a binomial
squared.

What would you add to $x^2 + 10x + \underline{25} = (x+5)^2$
to make it a p.s.t.

x	x	5
x	x^2	$5x$
5	$5x$	25

How about $x^2 - 12x + \underline{36} = (x-6)^2$

$$-\frac{12}{2} = -6, (-6)^2 = 36$$

How about $x^2 - 14x + \frac{49}{2} = (x - 7)^2$

$$x^2 - 14x + \frac{49}{2} = (x - 7)^2$$

\uparrow
 $\left(-\frac{14}{2}\right)^2$
 \downarrow
 -7^2

Completing
the square

$$x^2 + bx + \frac{\frac{b^2}{4}}{\left(\frac{b}{2}\right)^2} = \left(x + \frac{b}{2}\right)^2$$

Solve $x^2 + 4x - 1 = 0$ by completing the square.

STEP 1: Put constant on other side.

$$\begin{array}{rcl} x^2 + 4x - 1 & = & 0 \\ \cancel{x^2} + \cancel{4x} & & + 1 \\ \hline x^2 + 4x & = & 1 \end{array}$$

STEP 2: Make left side a p.s.t. by completing the square. Add that number to both sides

$$\begin{array}{rcl} x^2 + 4x + 4 & = & 1 + 4 \\ \cancel{x^2} + \cancel{4x} & & + 4 \\ (4)^2 & = & 4 \end{array}$$

STEP 3: Factor the P.s.t. and simplify right hand side.

$$(x + 2)^2 = 5$$

STEP 4: Square root both sides

$$x+2 = \pm \sqrt{5}$$

STEP 5 : Get x by itself and simplify

$$\begin{array}{r} x+2 = \pm \sqrt{5} \\ -2 \quad \quad \quad -2 \\ \hline x = -2 \pm \sqrt{5} \end{array}$$

Ex: $x^2 - 4x + 8 = 0$

$$\begin{array}{r} \cancel{-8} \quad \cancel{-8} \\ \hline x^2 - 4x + \underline{4} = -8 + \underline{4} \end{array}$$

$$\begin{array}{r} \cancel{4} \\ \cancel{-2} \quad \cancel{-2} \\ \cancel{-4} \end{array}$$

$$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

$$(x-2)^2 = -4$$

$$\begin{array}{r} x-2 = \pm \sqrt{-4} \\ +2 \quad \quad \quad +2 \\ \hline \end{array}$$

$$x = 2 \pm \sqrt{-4} = 2 \pm i\sqrt{4} = 2 \pm 2i$$