Because of permissions issues, some material (e.g., photographs) has been removed from this chapter, though reference to it may occur in the text. The omitted content was intentionally deleted and is not needed to meet the University's requirements for this course.

снартев

# **Exponents and Polynomials**

In one recent 24-hour period, the national debt grew by more than \$34.6 billion! Source: U.S. Department of the Treasury, Bureau of the Public Debt

istening to the radio on the way to campus, you hear politicians discussing the problem of the national debt that exceeds \$8 trillion. They state that it's more than the gross domestic product of China, the world's second-richest nation, and four times greater than the combined net worth of America's 691 billionaires. They make it seem like the national debt is a real problem, but later you realize that you don't really know what a number like 8 trillion means. If the national debt were evenly divided among all citizens of the country, how much would every man, woman, and child have to pay? Is economic doomsday about to arrive?

Literacy with numbers, called *numeracy*, is a prerequisite for functioning in a meaningful way personally, professionally, and as a citizen. In this chapter, you will learn to use exponents to provide a way of putting large and small numbers into perspective. The problem of the national debt appears as Example 10 in Section 5.7.



### **Objectives**

- Understand the vocabulary used to describe polynomials.
- 2 Add polynomials.
- 3 Subtract polynomials.
- **4** Graph equations defined by polynomials of degree 2.

## **Adding and Subtracting Polynomials**



More education results in a higher income. The mathematical models

$$M = -18x^{3} + 923x^{2} - 9603x + 48,446$$
  
and 
$$W = 17x^{3} - 450x^{2} + 6392x - 14,764$$

describe the median, or middlemost, annual income for men, M, and women, W, who have completed x years of education. We'll be working with these models and the data upon which they are based in the exercise set.

The algebraic expressions that appear on the right side of the models are examples of *polynomials*. A **polynomial** is a single term or the sum of two or more terms containing variables with whole-number exponents. These particular polynomials each contain four terms. Equations containing polynomials are used in such diverse areas as science, business, medicine, psychology, and sociology. In this section, we present basic ideas about polynomials. We then use our knowledge of combining like terms to find sums and differences of polynomials.

#### **Describing Polynomials**

Consider the polynomial

$$7x^3 - 9x^2 + 13x - 6.$$

We can express this polynomial as

$$7x^3 + (-9x^2) + 13x + (-6).$$

The polynomial contains four terms. It is customary to write the terms in the order of descending powers of the variables. This is the **standard form** of a polynomial.

We begin this chapter by limiting our discussion to polynomials containing only one variable. Each term of such a polynomial in x is of the form  $ax^n$ . The **degree** of  $ax^n$  is n. For example, the degree of the term  $7x^3$  is 3.

#### The Degree of ax<sup>n</sup>

If  $a \neq 0$  and *n* is a whole number, the degree of  $ax^n$  is *n*. The degree of a nonzero constant term is 0. The constant 0 has no defined degree.

Here is an example of a polynomial and the degree of each of its four terms:



Notice that the exponent on x for the term -2x, meaning  $-2x^1$ , is understood to be 1. For this reason, the degree of -2x is 1.

used to describe polynomials.

Understand the vocabulary

#### Study Tip

We can express 0 in many ways, including 0x,  $0x^2$ , and  $0x^3$ . It is impossible to assign a unique exponent to the variable. This is why 0 has no defined degree.

A polynomial is simplified when it contains no grouping symbols and no like terms. A simplified polynomial that has exactly one term is called a **monomial**. A **binomial** is a simplified polynomial that has two terms. A **trinomial** is a simplified polynomial with three terms. Simplified polynomials with four or more terms have no special names.

The **degree of a polynomial** is the greatest degree of all the terms of the polynomial. For example,  $4x^2 + 3x$  is a binomial of degree 2 because the degree of the first term is 2, and the degree of the other term is less than 2. Also,  $7x^5 - 2x^2 + 4$  is a trinomial of degree 5 because the degree of the first term is 5, and the degrees of the other terms are less than 5.

Up to now, we have used x to represent the variable in a polynomial. However, any letter can be used. For example,

- $7x^5 3x^3 + 8$  is a polynomial (in x) of degree 5. Because there are three terms, the polynomial is a trinomial.
- $6y^3 + 4y^2 y + 3$  is a polynomial (in y) of degree 3. Because there are four terms, the polynomial has no special name.
- $z^7 + \sqrt{2}$  is a polynomial (in z) of degree 7. Because there are two terms, the polynomial is a binomial.

#### 2 Add polynomials.

#### Adding Polynomials

Recall that *like terms* are terms containing exactly the same variables to the same powers. Polynomials are added by combining like terms. For example, we can add the monomials  $-9x^3$  and  $13x^3$  as follows:



#### EXAMPLE 1 Adding Polynomials

Add:  $(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6).$ 

**Solution** The like terms are  $-9x^3$  and  $13x^3$ , containing the same variable to the same power  $(x^3)$ , as well as  $7x^2$  and  $2x^2$  (both containing  $x^2$ ), -5x and -8x (both containing x), and the constant terms 3 and -6. We begin by grouping these pairs of like terms.

$$(-9x^{3} + 7x^{2} - 5x + 3) + (13x^{3} + 2x^{2} - 8x - 6)$$
  
=  $(-9x^{3} + 13x^{3}) + (7x^{2} + 2x^{2}) + (-5x - 8x) + (3 - 6)$  Group like terms.  
=  $4x^{3} + 9x^{2} + (-13x) + (-3)$  Combine like terms.  
=  $4x^{3} + 9x^{2} - 13x - 3$  Express addition of opposites as subtraction.

**CHECK POINT 1** Add:  $(-11x^3 + 7x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$ .

Polynomials can also be added by arranging like terms in columns. Then combine like terms, column by column.

EX/	AMPLE 2	Adding Polynomials Vertically	
Add:	$(-9x^3 + 7x^3)$	$x^{2} - 5x + 3$ ) + (13 $x^{3} + 2x^{2} - 8x - 6$ ).	

#### Solution

Study Tip

A vertical format often makes it easier to see the like terms.

$-9x^3 + 7x^2 - 5x + 3$	Line up like terms vertically.
$13x^3 + 2x^2 - 8x - 6$	
$4x^3 + 9x^2 - 13x - 3$	Add the like terms in each column.

This is the same answer that we found in Example 1.

CHECK POINT 2 Add the polynomials in Check Point 1 using a vertical format. Begin by arranging like terms in columns.

**3** Subtract polynomials.

#### Subtracting Polynomials

We subtract real numbers by adding the opposite, or additive inverse, of the number being subtracted. For example,

8 - 3 = 8 + (-3) = 5.

Subtraction of polynomials also involves opposites. If the sum of two polynomials is 0, the polynomials are **opposites**, or **additive inverses**, of each other. Here is an example:

$$(4x^2 - 6x - 7) + (-4x^2 + 6x + 7) = 0.$$
  
The opposite of  $4x^2 - 6x - 7$  is  $-4x^2 + 6x + 7$ , and vice-verse.

Observe that the opposite of  $4x^2 - 6x - 7$  can be obtained by changing the sign of each of its coefficients:

PolynomialChange 4 to -4,<br/>change -6 to 6,<br/>and change -7 to 7.Opposite $4x^2 - 6x - 7$ and change -7 to 7. $-4x^2 + 6x + 7$ 

In general, **the opposite of a polynomial is that polynomial with the sign of every coefficient changed**. Just as we did with real numbers, we subtract one polynomial from another by adding the opposite of the polynomial being subtracted.

#### Subtracting Polynomials

To subtract two polynomials, add the first polynomial and the opposite of the polynomial being subtracted.

#### **EXAMPLE 3** Subtracting Polynomials

Subtract:  $(7x^2 + 3x - 4) - (4x^2 - 6x - 7)$ .

#### **Solution**

$$(7x^{2} + 3x - 4) - (4x^{2} - 6x - 7)$$
Change the sign of each coefficient.  

$$= (7x^{2} + 3x - 4) + (-4x^{2} + 6x + 7)$$
Add the opposite of the polynomial being subtracted.  

$$= (7x^{2} - 4x^{2}) + (3x + 6x) + (-4 + 7)$$
Group like terms.  

$$= 3x^{2} + 9x + 3$$
Combine like terms.

#### Study Tip

Be careful of the order in Example 4. For example, subtracting 2 from 5 means 5 - 2. In general, subtracting *B* from *A* means *A* - *B*. The order of the resulting algebraic expression is not the same as the order in English.

# EXAMPLE 4 Subtracting Polynomials

Subtract  $2x^3 - 6x^2 - 3x + 9$  from  $7x^3 - 8x^2 + 9x - 6$ .

#### Solution

$(7x^3 - 8x^2 + 9x - 6) - (2x^3 - 6x^2 - 3x + 9)$	
Change the sign of each coefficient.	
$= (7x^3 - 8x^2 + 9x - 6) + (-2x^3 + 6x^2 + 3x - 9)$	Add the opposite of the polynomial being subtracted.
$= (7x^3 - 2x^3) + (-8x^2 + 6x^2)$	
+(9x+3x)+(-6-9)	Group like terms.
$= 5x^3 + (-2x^2) + 12x + (-15)$	Combine like terms.
$= 5x^3 - 2x^2 + 12x - 15$	Express addition of opposites as subtraction.

**CHECK POINT 4** Subtract  $3x^3 - 8x^2 - 5x + 6$  from  $10x^3 - 5x^2 + 7x - 2$ .

Subtraction can also be performed in columns.

#### EXAMPLE 5 Subtracting Polynomials Vertically

Use the method of subtracting in columns to find

$$(12y^3 - 9y^2 - 11y - 3) - (4y^3 - 5y + 8)$$

**Solution** Arrange like terms in columns.



**CHECK POINT 5** Use the method of subtracting in columns to find

$$(8y^3 - 10y^2 - 14y - 2) - (5y^3 - 3y + 6).$$

#### **Graphing Equations Defined by Polynomials**

Look at the picture of this gymnast. He has created a perfect balance in which the two halves of his body are mirror images of each other. Graphs of equations defined by polynomials of degree 2, such as  $y = x^2 - 4$ , have this mirrorlike quality. We can obtain their graphs, shaped like bowls or inverted bowls, using the point-plotting method for graphing an equation in two variables.

NOT AVAILABLE FOR ELECTRONIC VIEWING Graph equations defined by polynomials of degree 2.

#### EXAMPLE 6 Graphing an Equation Defined by a Polynomial of Degree 2

Graph the equation:  $y = x^2 - 4$ .

**Solution** The given equation involves two variables, *x* and *y*. However, because the variable *x* is squared, it is not a linear equation in two variables.

$$y = x^2 - 4$$
  
This is not in the form  $y = mx + b$   
because x is squared.

Although the graph is not a line, it is still a picture of all the ordered-pair solutions of  $y = x^2 - 4$ . Thus, we can use the point-plotting method to obtain the graph.

Step 1. Find several ordered pairs that are solutions of the equation. To find some solutions of  $y = x^2 - 4$ , we select integers for x, starting with -3 and ending with 3.

Star	rt with <i>x</i>	. Compute y.	Form	the ordered pair (x, y).
	×	$y = x^2 - 4$		[x, y]
-	-3	$y = (-3)^2 - 4 = 9 - 4 = 5$	5	(-3, 5)
-	-2	$y = (-2)^2 - 4 = 4 - 4 = 0$	)	(-2, 0)
	-1	$y = (-1)^2 - 4 = 1 - 4 = -$	-3	(-1, -3)
	0	$y = 0^2 - 4 = 0 - 4 = -4$		(0, -4)
-	1	$y = 1^2 - 4 = 1 - 4 = -3$		(1, -3)
-	2	$y = 2^2 - 4 = 4 - 4 = 0$		(2, 0)
-	3	$y = 3^2 - 4 = 9 - 4 = 5$		(3,5)

Step 2. Plot these ordered pairs as points in the rectangular coordinate system. The seven ordered pairs in the table of values are plotted in Figure 5.1(a).

Step 3. Connect the points with a smooth curve. The seven points are joined with a smooth curve in Figure 5.1(b). The graph of  $y = x^2 - 4$  is a curve where the part of the graph to the right of the y-axis is a reflection of the part to the left of it, and vice versa. The arrows on both ends of the curve indicate that it extends indefinitely in both directions.



**CHECK POINT 6** Graph the equation:  $y = x^2 - 1$ . Select integers for x, starting with -3 and ending with 3.

## Study Tip

If the graph of an equation is not a straight line, use more solutions than when graphing lines. These extra solutions are needed to get a better general idea of the graph's shape. 5.1 EXERCISE SET

#### **Practice Exercises**

In Exercises 1–16, identify each polynomial as a monomial, a binomial, or a trinomial. Give the degree of the polynomial.

**MyMathLab** 

<b>1.</b> 3 <i>x</i> + 7	<b>2.</b> $5x - 2$
<b>3.</b> $x^3 - 2x$	<b>4.</b> $x^5 - 7x$
<b>5.</b> $8x^2$	<b>6.</b> $10x^2$
<b>7</b> . 5	<b>8</b> . 9
<b>9.</b> $x^2 - 3x + 4$	<b>10.</b> $x^2 - 9x + 2$
<b>11.</b> $7y^2 - 9y^4 + 5$	<b>12.</b> $3y^2 - 14y^5 + 6$
<b>13.</b> $15x - 7x^3$	<b>14.</b> $9x - 5x^3$
<b>15</b> . −9 <i>y</i> <sup>23</sup>	<b>16.</b> $-11y^{26}$

In Exercises 17–38, add the polynomials.

**17.** (9x + 8) + (-17x + 5) **18.** (8x - 5) + (-13x + 9) **19.**  $(4x^2 + 6x - 7) + (8x^2 + 9x - 2)$  **20.**  $(11x^2 + 7x - 4) + (27x^2 + 10x - 20)$  **21.**  $(7x^2 - 11x) + (3x^2 - x)$  **22.**  $(-3x^2 + x) + (4x^2 + 8x)$  **23.**  $(4x^2 - 6x + 12) + (x^2 + 3x + 1)$  **24.**  $(-7x^2 + 8x + 3) + (2x^2 + x + 8)$  **25.**  $(4y^3 + 7y - 5) + (10y^2 - 6y + 3)$  **26.**  $(2y^3 + 3y + 10) + (3y^2 + 5y - 22)$  **27.**  $(2x^2 - 6x + 7) + (3x^3 - 3x)$  **28.**  $(4x^3 + 5x + 13) + (-4x^2 + 22)$ **29.**  $(4y^2 + 8y + 11) + (-2y^3 + 5y + 2)$ 

**30.** 
$$(7y^3 + 5y - 1) + (2y^2 - 6y + 3)$$
  
**31.**  $(-2y^6 + 3y^4 - y^2) + (-y^6 + 5y^4 + 2y^2)$ 

32. 
$$(7r^4 + 5r^2 + 2r) + (-18r^4 - 5r^2 - r)$$
  
33.  $(9x^3 - x^2 - x - \frac{1}{3}) + (x^3 + x^2 + x + \frac{4}{3})$   
34.  $(12x^3 - x^2 - x + \frac{4}{3}) + (x^3 + x^2 + x - \frac{4}{3})$   
35.  $(\frac{1}{5}x^4 + \frac{1}{3}x^3 + \frac{3}{8}x^2 + 6) + (-\frac{3}{5}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 6)$   
36.  $(\frac{2}{5}x^4 + \frac{2}{3}x^3 + \frac{5}{8}x^2 + 7) + (-\frac{4}{5}x^4 + \frac{1}{3}x^3 - \frac{1}{4}x^2 - 7)$   
37.  $(0.03x^5 - 0.1x^3 + x + 0.03) + (-0.02x^5 + x^4 - 0.7x + 0.3)$ 

**38.**  $(0.06x^5 - 0.2x^3 + x + 0.05) + (-0.04x^5 + 2x^4 - 0.8x + 0.5)$ 

Mathar

PRACTICE

In Exercises 39–54, use a vertical format to add the polynomials.

READ

-

REVIEW

**39.** 
$$5y^3 - 7y^2$$
  
 $6y^3 + 4y^2$   
**40.**  $13x^4 - x^2$   
 $7x^4 + 2x^2$   
**41.**  $3x^2 - 7x + 4$   
 $-5x^2 + 6x - 3$   
**42.**  $7x^2 - 5x - 6$   
 $-9x^2 + 4x + 6$   
**43.**  $\frac{1}{4}x^4 - \frac{2}{3}x^3 - 5$   
 $-\frac{1}{2}x^4 + \frac{1}{5}x^3 + 4.7$   
**44.**  $\frac{1}{3}x^9 - \frac{1}{5}x^5 - 2.7$   
 $-\frac{3}{4}x^9 + \frac{2}{3}x^5 + 1$   
**45.**  $y^3 + 5y^2 - 7y - 3$   
 $-2y^3 + 3y^2 + 4y - 11$   
**46.**  $y^3 + y^2 - 7y + 9$   
 $-y^3 - 6y^2 - 8y + 11$   
**47.**  $4x^3 - 6x^2 + 5x - 7$   
 $-9x^3 - 4x + 3$   
**48.**  $-4y^3 + 6y^2 - 8y + 11$   
 $2y^3 + 9y - 3$   
**49.**  $7x^4 - 3x^3 + x^2$   
 $x^3 - x^2 + 4x - 2$   
**50.**  $7y^5 - 3y^3 + y^2$   
**51.**  $7x^2 - 9x + 3$   
 $4x^2 + 11x - 2$   
 $-3x^2 + 5x - 6$   
**52.**  $7y^2 - 11y - 6$   
 $8y^2 + 3y + 4$   
 $-9y^2 - 5y + 2$   
**53.**  $1.2x^3 - 3x^2 + 9.1$   
 $7.8x^3 - 3.1x^2 + 8$   
 $1.2x^2 - 6$   
**54.**  $7.9x^3 - 6.8x^2 + 3.3$   
 $6.1x^3 - 2.2x^2 + 7$   
 $-3x^2 - 5$ 

In Exercises 55–74, subtract the polynomials.

**55.** (x - 8) - (3x + 2) **56.** (x - 2) - (7x + 9) **57.**  $(x^2 - 5x - 3) - (6x^2 + 4x + 9)$  **58.**  $(3x^2 - 8x - 2) - (11x^2 + 5x + 4)$  **59.**  $(x^2 - 5x) - (6x^2 - 4x)$  **60.**  $(3x^2 - 2x) - (5x^2 - 6x)$ **61.**  $(x^2 - 8x - 9) - (5x^2 - 4x - 3)$  0)

**62.** 
$$(x - 3x + 3) - (x - 6x - 8)$$
  
**63.**  $(y - 8) - (3y - 2)$   
**64.**  $(y - 2) - (7y - 9)$   
**65.**  $(6y^3 + 2y^2 - y - 11) - (y^2 - 8y + 9)$   
**66.**  $(5y^3 + y^2 - 3y - 8) - (y^2 - 8y + 11)$   
**67.**  $(7n^3 - n^7 - 8) - (6n^3 - n^7 - 10)$   
**68.**  $(2n^2 - n^7 - 6) - (2n^3 - n^7 - 8)$   
**69.**  $(y^6 - y^3) - (y^2 - y)$   
**70.**  $(y^5 - y^3) - (y^4 - y^2)$   
**71.**  $(7x^4 + 4x^2 + 5x) - (-19x^4 - 5x^2 - x)$   
**72.**  $(-3x^6 + 3x^4 - x^2) - (-x^6 + 2x^4 + 2x^2)$   
**73.**  $(\frac{3}{7}x^3 - \frac{1}{7}x - \frac{1}{7}) - (-\frac{2}{7}x^3 + \frac{1}{7}x - \frac{1}{7})$ 

1.2

1.2

- 2)

 $\begin{pmatrix} 7^x & 5^x & 3 \end{pmatrix}$   $\begin{pmatrix} 7^x & 4^x \end{pmatrix}$ **74.**  $\left(\frac{3}{8}x^2 - \frac{1}{3}x - \frac{1}{4}\right) - \left(-\frac{1}{8}x^2 + \frac{1}{2}x - \frac{1}{4}\right)$ 

In Exercises 75-88, use a vertical format to subtract the polynomials.

75.	7x + 1	76.	4x + 2
	-(3x-5)		-(3x-5)
77.	$7x^2 - 3$	78.	$9y^2 - 6$
	$-(3x^2+4)$		$-(-5y^2+2)$
79.	$7y^2 - 5y + 2$		
	$\frac{-(11y^2+2y-3)}{2}$		
80.	$3x^5 - 5x^3 + 6$		
	$\frac{-(7x^5+4x^3-2)}{2}$		
81.	$7x^3 + 5x^2 - 3$		
	$\frac{-(-2x^3-6x^2+5)}{2}$		
82.	$3y^4 - 4y^2 + 7$		
	$-(-5y^4-6y^2-13)$		
83.	$5y^3 + 6y^2 - 3y + 10$		
	$-(6y^3 - 2y^2 - 4y - 4)$		
84.	$4y^3 + 5y^2 + 7y + 11$		
	$\frac{-(-5y^3+6y^2-9y-3)}{-(-5y^3+6y^2-9y-3)}$	3)	
85.	$7x^4 - 3x^3 + 2x^2$		
	$-($ $-x^3 - x^2 + x -$	- 2)	
86.	$5y^6 - 3y^3 + 2y^2$		
	$-(-y^3-y^2-y^2)$	- 1)	
87.	$0.07x^3 - 0.01x^2 + 0.02$	2x	
	$-(0.02x^3 - 0.03x^2 -$	x)	
88.	$0.04x^3 - 0.03x^2 + 0.05$	<i>x</i>	
	$-(0.02x^3 - 0.06x^2 -$	x)	

Graph each equation in Exercises 89–94. Find seven solutions in your table of values for each equation by using integers for x, starting with -3 and ending with 3.

<b>89.</b> $y = x^2$	<b>90.</b> $y = x^2 - 2$
<b>91.</b> $y = x^2 + 1$	<b>92.</b> $y = x^2 + 2$
<b>93.</b> $y = 4 - x^2$	<b>94.</b> $y = 9 - x^2$

#### **Practice PLUS**

In Exercises 95–98, perform the indicated operations. **95.**  $[(4x^2 + 7x - 5) - (2x^2 - 10x + 3)] - (x^2 + 5x - 8)$ 

- **96.**  $[(10x^3 5x^2 + 4x + 3) (-3x^3 4x^2 + x)] (7x^3 - 5x + 4)$
- **97.**  $[(4y^2 3y + 8) (5y^2 + 7y 4)] [(8y^2 + 5y - 7) + (-10y^2 + 4y + 3)]$

**98.** 
$$[(7y^2 - 4y + 2) - (12y^2 + 3y - 5)] - [(5y^2 - 2y - 8) + (-7y^2 + 10y - 13)]$$

- **99.** Subtract  $x^3 2x^2 + 2$  from the sum of  $4x^3 + x^2$  and  $-x^3 + 7x 3$ .
- **100.** Subtract  $-3x^3 7x + 5$  from the sum of  $2x^2 + 4x 7$ and  $-5x^3 - 2x - 3$ .
- **101.** Subtract  $-y^2 + 7y^3$  from the difference between  $-5 + y^2 + 4y^3$  and  $-8 y + 7y^3$ . Express the answer in standard form.
- **102.** Subtract  $-2y^2 + 8y^3$  from the difference between  $-6 + y^2 + 5y^3$  and  $-12 y + 13y^3$ . Express the answer in standard form.

#### **Application Exercises**

As you complete more years of education, you can count on a greater income. The bar graph shows the median, or middlemost, annual income for Americans, by level of education, in 2004.



Source: Bureau of the Census

Here are polynomial models that describe the median annual income for men, M, and for women, W, who have completed x years of education:

> $M = 177x^2 + 288x + 7075$  $W = 255x^2 - 2956x + 24,336$  $M = -18x^3 + 923x^2 - 9603x + 48,446$  $W = 17x^3 - 450x^2 + 6392x - 14.764.$

## *Exercises* 103–106 *are based on these models and the data displayed by the graph.*

- **103. a.** Use the equations defined by polynomials of degree 3 to find a mathematical model for M W.
  - **b.** According to the model in part (a), what is the difference in the median annual income between men and women with 14 years of education?
  - **c.** According to the data displayed by the graph on the previous page, what is the actual difference in the median annual income between men and women with 14 years of education? Did the model in part (b) underestimate or overestimate this difference? By how much?
- **104. a.** Use the equations defined by polynomials of degree 3 to find a mathematical model for M W.
  - **b.** According to the model in part (a), what is the difference in the median annual income between men and women with 16 years of education?
  - **c.** According to the data displayed by the graph on the previous page, what is the actual difference in the median annual income between men and women with 16 years of education? Did the model in part (b) underestimate or overestimate this difference? By how much?
- **105. a.** Use the equation defined by a polynomial of degree 2 to find the median annual income for a man with 16 years of education. Does this underestimate or overestimate the median income shown by the bar graph on the previous page? By how much?
  - **b.** Shown in a rectangular coordinate system are the graphs of the polynomial models of degree 2 that describe median annual income, by level of education. Identify your solution from part (a) as a point on the appropriate graph.



- **c.** Use the appropriate graph in part (b) to estimate, to the nearest thousand dollars, the median annual income for a woman with 16 years of education.
- **106. a.** Use the equation defined by a polynomial of degree 2 to find the median annual income for a woman with 18 years of education. Does this underestimate or overestimate the median income shown by the bar graph on the previous page? By how much?

- **b.** Shown in Exercise 105(b) are rectangular coordinate graphs of the polynomial models of degree 2 that describe median annual income, by level of education. Identify your solution from part (a) as a point on the appropriate graph.
- **c.** Use the appropriate graph in Exercise 105(b) to estimate, to the nearest thousand dollars, the median annual income for a man with 18 years of education.

#### Writing in Mathematics

**107**. What is a polynomial?

- **108.** What is a monomial? Give an example with your explanation.
- **109.** What is a binomial? Give an example with your explanation.
- **110.** What is a trinomial? Give an example with your explanation.
- **111.** What is the degree of a polynomial? Provide an example with your explanation.
- 112. Explain how to add polynomials.
- 113. Explain how to subtract polynomials.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 114–117, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

- **114.** I add like monomials by adding both their coefficients and the exponents that appear on their common variable factor.
- **115.** By looking at the first terms of a polynomial, I can determine its degree.
- **116.** As long as I understand how to add and subtract polynomials, I can select the format, horizontal or vertical, that works best for me.
- **117.** I used two points and a checkpoint to graph  $y = x^2 4$ .

# In Exercises 118–121, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

**118.** It is not possible to write a binomial with degree 0.

- **119.**  $\frac{1}{5x^2} + \frac{1}{3x}$  is a binomial.
- **120.**  $(2x^2 8x + 6) (x^2 3x + 5) = x^2 5x + 1$  for any value of *x*.
- **121.** In the polynomial  $3x^2 5x + 13$ , the coefficient of x is 5.
- **122.** What polynomial must be subtracted from  $5x^2 2x + 1$  so that the difference is  $8x^2 x + 3$ ?
- **123.** The number of people who catch a cold *t* weeks after January 1 is  $5t 3t^2 + t^3$ . The number of people who recover *t* weeks after January 1 is  $t t^2 + \frac{1}{3}t^3$ . Write a polynomial in standard form for the number of people who are still ill with a cold *t* weeks after January 1.

**124.** Explain why it is not possible to add two polynomials of degree 3 and get a polynomial of degree 4.

#### **Review Exercises**

ECTIO

**Objectives** 

exponents.

exponents.

rule.

1 Use the product rule for

2 Use the power rule for

4 Multiply monomials.

polynomial.

**3** Use the products-to-powers

5 Multiply a monomial and a

6 Multiply polynomials when neither is a monomial.

- **125.** Simplify:  $(-10)(-7) \div (1 8)$ . (Section 1.8, Example 8)
- **126.** Subtract: -4.6 (-10.2). (Section 1.6, Example 2)
- **127.** Solve: 3(x 2) = 9(x + 2). (Section 2.3, Example 3)

#### **Preview Exercises**

*Exercises* 128–130 *will help you prepare for the material covered in the next section.* 

**128.** Find the missing exponent, designated by the question mark, in the final step.

$$x^3 \cdot x^4 = (x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) = x^?$$

**129.** Use the distributive property to multiply: 3x(x + 5).

**130.** Simplify: x(x + 2) + 3(x + 2).

## **Multiplying Polynomials**



The ancient Greeks believed that the most visually pleasing rectangles have a ratio of length to width of approximately 1.618 to 1. With the exception of the squares on the lower left and the upper right, the interior of this geometric figure is filled entirely with these *golden rectangles*. Furthermore, the large rectangle is also a golden rectangle.

The total area of the large rectangle shown above can be found in many ways. This is because the area of any large rectangular region is related to the areas of the smaller rectangles that make up that region. In this section, we apply areas of rectangles as a way to picture the multiplication of polynomials. Before studying how polynomials are multiplied, we must develop some rules for working with exponents.

#### **The Product Rule for Exponents**

We have seen that exponents are used to indicate repeated, multiplication. For example,  $2^4$ , where 2 is the base and 4 is the exponent, indicates that 2 occurs as a factor four times:

```
2^4 = 2 \cdot 2 \cdot 2 \cdot 2.
```

Use the product rule for exponents.

Now consider the multiplication of two exponential expressions, such as  $2^4 \cdot 2^3$ . We are multiplying 4 factors of 2 and 3 factors of 2. We have a total of 7 factors of 2:

4 factors of 2  

$$2^4 \cdot 2^3 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$$
  
Total: 7 factors of 2  
 $2^4 \cdot 2^3 = 2^7$ . Caution:  $2^4 \cdot 2^3$  is not  
equal to  $2^{4 \cdot 3}$ , or  $2^{12}$ , as  
might be expected.

Thus,

We can quickly find the exponent, 7, of the product by adding 4 and 3, the original exponents:

$$2^4 \cdot 2^3 = 2^{4+3} = 2^7.$$

This suggests the following rule:

#### The Product Rule

 $b^m \cdot b^n = b^{m+n}$ 

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

#### Using the Product Rule EXAMPLE 1

Multiply each expression using the product rule:

**a.** 
$$2^2 \cdot 2^3$$
 **b.**  $x^7 \cdot x^9$  **c.**  $y \cdot y^5$  **d.**  $y^3 \cdot y^2 \cdot y^5$ 

#### Solution

a.  $2^2 \cdot 2^4$ 

- a.  $2^2 \cdot 2^3 = 2^{2+3} = 2^5$  or 32
- **b.**  $x^7 \cdot x^9 = x^{7+9} = x^{16}$
- **c.**  $y \cdot y^5 = y^1 \cdot y^5 = y^{1+5} = y^6$ **d.**  $v^3 \cdot v^2 \cdot v^5 = v^{3+2+5} = v^{10}$
- **CHECK POINT 1** Multiply each expression using the product rule:

**b.**  $x^6 \cdot x^4$ **d.**  $v^4 \cdot v^3 \cdot v^2$ . c.  $v \cdot v^7$ 

#### The Power Rule for Exponents

The next property of exponents applies when an exponential expression is raised to a power. Here is an example:

 $(3^2)^4$ .

The exponential expression 3<sup>2</sup> is raised to the fourth power.

There are 4 factors  $3^2$ . Thus,

$$(3^{2})^{4} = 3^{2} \cdot 3^{2} \cdot 3^{2} \cdot 3^{2} = 3^{2+2+2+2} = 3^{8}.$$

Add exponents when multiplying with the same base.

We can obtain the answer,  $3^8$ , by multiplying the exponents:

$$(3^2)^4 = 3^{2 \cdot 4} = 3^8.$$

Study Tip

The product rule does not apply to exponential expressions with different bases:

```
• x^7 \cdot y^9, or x^7 y^9, cannot be
  simplified.
```

2 Use the power rule for exponents.

By generalizing  $(3^2)^4 = 3^{2 \cdot 4} = 3^8$ , we obtain the following rule:

The Power Rule (Powers to Powers)

 $(b^m)^n = b^{mn}$ 

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

#### Using the Power Rule EXAMPLE 2

Simplify each expression using the power rule:

a. 
$$(2^3)^5$$
 b.  $(x^6)^4$  c.  $[(-3)^7]^5$ .  
Solution  
a.  $(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$   
b.  $(x^6)^4 = x^{6 \cdot 4} = x^{24}$   
c.  $[(-3)^7]^5 = (-3)^{7 \cdot 5} = (-3)^{35}$   
 $\checkmark$  CHECK POINT 2 Simplify each expression using the power rule:

**\** 

a.  $(3^4)^5$  b.  $(x^9)^{10}$  c.  $[(-5)^7]^3$ .

Use the products-to-powers rule.

#### The Products-to-Powers Rule for Exponents

The next property of exponents applies when we are raising a product to a power. Here is an example:

> $(2x)^4$ . The product  $\mathbf{2}x$  is raised to the fourth power.

There are four factors of 2x. Thus,

 $(2x)^4 = 2x \cdot 2x \cdot 2x \cdot 2x = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x = 2^4 x^4.$ 

We can obtain the answer,  $2^4x^4$ , by raising each factor within the parentheses to the fourth power:

$$(2x)^4 = 2^4 x^4.$$

This suggests the following rule:

#### **Products to Powers**

 $(ab)^n = a^n b^n$ 

When a product is raised to a power, raise each factor to the power.

#### EXAMPLE 3 Using the Products-to-Powers Rule

Simplify each expression using the products-to-powers rule:

**b.**  $(-2v^4)^5$ . a.  $(5x)^3$ 

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#### Study Tip

Do not confuse the product and power rules. Note the following differences:

- $x^4 \cdot x^7 = x^{4+7} = x^{11}$
- $(x^4)^7 = x^{4\cdot7} = x^{28}$ .

#### Solution

a. 
$$(5x)^3 = 5^3x^3$$
  
 $= 125x^3$   
b.  $(-2y^4)^5 = (-2)^5(y^4)^5$   
 $= (-2)^5y^{4\cdot5}$   
 $= (-2)^5y^{4\cdot5}$   
 $= (-2)^5y^{4\cdot5}$   
Raise each factor to the fifth power.  
 $= (-2)^5y^{4\cdot5}$   
To raise an exponential expression to a power, multiply  
exponents:  $(b^m)^n = b^{mn}$ .  
 $= -32y^{20}$   
 $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$ 

 $\checkmark$ rule: s

**b.**  $(-4y^2)^3$ . a.  $(2x)^4$ 

Study Tip

Try to avoid the following common errors that can occur when simplifying exponential expressions.

Correct	Incorrect	Description of Error
$b^3 \cdot b^4 = b^{3+4} = b^7$	$b^3 \cdot b^4 = b^{12}$	Exponents should be added, not multiplied.
$3^2 \cdot 3^4 = 3^{2+4} = 3^6$	$3^2 \cdot 3^4 = 9^{2+4} \neq 9^6$	The common base should be retained, not multiplied.
$(x^5)^3 = x^{5\cdot 3} = x^{15}$	$(x^5)^3 = x^{5+3} = x^8$	Exponents should be multiplied, not added, when raising a power to a power.
$(4x)^3 = 4^3x^3 = 64x^3$	$(4x)^3 = 4x^3$	Both factors should be cubed.

4 Multiply monomials.

#### **Multiplying Monomials**

Now that we have developed three properties of exponents, we are ready to turn to polynomial multiplication. We begin with the product of two monomials, such as  $-8x^6$ and  $5x^3$ . This product is obtained by multiplying the coefficients, -8 and 5, and then multiplying the variables using the product rule for exponents.

$$(-8x^6)(5x^3) = -8 \cdot 5 \cdot x^6 \cdot x^3 = -8 \cdot 5x^{6+3} = -40x^9$$

Multiply coefficients and add exponents.

#### **Multiplying Monomials**

To multiply monomials with the same variable base, multiply the coefficients and then multiply the variables. Use the product rule for exponents to multiply the variables: Keep the variable and add the exponents.

EXAMPLE 4 Multiplying Monomials

Multiply: **a.**  $(2x)(4x^2)$ 

Solution

#### **b.** $(-10x^6)(6x^{10})$ .

Study Tip

Don't confuse adding and multiplying monomials.

**Addition:** 

$$5x^4 + 6x^4 = 11x^4$$

**Multiplication:** 

 $(5x^4)(6x^4) = (5 \cdot 6)(x^4 \cdot x^4)$  $= 30x^{4+4}$  $= 30x^8$ 

Only like terms can be added or subtracted, but unlike terms may be multiplied.

#### Addition:

5

 $5x^4 + 3x^2$  cannot be simplified.

#### **Multiplication:**

polynomial.

 $(5x^4)(3x^2) = (5\cdot 3)(x^4 \cdot x^2)$  $= 15x^{4+2}$  $= 15x^{6}$ 

Multiply a monomial and a

a. 
$$(2x)(4x^2) = (2 \cdot 4)(x \cdot x^2)$$
  
 $= 8x^{1+2}$   
 $= 8x^3$   
b.  $(-10x^6)(6x^{10}) = (-10 \cdot 6)(x^6 \cdot x^{10})$   
 $= -60x^{6+10}$   
 $= -60x^{16}$   
Multiply the coefficients and multiply the variables.  
 $b^m \cdot b^n = b^{m+n}$ .  
 $b^m \cdot b^n = b^{m+n}$ .  
 $b^m \cdot b^n = b^{m+n}$ .  
 $= -60x^{16}$   
Simplify.  
**CHECK POINT 4**  
Multiply: a.  $(7x^2)(10x)$   
b.  $(-5x^4)(4x^5)$ .

#### Multiplying a Monomial and a Polynomial That Is Not a Monomial

We use the distributive property to multiply a monomial and a polynomial that is not a monomial. For example,

$$3x^{2}(2x^{3} + 5x) = 3x^{2} \cdot 2x^{3} + 3x^{2} \cdot 5x = 3 \cdot 2x^{2+3} + 3 \cdot 5x^{2+1} = 6x^{5} + 15x^{3}.$$
  
Monomial Binomial Multiply coefficients and add exponents.

#### Multiplying a Monomial and a Polynomial That Is Not a Monomial

To multiply a monomial and a polynomial, use the distributive property to multiply each term of the polynomial by the monomial.

EXAMPLE 5 Multiplying a Monomial and a Polynomial

Multiply: a. 2x(x + 4)

## **b.** $3x^2(4x^3 - 5x + 2)$ .

Solution

a. 
$$2x(x + 4) = 2x \cdot x + 2x \cdot 4$$
 Use the distributive property.  
 $= 2 \cdot 1x^{1+1} + 2 \cdot 4x$  To multiply the monomials, multiply coefficients and add exponents.  
 $= 2x^2 + 8x$  Simplify.  
b.  $3x^2(4x^3 - 5x + 2)$   
 $= 3x^2 \cdot 4x^3 - 3x^2 \cdot 5x + 3x^2 \cdot 2$  Use the distributive property.  
 $= 3 \cdot 4x^{2+3} - 3 \cdot 5x^{2+1} + 3 \cdot 2x^2$  To multiply the monomials, multiply coefficients and add exponents.  
 $= 12x^5 - 15x^3 + 6x^2$  Simplify.

Rectangles often make it possible to visualize polynomial multiplication. For example, **Figure 5.2** shows a rectangle with length 2x and width x + 4. The area of the large rectangle is

2x(x + 4).

The sum of the areas of the two smaller rectangles is

$$2x^2 + 8x$$
.

FIGURE 5.2

 $2x^2$ 

x

x + 4

8*x* 

4

2x

Conclusion:

$$2x(x+4) = 2x^2 + 8x$$

CHECK POINT 5 Multiply:

**a.** 3x(x + 5)

6 Multiply polynomials when neither is a monomial.

#### Multiplying Polynomials When Neither Is a Monomial

How do we multiply two polynomials if neither is a monomial? For example, consider



**b.**  $6x^2(5x^3 - 2x + 3)$ .

One way to perform this multiplication is to distribute 2x throughout the trinomial

$$2x(x^2 + 4x + 5)$$

and 3 throughout the trinomial

$$3(x^2 + 4x + 5).$$

Then combine the like terms that result. In general, the product of two polynomials is the polynomial obtained by multiplying each term of one polynomial by each term of the other polynomial and then combining like terms.

#### Using Technology

**Graphic Connections** The graphs of

 $y_1 = (x + 3)(x + 2)$ and  $y_2 = x^2 + 5x + 6$ 

are the same.



Multiplying Polynomials When Neither Is a Monomial

Multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.

#### EXAMPLE 6 Multiplying Binomials

Multiply: a. (x + 3)(x + 2) b. (3x + 7)(2x - 4).

**Solution** We begin by multiplying each term of the second binomial by each term of the first binomial.

a. 
$$(x + 3)(x + 2)$$
  
 $= x(x + 2) + 3(x + 2)$   
 $= x \cdot x + x \cdot 2 + 3 \cdot x + 3 \cdot 2$   
 $= x^{2} + 2x + 3x + 6$   
 $= x^{2} + 5x + 6$ 

Multiply the second binomial by each term of the first binomial.

Use the distributive property. Multiply. Note that  $x \cdot x = x^1 \cdot x^1 = x^{1+1} = x^2$ . Combine like terms.



You can visualize the polynomial multiplication in Example 6(a),  $(x + 3)(x + 2) = x^2 + 5x + 6$ , by analyzing the areas in **Figure 5.3**.





Conclusion:

FIGURE 5.3

 $(x+3)(x+2) = x^2 + 5x + 6$ 

#### EXAMPLE 7 Multiplying a Binomial and a Trinomial

Multiply:  $(2x + 3)(x^2 + 4x + 5)$ .

#### **Solution**

 $(2x + 3)(x^{2} + 4x + 5) = 2x(x^{2} + 4x + 5) + 3(x^{2} + 4x + 5)$   $= 2x(x^{2} + 4x + 5) + 3(x^{2} + 4x + 5)$   $= 2x \cdot x^{2} + 2x \cdot 4x + 2x \cdot 5 + 3x^{2} + 3 \cdot 4x + 3 \cdot 5$   $= 2x^{3} + 8x^{2} + 10x + 3x^{2} + 12x + 15$   $= 2x^{3} + 11x^{2} + 22x + 15$   $= 2x^{3} + 11x^{2} + 12x^{2} + 15$   $= 2x^{3} + 11x^{2} + 12x^{2} + 15$  $= 2x^{3} + 11x^{2} + 12x^{2} + 15$ 

Another method for solving Example 7 is to use a vertical format similar to that used for multiplying whole numbers.

 $\begin{array}{rl} x^{2} + 4x + 5 \\ 2x + 3 \\ 3x^{2} + 12x + 15 \\ \text{same column.} \end{array} \begin{array}{r} 3(x^{2} + 4x + 5) \\ 2x^{3} + 8x^{2} + 10x \\ 2x^{3} + 11x^{2} + 22x + 15 \end{array} \begin{array}{r} 3(x^{2} + 4x + 5) \\ 2x(x^{2} + 4x + 5) \\ 2x(x^{2} + 4x + 5) \end{array}$ Combine like terms. EXAMPLE 8 Multiplying Polynomials Using a Vertical Format Multiply:  $(2x^{2} - 3x)(5x^{3} - 4x^{2} + 7x).$ 

ISBN 0-558-95172-4

**Solution** To use the vertical format, it is most convenient to write the polynomial with the greater number of terms in the top row.

$$5x^3 - 4x^2 + 7x$$
$$2x^2 - 3x$$

We now multiply each term in the top polynomial by the last term in the bottom polynomial.

$$\frac{5x^3 - 4x^2 + 7x}{2x^2 - 3x}$$
  
-15x<sup>4</sup> + 12x<sup>3</sup> - 21x<sup>2</sup> -3x(5x<sup>3</sup> - 4x<sup>2</sup> + 7x)

Then we multiply each term in the top polynomial by  $2x^2$ , the first term in the bottom polynomial. Like terms are placed in columns because the final step involves combining them.

	$5x^3 - 4x^2 + 7x$	
	$2x^2 - 3x$	$-3x(5x^3 - 4x^2 + 7x)$
Write like terms in the same column.	$-15x^4 + 12x^3 - 21x^2$ $10x^5 - 8x^4 + 14x^3$	$2x^2(5x^3-4x^2+7x)$
	$10x^5 - 23x^4 + 26x^3 - 21x^2$	Combine like terms, which are lined up in columns.

CHECK POINT

**T 8** Multiply using a vertical format: 
$$(3x^2-2x)(2x^3-5x^2+4x)$$
.

5.2 EXERCISE SET MyMathLab PRACTICE WATCH DOWNLOAD READ

#### **Practice Exercises**

In Exercises 1–8, multiply each expression using the product rule.

<b>1.</b> $x^{15} \cdot x^3$	<b>2.</b> $x^{12} \cdot x^4$
<b>3.</b> $y \cdot y^{11}$	<b>4.</b> $y \cdot y^{19}$
<b>5.</b> $x^2 \cdot x^6 \cdot x^3$	<b>6.</b> $x^4 \cdot x^3 \cdot x^5$

**7.**  $7^9 \cdot 7^{10}$  **8.**  $8^7 \cdot 8^{10}$ 

In Exercises 9–14, simplify each expression using the power rule.

9.	$(6^9)^{10}$	10.	$(6^7)^{10}$
11.	$(x^{15})^3$	12.	$(x^{12})^4$
13.	$[(-20)^3]^3$	14.	$[(-50)^4]^4$

*In Exercises 15–24, simplify each expression using the products-to-powers rule.* 

<b>15.</b> $(2x)^3$	16.	$(4x)^3$
<b>17</b> . (-5 <i>x</i>	<sup>2</sup> ) <sup>2</sup> 18.	$(-6x)^2$
<b>19.</b> (4 <i>x</i> <sup>3</sup> )	<sup>2</sup> 20.	$(6x^3)^2$
<b>21</b> . (-2 <i>y</i>	<sup>6</sup> ) <sup>4</sup> 22.	$(-2y^5)^4$
<b>23.</b> (-2 <i>x</i>	<sup>7</sup> ) <sup>5</sup> <b>24</b> .	$(-2x^{11})^5$
		• 1

In Exercises 25–34, multiply the monomials.

<b>25.</b> (7 <i>x</i> )(2 <i>x</i> )	<b>26.</b> (8 <i>x</i> )(3 <i>x</i> )
<b>27.</b> $(6x)(4x^2)$	<b>28.</b> $(10x)(3x^2)$
<b>29.</b> $(-5y^4)(3y^3)$	<b>30.</b> $(-6y^4)(2y^3)$

<b>31.</b> $\left(-\frac{1}{2}a^3\right)\left(-\frac{1}{4}a^2\right)$	<b>32.</b> $\left(-\frac{1}{3}a^4\right)\left(-\frac{1}{2}a^2\right)$
<b>33.</b> $(2x^2)(-3x)(8x^4)$	<b>34.</b> $(3x^3)(-2x)(5x^6)$

In Exercises 35–54, find each product of the monomial and the polynomial.

Ż

REVIEW

35.	4x(x + 3)	36.	6x(x + 5)
37.	x(x - 3)	38.	x(x - 7)
39.	2x(x-6)	40.	3x(x - 5)
41.	-4y(3y + 5)		
42.	-5y(6y + 7)		
43.	$4x^2(x+2)$		
44.	$5x^2(x+6)$		
45.	$2y^2(y^2+3y)$		
46.	$4y^2(y^2+2y)$		
47.	$2y^2(3y^2-4y+7)$		
48.	$4y^2(5y^2 - 6y + 3)$		
49.	$(3x^3 + 4x^2)(2x)$		
50.	$(4x^3 + 5x^2)(2x)$		
51.	$(x^2 + 5x - 3)(-2x)$		
52.	$(x^3 - 2x + 2)(-4x)$		
53.	$-3x^2(-4x^2 + x - 5)$		
54.	$-6x^2(3x^2 - 2x - 7)$		

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In Exercises 55–78, find each product. In each case, neither factor is a monomial.

**55.** (x + 3)(x + 5)**56.** (x + 4)(x + 6)**57.** (2x + 1)(x + 4)**58.** (2x + 5)(x + 3)**59.** (x + 3)(x - 5)**60.** (x + 4)(x - 6)**61.** (x - 11)(x + 9)**62.** (x - 12)(x + 8)**63.** (2x - 5)(x + 4)**64.** (3x - 4)(x + 5)**65.**  $\left(\frac{1}{4}x + 4\right)\left(\frac{3}{4}x - 1\right)$ **66.**  $\left(\frac{1}{5}x + 5\right)\left(\frac{3}{5}x - 1\right)$ **67.**  $(x + 1)(x^2 + 2x + 3)$ **68.**  $(x + 2)(x^2 + x + 5)$ **69.**  $(y-3)(y^2-3y+4)$ **70.**  $(y-2)(y^2-4y+3)$ **71.**  $(2a - 3)(a^2 - 3a + 5)$ **72.**  $(2a - 1)(a^2 - 4a + 3)$ **73.**  $(x + 1)(x^3 + 2x^2 + 3x + 4)$ **74.**  $(x + 1)(x^3 + 4x^2 + 7x + 3)$ **75.**  $\left(x - \frac{1}{2}\right)(4x^3 - 2x^2 + 5x - 6)$ 

**76.** 
$$\left(x - \frac{1}{3}\right)(3x^3 - 6x^2 + 5x - 9)$$

**77.**  $(x^2 + 2x + 1)(x^2 - x + 2)$ **78.**  $(x^2 + 3x + 1)(x^2 - 2x - 1)$ 

In Exercises 79–92, use a vertical format to find each product.

**79.**  $x^2 - 5x + 3$  **80.**  $\frac{x+8}{x^2 - 7x + 9}$  **81.**  $\frac{x+4}{x^2 - 3x + 9}$  **82.**  $\frac{2x-3}{y^2 - 5y + 3}$  **83.**  $\frac{4y-5}{2x^3 + x^2 + 2x + 3}$  **84.**  $\frac{3y^3 + 2y^2 + y + 4}{3y^3 + 2y^2 + y + 4}$  **85.**  $\frac{4z^3 - 2z^2 + 5z - 4}{3z - 2}$  **86.**  $\frac{3z-2}{5z^3 - 3z^2 + 4z - 3}$ **87.** 2z - 4

**87.** 
$$7x^3 - 5x^2 + 6x$$
  
**88.**  $\overline{9y^3 - 7y^2 + 5y}$   
**89.**  $\overline{2y^5 - 3y^3 + y^2 - 2y + 3}$   
**90.**  $n^4 - n^3 + n^2 - n + 1$   
**91.**  $x^2 + 7x - 3$   
**92.**  $\frac{x^2 - x - 1}{x^2 + 6x - 4}$ 

#### Practice PLUS

In Exercises 93–100, perform the indicated operations. 93. (x + 4)(x - 5) - (x + 3)(x - 6)94. (x + 5)(x - 6) - (x + 2)(x - 9)95.  $4x^2(5x^3 + 3x - 2) - 5x^3(x^2 - 6)$ 96.  $3x^2(6x^3 + 2x - 3) - 4x^3(x^2 - 5)$ 97.  $(y + 1)(y^2 - y + 1) + (y - 1)(y^2 + y + 1)$ 98.  $(y + 1)(y^2 - y + 1) - (y - 1)(y^2 + y + 1)$ 99.  $(y + 6)^2 - (y - 2)^2$ 100.  $(y + 5)^2 - (y - 4)^2$ 

#### **Application Exercises**

**101.** Find a trinomial for the area of the rectangular rug shown below whose sides are x + 5 feet and 2x - 3 feet.



**102.** The base of a triangular sail is 4x feet and its height is 3x + 10 feet. Write a binomial in terms of x for the area of the sail.



In Exercises 103-104,

- **a**. *Express the area of the large rectangle as the product of two binomials.*
- **b**. Find the sum of the areas of the four smaller rectangles.
- **c.** Use polynomial multiplication to show that your expressions for area in parts (a) and (b) are equal.



#### Writing in Mathematics

- **105.** Explain the product rule for exponents. Use  $2^3 \cdot 2^5$  in your explanation.
- **106.** Explain the power rule for exponents. Use  $(3^2)^4$  in your explanation.
- **107.** Explain how to simplify an expression that involves a product raised to a power. Provide an example with your explanation.
- 108. Explain how to multiply monomials. Give an example.
- **109.** Explain how to multiply a monomial and a polynomial that is not a monomial. Give an example.
- **110.** Explain how to multiply polynomials when neither is a monomial. Give an example.
- **111.** Explain the difference between performing these two operations:

$$2x^2 + 3x^2$$
 and  $(2x^2)(3x^2)$ .

**112.** Discuss situations in which a vertical format, rather than a horizontal format, is useful for multiplying polynomials.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 113–116, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

- **113.** I'm working with two monomials that I cannot add, although I can multiply them.
- **114.** I'm working with two monomials that I can add, although I cannot multiply them.
- **115.** Other than multiplying monomials, the distributive property is used to multiply other kinds of polynomials.
- **116.** I used the product rule for exponents to multiply  $x^7$  and  $y^9$ .

In Exercises 117–120, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

**117.** 
$$4x^3 \cdot 3x^4 = 12x^{12}$$

**118.** 
$$5x^2 \cdot 4x^6 = 9x^8$$

**119.**  $(y - 1)(y^2 + y + 1) = y^3 - 1$ 

- **120.** Some polynomial multiplications can only be performed by using a vertical format.
- **121.** Find a polynomial in descending powers of *x* representing the area of the shaded region.



**122.** Find each of the products in parts (a)–(c).

a. 
$$(x-1)(x+1)$$

**b.** 
$$(x - 1)(x^2 + x + 1)$$

(

c. 
$$(x-1)(x^3 + x^2 + x + 1)$$

- d. Using the pattern found in parts (a)–(c), find  $(x-1)(x^4 + x^3 + x^2 + x + 1)$  without actually multiplying.
- **123**. Find the missing factor.

$$\underline{\qquad})\left(-\frac{1}{4}xy^3\right) = 2x^5y^3$$

#### **Review Exercises**

- **124.** Solve: 4x 7 > 9x 2. (Section 2.7, Example 7)
- **125.** Graph 3x 2y = 6 using intercepts. (Section 3.2, Example 4)
- **126.** Find the slope of the line passing through the points (-2, 8) and (1, 6). (Section 3.3, Example 1)

#### **Preview Exercises**

Exercises 127–129 will help you prepare for the material covered in the next section. In each exercise, find the indicated products. Then, if possible, state a fast method for finding these products. (You may already be familiar with some of these methods from a high school algebra course.)

**127.** a. 
$$(x + 3)(x + 4)$$
  
b.  $(x + 5)(x + 20)$   
**128.** a.  $(x + 3)(x - 3)$   
b.  $(x + 5)(x - 5)$   
**129.** a.  $(x + 3)^2$   
b.  $(x + 5)^2$ 

# **5.3**

## **Objectives**



4 Find the square of a binomial difference.

## **Special Products**



Let's cut to the chase. Are there fast methods for finding products of polynomials? Yes. In this section, we use the distributive property to develop patterns that will let you multiply certain binomials quite rapidly.

## 1 Use FOIL in polynomial multiplication.

#### The Product of Two Binomials: FOIL

Frequently, we need to find the product of two binomials. One way to perform this multiplication is to distribute each term in the first binomial through the second binomial. For example, we can find the product of the binomials 3x + 2 and 4x + 5 as follows:

$$(3x + 2)(4x + 5) = 3x(4x + 5) + 2(4x + 5)$$
  
Distribute 3x  
over 4x + 5.  
Distribute 2  
over 4x + 5.  
Distribute 3x  
over 4x

We can also find the product of 3x + 2 and 4x + 5 using a method called FOIL, which is based on our work shown above. Any two binomials can be quickly multiplied by using the FOIL method, in which **F** represents the product of the **first** terms in each binomial, **O** represents the product of the **outside** terms, **I** represents the product of the **inside** terms, and **L** represents the product of the **last**, or second, terms in each binomial. For example, we can use the FOIL method to find the product of the binomials 3x + 2and 4x + 5 as follows:



ISBN 0-558-95172-

In general, here's how to use the FOIL method to find the product of ax + b and cx + d:

#### Using the FOIL Method to Multiply Binomials



#### EXAMPLE 1 Using the FOIL Method

Multiply: (x + 3)(x + 4).

#### Solution



CHECK POINT 1 Multiply: (x + 5)(x + 6).

#### EXAMPLE 2 Using the FOIL Method

Multiply: (3x + 4)(5x - 3).

#### Solution



**EXAMPLE 3** Using the FOIL Method Multiply: (2 - 5x)(3 - 4x).

Solution

The product can also be expressed in standard form as  $20x^2 - 23x + 6$ .

**CHECK POINT 3** Multiply: (4 - 2x)(5 - 3x).

 Multiply the sum and difference of two terms.

#### Multiplying the Sum and Difference of Two Terms

We can use the FOIL method to multiply A + B and A - B as follows:

**F 0 I L**  $(A + B)(A - B) = A^2 - AB + AB - B^2 = A^2 - B^2.$ 

Notice that the outside and inside products have a sum of 0 and the terms cancel. The FOIL multiplication provides us with a quick rule for multiplying the sum and difference of two terms, referred to as a *special-product formula*.



#### EXAMPLE 4 Finding the Product of the Sum and Difference of Two Terms

Multiply: a. (4y + 3)(4y - 3) b. (3x - 7)(3x + 7) c.  $(5a^4 + 6)(5a^4 - 6)$ .

**Solution** Use the special-product formula shown.

$$(A + B)(A - B) = A^{2} - B^{2}$$
First term squared - Second term squared - Roduct
a.  $(4y + 3)(4y - 3) = (4y)^{2} - 3^{2} = 16y^{2} - 9$ 
b.  $(3x - 7)(3x + 7) = (3x)^{2} - 7^{2} = 9x^{2} - 49$ 
c.  $(5a^{4} + 6)(5a^{4} - 6) = (5a^{4})^{2} - 6^{2} = 25a^{8} - 36$ 
  
CHECK POINT 4 Multiply: a.  $(7y + 8)(7y - 8)$ 
b.  $(4x - 5)(4x + 5)$ 
c.  $(2a^{3} + 3)(2a^{3} - 3)$ .

**3** Find the square of a binomial sum.

#### The Square of a Binomial

Let's now find  $(A + B)^2$ , the square of a binomial sum. To do so, we begin with the FOIL method and look for a general rule.

$$(A + B)^2 = (A + B)(A + B) = A \cdot A + A \cdot B + A \cdot B + B \cdot B$$
$$= A^2 + 2AB + B^2$$

This result implies the following rule, which is another example of a special-product formula:





#### Finding the Square of a Binomial Sum

Multiply:

**a.**  $(x + 3)^2$  **b.**  $(3x + 7)^2$ .

**Solution** Use the special-product formula shown.

$(A + B)^2 =$	$A^2$	+	2AB	+	$B^2$	
	(First Term) <sup>2</sup>	+	2 • Product of the Terms	+	(Last Term) <sup>2</sup>	= Product
<b>a.</b> $(x + 3)^2 =$	<i>x</i> <sup>2</sup>	+	$2 \cdot x \cdot 3$	+	3 <sup>2</sup>	$= x^2 + 6x + 9$
<b>b.</b> $(3x + 7)^2 =$	$(3x)^2$	+	2(3x)(7)	+	7 <sup>2</sup>	$= 9x^2 + 42x + 49$

#### CHECK POINT 5 Multiply:

**a.**  $(x + 10)^2$  **b.**  $(5x + 4)^2$ .

The formula for the square of a binomial sum can be interpreted geometrically by analyzing the areas in **Figure 5.4**. A



## Study Tip

is not the sum of the squares.  $(A + B)^2 \neq A^2 + B^2$ The middle term 2AB is missing.  $(x + 3)^2 \neq x^2 + 9$ Incorrect!

Caution! The square of a sum

Show that  $(x + 3)^2$  and  $x^2 + 9$  are not equal by substituting 5 for x in each expression and simplifying.

4 Find the square of a binomial difference.

A similar pattern occurs for  $(A - B)^2$ , the square of a binomial difference. Using the FOIL method on  $(A - B)^2$ , we obtain the following rule:



#### EXAMPLE 6 Finding the Square of a Binomial Difference

Multiply:

a.  $(x - 4)^2$  b.  $(5y - 6)^2$ .

**Solution** Use the special-product formula shown.

$(A - B)^2 =$	$A^2$	_	2AB	+	$B^2$	
	(First Term) <sup>2</sup>	_	2 • Product of the Terms	+	(Last Term) <sup>2</sup>	= Product
<b>a.</b> $(x - 4)^2 =$	<i>x</i> <sup>2</sup>	_	$2 \cdot x \cdot 4$	+	4 <sup>2</sup>	$= x^2 - 8x + 16$
<b>b.</b> $(5y - 6)^2 =$	$(5y)^2$	-	2(5y)(6)	+	6 <sup>2</sup>	$= 25y^2 - 60y + 36$

CHECK POINT 6 Multiply:

**a.**  $(x-9)^2$  **b.**  $(7x-3)^2$ .

The following table summarizes the FOIL method and the three special products. The special products occur so frequently in algebra that it is convenient to memorize the form or pattern of these formulas.

FOIL and Special Products Let *A*, *B*, *C*, and *D* be real numbers, variables, or algebraic expressions.

FOIL	Example
$\mathbf{F}  0  \mathbf{I}  \mathbf{L}$ $(A + B)(C + D) = AC + AD + BC + BD$	F 0 I L (2x + 3)(4x + 5) = (2x)(4x) + (2x)(5) + (3)(4x) + (3)(5) $= 8x^{2} + 10x + 12x + 15$ $= 8x^{2} + 22x + 15$
Sum and Difference of Two Terms	Example
$(A + B)(A - B) = A^2 - B^2$	$(2x + 3)(2x - 3) = (2x)^2 - 3^2$
	$=4x^2-9$
Square of a Binomial	Example
$(A + B)^2 = A^2 + 2AB + B^2$	$(2x + 3)^2 = (2x)^2 + 2(2x)(3) + 3^2$
	$= 4x^2 + 12x + 9$
$(A - B)^2 = A^2 - 2AB + B^2$	$(2x - 3)^2 = (2x)^2 - 2(2x)(3) + 3^2$
	$=4x^2-12x+9$

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REVIEW

#### 5.3 EXERCISE SET

## MyMathLab

Mathing Practice Watch

**Practice Exercises** 

In Exercises 1–24, use the FOIL method to find each product. Express the product in descending powers of the variable.

**1.** (x + 4)(x + 6)**2.** (x + 8)(x + 2)**3.** (y - 7)(y + 3)**4.** (y - 3)(y + 4)**5.** (2x - 3)(x + 5)**6.** (3x - 5)(x + 7)7. (4y + 3)(y - 1)**8.** (5y + 4)(y - 2)**9.** (2x - 3)(5x + 3)**10.** (2x - 5)(7x + 2)**11.** (3v - 7)(4v - 5)**12.** (4y - 5)(7y - 4)**13.** (7 + 3x)(1 - 5x)**14.** (2 + 5x)(1 - 4x)**15.** (5 - 3y)(6 - 2y)**16.** (7 - 2y)(10 - 3y)**17.**  $(5x^2 - 4)(3x^2 - 7)$ **18.**  $(7x^2 - 2)(3x^2 - 5)$ **19.** (6x - 5)(2 - x)**20.** (4x - 3)(2 - x)**21.**  $(x + 5)(x^2 + 3)$ **22.**  $(x + 4)(x^2 + 5)$ **23.**  $(8x^3 + 3)(x^2 + 5)$ **24.**  $(7x^3 + 5)(x^2 + 2)$ 

In Exercises 25–44, multiply using the rule for finding the product of the sum and difference of two terms.

25. (x + 3)(x - 3)26. (y + 5)(y - 5)27. (3x + 2)(3x - 2)28. (2x + 5)(2x - 5)29. (3r - 4)(3r + 4)30. (5z - 2)(5z + 2)31. (3 + r)(3 - r)32. (4 + s)(4 - s)33. (5 - 7x)(5 + 7x)34. (4 - 3y)(4 + 3y)35.  $\left(2x + \frac{1}{2}\right)\left(2x - \frac{1}{2}\right)$ 36.  $\left(3y + \frac{1}{3}\right)\left(3y - \frac{1}{3}\right)$ 37.  $(y^2 + 1)(y^2 - 1)$ 38.  $(y^2 + 2)(y^2 - 2)$ 39.  $(r^3 + 2)(r^3 - 2)$  **40.**  $(m^3 + 4)(m^3 - 4)$  **41.**  $(1 - y^4)(1 + y^4)$  **42.**  $(2 - s^5)(2 + s^5)$  **43.**  $(x^{10} + 5)(x^{10} - 5)$ **44.**  $(x^{12} + 3)(x^{12} - 3)$ 

In Exercises 45–62, multiply using the rules for the square of a binomial.

DOWNLOA

**45**.  $(x + 2)^2$ **46.**  $(x + 5)^2$ **47.**  $(2x + 5)^2$ **48.**  $(5x + 2)^2$ **49.**  $(x - 3)^2$ **50.**  $(x - 6)^2$ **51.**  $(3y - 4)^2$ **52.**  $(4y - 3)^2$ **53.**  $(4x^2 - 1)^2$ **54.**  $(5x^2 - 3)^2$ **55.**  $(7 - 2x)^2$ **56.**  $(9 - 5x)^2$ **57.**  $\left(2x+\frac{1}{2}\right)^2$ **58.**  $\left(3x + \frac{1}{3}\right)^2$ **59.**  $\left(4y - \frac{1}{4}\right)^2$ **60.**  $\left(2y - \frac{1}{2}\right)^2$ **61.**  $(x^8 + 3)^2$ **62.**  $(x^8 + 5)^2$ 

In Exercises 63-82, multiply using the method of your choice.

**63.**  $(x - 1)(x^2 + x + 1)$  **64.**  $(x + 1)(x^2 - x + 1)$  **65.**  $(x - 1)^2$  **66.**  $(x + 1)^2$  **67.** (3y + 7)(3y - 7) **68.** (4y + 9)(4y - 9) **69.**  $3x^2(4x^2 + x + 9)$  **70.**  $5x^2(7x^2 + x + 6)$  **71.** (7y + 3)(10y - 4) **72.** (8y + 3)(10y - 5) **73.**  $(x^2 + 1)^2$  **74.**  $(x^2 + 2)^2$  **75.**  $(x^2 + 1)(x^2 + 2)$  **76.**  $(x^2 + 2)(x^2 + 3)$ **77.**  $(x^2 + 4)(x^2 - 4)$ 

**78.** 
$$(x^2 + 5)(x^2 - 5)$$
  
**79.**  $(2 - 3x^5)^2$   
**80.**  $(2 - 3x^6)^2$   
**81.**  $\left(\frac{1}{4}x^2 + 12\right)\left(\frac{3}{4}x^2 - 8\right)$   
**82.**  $\left(\frac{1}{4}x^2 + 16\right)\left(\frac{3}{4}x^2 - 4\right)$ 

In Exercises 83–88, find the area of each shaded region. Write the answer as a polynomial in descending powers of x.









### Practice PLUS

In Exercises 89–96, multiply by the method of your choice. 89.  $[(2x + 3)(2x - 3)]^2$ 90.  $[(3x + 2)(3x - 2)]^2$ 91.  $(4x^2 + 1)[(2x + 1)(2x - 1)]$ 92.  $(9x^2 + 1)[(3x + 1)(3x - 1)]$ 93.  $(x + 2)^3$ 94.  $(x + 4)^3$ 95. [(x + 3) - y][(x + 3) + y]96. [(x + 5) - y][(x + 5) + y]

## Application Exercises

The square garden shown in the figure measures x yards on each side. The garden is to be expanded so that one side is increased by 2 yards and an adjacent side is increased by 1 yard. The graph shows the area of the expanded garden, y, in terms of the length of one of its original sides, x. Use this information to solve Exercises 97–100.



- **97.** Write a product of two binomials that expresses the area of the larger garden.
- **98**. Write a polynomial in descending powers of *x* that expresses the area of the larger garden.
- **99.** If the original garden measures 6 yards on a side, use your expression from Exercise 97 to find the area of the larger garden. Then identify your solution as a point on the graph shown.
- **100.** If the original garden measures 8 yards on a side, use your polynomial from Exercise 98 to find the area of the larger garden. Then identify your solution as a point on the graph shown.

The square painting in the figure measures x inches on each side. The painting is uniformly surrounded by a frame that measures 1 inch wide. Use this information to solve Exercises 101–102.



- **101.** Write a polynomial in descending powers of x that expresses the area of the square that includes the painting and the frame.
- **102.** Write an algebraic expression that describes the area of the frame. (*Hint*: The area of the frame is the area of the square that includes the painting and the frame minus the area of the painting.)

#### Writing in Mathematics

- **103.** Explain how to multiply two binomials using the FOIL method. Give an example with your explanation.
- **104.** Explain how to find the product of the sum and difference of two terms. Give an example with your explanation.
- **105.** Explain how to square a binomial sum. Give an example with your explanation.
- **106.** Explain how to square a binomial difference. Give an example with your explanation.
- **107.** Explain why the graph for Exercises 97–100 is shown only in quadrant I.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 108–111, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

- **108.** Squaring a binomial sum is as simple as squaring each of the two terms and then writing their sum.
- **109.** I can distribute the exponent 2 on each factor of  $(5x)^2$ , but I cannot do the same thing on each term of  $(x + 5)^2$ .
- **110.** Instead of using the formula for the square of a binomial sum, I prefer to write the binomial sum twice and then apply the FOIL method.
- **111.** Special-product formulas for (A + B)(A B),  $(A + B)^2$ , and  $(A B)^2$  have patterns that make their multiplications quicker than using the FOIL method.

In Exercises 112–115, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

- **112.**  $(3 + 4)^2 = 3^2 + 4^2$
- **113.**  $(2y + 7)^2 = 4y^2 + 28y + 49$
- **114.**  $(3x^2 + 2)(3x^2 2) = 9x^2 4$
- **115.**  $(x 5)^2 = x^2 5x + 25$
- **116.** What two binomials must be multiplied using the FOIL method to give a product of  $x^2 8x 20$ ?
- **117.** Express the volume of the box as a polynomial in standard form.



**118.** Express the area of the plane figure shown as a polynomial in standard form.



#### **Technology Exercises**

In Exercises 119–122, use a graphing utility to graph each side of the equation in the same viewing rectangle. (Call the left side  $y_1$ and the right side  $y_2$ .) If the graphs coincide, verify that the multiplication has been performed correctly. If the graphs do not appear to coincide, this indicates that the multiplication is incorrect. In these exercises, correct the right side of the equation. Then graph the left side and the corrected right side to verify that the graphs coincide.

- **119.**  $(x + 1)^2 = x^2 + 1$ ; Use a [-5, 5, 1] by [0, 20, 1] viewing rectangle.
- **120.**  $(x + 2)^2 = x^2 + 2x + 4$ ; Use a [-6, 5, 1] by [0, 20, 1] viewing rectangle.
- **121.**  $(x + 1)(x 1) = x^2 1$ ; Use a [-6, 5, 1] by [-2, 18, 1] viewing rectangle.
- **122.**  $(x 2)(x + 2) + 4 = x^2$ ; Use a [-6, 5, 1] by [-2, 18, 1] viewing rectangle.

#### **Review Exercises**

In Exercises 123–124, solve each system by the method of your choice.

- **123.** 2x + 3y = 1y = 3x - 7(Section 4.2, Example 1)
- **124.** 3x + 4y = 72x + 7y = 9(Section 4.3, Example 3)
- **125.** Graph:  $y = \frac{1}{3}x$ . (Section 3.4, Example 3)

#### **Preview Exercises**

*Exercises* 126–128 *will help you prepare for the material covered in the next section.* 

**126**. Use the order of operations to evaluate

$$x^3y + 2xy^2 + 5x - 2$$

for 
$$x = -2$$
 and  $y = 3$ .

**127**. Use the second step to combine the like terms.

$$5xy + 6xy = (5 + 6)xy = ?$$

**128.** Multiply using FOIL: (x + 2y)(3x + 5y).

# **5.4**

## **Objectives**

- 1 Evaluate polynomials in several variables.
- 2 Understand the vocabulary of polynomials in two variables.
- 3 Add and subtract polynomials in several variables.
- 4 Multiply polynomials in several variables.

Evaluate polynomials in several variables.

## **Polynomials in Several Variables**



The next time you visit a lumberyard and go rummaging through piles of wood, think *polynomials*, although polynomials a bit different from those we have encountered so far. The construction industry uses a polynomial in two variables to determine the number of board feet that can be manufactured from a tree with a diameter of x inches and a length of y feet. This polynomial is

$$\frac{1}{4}x^2y - 2xy + 4y.$$

We call a polynomial containing two or more variables a **polynomial in several variables**. These polynomials can be evaluated, added, subtracted, and multiplied just like polynomials that contain only one variable.

#### **Evaluating a Polynomial in Several Variables**

Two steps can be used to evaluate a polynomial in several variables.

#### Evaluating a Polynomial in Several Variables

- 1. Substitute the given value for each variable.
- 2. Perform the resulting computation using the order of operations.

#### EXAMPLE 1 Evaluating a Polynomial in Two Variables

Evaluate  $2x^{3}y + xy^{2} + 7x - 3$  for x = -2 and y = 3.

**Solution** We begin by substituting -2 for x and 3 for y in the polynomial.

 $2x^{3}y + xy^{2} + 7x - 3$   $= 2(-2)^{3} \cdot 3 + (-2) \cdot 3^{2} + 7(-2) - 3$   $= 2(-8) \cdot 3 + (-2) \cdot 9 + 7(-2) - 3$ This is the given polynomial. Replace x with -2 and y with 3. Evaluate exponential expressions:  $(-2)^{3} = (-2)(-2)(-2) = -8 \text{ and}$   $3^{2} = 3 \cdot 3 = 9.$  = -48 + (-18) + (-14) - 3 = -83Perform the indicated multiplications. Add from left to right. 2 Understand the vocabulary of polynomials in two variables.

#### **Describing Polynomials in Two Variables**

In this section, we will limit our discussion of polynomials in several variables to two variables.

In general, a **polynomial in two variables**, x and y, contains the sum of one or more monomials in the form  $ax^ny^m$ . The constant, a, is the **coefficient**. The exponents, n and m, represent whole numbers. The **degree** of the monomial  $ax^ny^m$  is n + m. We'll use the polynomial from the construction industry to illustrate these ideas.



The **degree of a polynomial in two variables** is the highest degree of all its terms. For the preceding polynomial, the degree is 3.

#### EXAMPLE 2 Using the Vocabulary of Polynomials

Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$7x^2y^3 - 17x^4y^2 + xy - 6y^2 + 9.$$

Solution

	Term	Coefficient	Degree (Sum of Exponents on the Variables)
	$7x^2y^3$	7	2 + 3 = 5
	$-17x^4y^2$	-17	4 + 2 = 6
Think of xy	xy	1	1 + 1 = 2
as if $y$ .	$-6y^{2}$	-6	2
	9	9	0

The degree of the polynomial is the highest degree of all its terms, which is 6.

CHECK POINT 2 Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$8x^4y^5 - 7x^3y^2 - x^2y - 5x + 11.$$

3 Add and subtract polynomials in several variables.

#### Adding and Subtracting Polynomials in Several Variables

Polynomials in several variables are added by combining like terms. For example, we can add the monomials  $-7xy^2$  and  $13xy^2$  as follows:

$$-7xy^2 + 13xy^2 = (-7 + 13)xy^2 = 6xy^2.$$

These like terms both contain the variable factors x and  $y^2$ . Same variable EXAMPLE 3 Adding Polynomials in Two Variables Add:  $(6xy^2 - 5xy + 7) + (9xy^2 + 2xy - 6)$ . Solution  $(6xy^2 - 5xy + 7) + (9xy^2 + 2xy - 6)$   $= (6xy^2 + 9xy^2) + (-5xy + 2xy) + (7 - 6)$  Group like terms.  $= 15xy^2 - 3xy + 1$  Combine like terms by adding coefficients and keeping the same variable factors.

We subtract polynomials in two variables just as we did when subtracting polynomials in one variable. Add the first polynomial and the opposite of the polynomial being subtracted.

#### EXAMPLE 4 Subtracting Polynomials in Two Variables

Subtract:

$$(5x^3 - 9x^2y + 3xy^2 - 4) - (3x^3 - 6x^2y - 2xy^2 + 3).$$

#### Solution

$$(5x^3 - 9x^2y + 3xy^2 - 4) - (3x^3 - 6x^2y - 2xy^2 + 3)$$
  
Change the sign of each coefficient.

$$= (5x^3 - 9x^2y + 3xy^2 - 4) + (-3x^3 + 6x^2y + 2xy^2 - 3)$$

$$= (5x^3 - 3x^3) + (-9x^2y + 6x^2y) + (3xy^2 + 2xy^2) + (-4 - 3)$$
  
= 2x<sup>3</sup> - 3x<sup>2</sup>y + 5xy<sup>2</sup> - 7

Add the opposite of the polynomial being subtracted. Group like terms. Combine like terms by adding coefficients and keeping the same variable factors.

**CHECK POINT 4** Subtract:

 $(7x^3 - 10x^2y + 2xy^2 - 5) - (4x^3 - 12x^2y - 3xy^2 + 5).$ 

#### Multiplying Polynomials in Several Variables

The product of monomials forms the basis of polynomial multiplication. As with monomials in one variable, multiplication can be done mentally by multiplying coefficients and adding exponents on variables with the same base.

Multiply:  $(7x^2y)(5x^3y^2)$ . Solution

$$(7x^2y)(5x^3y^2) = (7 \cdot 5)(x^2 \cdot x^3)(y \cdot y^2) = 35x^{2+3}y^{1+2}$$

) This regrouping can be worked mentally. Multiply coefficients and add exponents on variables with same base. SBN 0-558-95172-4

Multiply polynomials in several variables.

 $= 35x^5y^3$  Simplify.

Simplify.

**CHECK POINT 5** Multiply:  $(6xy^3)(10x^4y^2)$ .

How do we multiply a monomial and a polynomial that is not a monomial? As we did with polynomials in one variable, multiply each term of the polynomial by the monomial.

#### EXAMPLE 6 Multiplying a Monomial and a Polynomial

Multiply:  $3x^2y(4x^3y^2 - 6x^2y + 2)$ .

Solution

 $3x^2y(4x^3y^2 - 6x^2y + 2)$  $= 3x^2y \cdot 4x^3y^2 - 3x^2y \cdot 6x^2y + 3x^2y \cdot 2$  Use the distributive property.  $= 12x^{2+3}y^{1+2} - 18x^{2+2}y^{1+1} + 6x^2y$  Multiply coefficients and add exponents  $= 12x^5y^3 - 18x^4y^2 + 6x^2y$ 

on variables with the same base.

**CHECK POINT 6** Multiply:  $6xy^2(10x^4y^5 - 2x^2y + 3)$ .

FOIL and the special-products formulas can be used to multiply polynomials in several variables.

#### EXAMPLE 7 Multiplying Polynomials in Two Variables

Multiply: a. (x + 4y)(3x - 5y) b.  $(5x + 3y)^2$ .

**Solution** We will perform the multiplication in part (a) using the FOIL method. We will multiply in part (b) using the formula for the square of a binomial,  $(A + B)^2$ .

a. (x + 4y)(3x - 5y)Multiply these binomials using the FOIL method. F O I L = (x)(3x) + (x)(-5y) + (4y)(3x) + (4y)(-5y) $= 3x^2 - 5xy + 12xy - 20y^2$  $= 3x^2 + 7xy - 20y^2$ Combine like terms.  $(A + B)^2 = A^2 + 2 \cdot A \cdot B + B^2$ b.  $(5x + 3y)^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$  $= 25x^2 + 30xy + 9y^2$ **CHECK POINT 7** Multiply:

a. (7x - 6y)(3x - y)

**b.**  $(2x + 4y)^2$ .

#### EXAMPLE 8 Multiplying Polynomials in Two Variables Multiply: **a.** $(4x^2y + 3y)(4x^2y - 3y)$ **b.** $(x + y)(x^2 - xy + y^2)$ .

**Solution** We perform the multiplication in part (a) using the formula for the product of the sum and difference of two terms. We perform the multiplication in part (b) by multiplying each term of the trinomial,  $x^2 - xy + y^2$ , by x and y, respectively, and then adding like terms.

$$(A + B) \cdot (A - B) = A^{2} - B^{2}$$
a.  $(4x^{2}y + 3y)(4x^{2}y - 3y) = (4x^{2}y)^{2} - (3y)^{2}$   
 $= 16x^{4}y^{2} - 9y^{2}$ 
b.  $(x + y)(x^{2} - xy + y^{2})$   
 $= x(x^{2} - xy + y^{2}) + y(x^{2} - xy + y^{2})$ 
Multiply the trinomial by each term of the binomial.  
 $= x \cdot x^{2} - x \cdot xy + x \cdot y^{2} + y \cdot x^{2} - y \cdot xy + y \cdot y^{2}$ 
Use the distributive property.  
 $= x^{3} - x^{2}y + xy^{2} + x^{2}y - xy^{2} + y^{3}$ 
Add exponents on variables with the same base.  
 $= x^{3} + y^{3}$ 
CHECK POINT 8 Multiply:

**b.** 
$$(x - y)(x^2 + xy + y^2)$$



a.  $(6xy^2 + 5x)(6xy^2 - 5x)$ 

#### **Practice Exercises**

In Exercises 1–6, evaluate each polynomial for x = 2 and y = -3.

<b>1.</b> $x^2 + 2xy + y^2$	<b>2.</b> $x^2 + 3xy + y^2$
<b>3.</b> $xy^3 - xy + 1$	<b>4.</b> $x^3y - xy + 2$
<b>5.</b> $2x^2y - 5y + 3$	<b>6.</b> $3x^2y - 4y + 5$

*In Exercises 7–8, determine the coefficient of each term, the degree of each term, and the degree of the polynomial.* 

**7.**  $x^3y^2 - 5x^2y^7 + 6y^2 - 3$ **8.**  $12x^4y - 5x^3y^7 - x^2 + 4$ 

In Exercises 9–20, add or subtract as indicated.

**9.** 
$$(5x^2y - 3xy) + (2x^2y - xy)$$
  
**10.**  $(-2x^2y + xy) + (4x^2y + 7xy)$   
**11.**  $(4x^2y + 8xy + 11) + (-2x^2y + 5xy + 2)$ 

**12.** 
$$(7x^2y + 5xy + 13) + (-3x^2y + 6xy + 4)$$

**13.** 
$$(7x^4y^2 - 5x^2y^2 + 3xy) + (-18x^4y^2 - 6x^2y^2 - xy)$$

**14.** 
$$(6x^4y^2 - 10x^2y^2 + 7xy) + (-12x^4y^2 - 3x^2y^2 - xy)$$

**15.**  $(x^3 + 7xy - 5y^2) - (6x^3 - xy + 4y^2)$ 

**16.**  $(x^4 - 7xy - 5y^3) - (6x^4 - 3xy + 4y^3)$  **17.**  $(3x^4y^2 + 5x^3y - 3y) - (2x^4y^2 - 3x^3y - 4y + 6x)$  **18.**  $(5x^4y^2 + 6x^3y - 7y) - (3x^4y^2 - 5x^3y - 6y + 8x)$  **19.**  $(x^3 - y^3) - (-4x^3 - x^2y + xy^2 + 3y^3)$ **20.**  $(x^3 - y^3) - (-6x^3 + x^2y - xy^2 + 2y^3)$ 

**21.** Add: 
$$5x^2y^2 - 4xy^2 + 6y^2$$
  
 $-8x^2y^2 + 5xy^2 - y^2$   
**22.** Add:  $7a^2b^2 - 5ab^2 + 6b^2$   
 $-10a^2b^2 + 6ab^2 + 6b^2$   
**23.** Subtract:  $3a^2b^4 - 5ab^2 + 7ab$   
 $-(-5a^2b^4 - 8ab^2 - ab)$ 

**24.** Subtract: 
$$\frac{13x^2y^4 - 17xy^2 + xy}{-(-7x^2y^4 - 8xy^2 - xy)}$$

- **25.** Subtract 11x 5y from the sum of 7x + 13y and -26x + 19y.
- **26.** Subtract 23x 5y from the sum of 6x + 15y and x 19y.

*In Exercises* 27–76, *find each product.* **27.** (5x<sup>2</sup>y)(8xy) **28.** (10x<sup>2</sup>y)(5xy) **29.**  $(-8x^3y^4)(3x^2y^5)$ **30.**  $(7x^4y^5)(-10x^7y^{11})$ **31.** 9xy(5x + 2y)**32.** 7xy(8x + 3y)**33.**  $5xy^2(10x^2 - 3y)$ **34.**  $6x^2y(5x^2 - 9y)$ **35.**  $4ab^2(7a^2b^3 + 2ab)$ **36.**  $2ab^2(20a^2b^3 + 11ab)$ **37.**  $-b(a^2 - ab + b^2)$ **38.**  $-b(a^3 - ab + b^3)$ **39.** (x + 5y)(7x + 3y)**40.** (x + 9y)(6x + 7y)**41.** (x - 3y)(2x + 7y)**42.** (3x - y)(2x + 5y)**43.** (3xy - 1)(5xy + 2)**44.** (7xy + 1)(2xy - 3)**45.**  $(2x + 3y)^2$ **46.**  $(2x + 5y)^2$ **47.**  $(xy - 3)^2$ **48.**  $(xy - 5)^2$ **49.**  $(x^2 + y^2)^2$ **50.**  $(2x^2 + y^2)^2$ **51.**  $(x^2 - 2y^2)^2$ **52.**  $(x^2 - y^2)^2$ **53.** (3x + y)(3x - y)**54.** (x + 5y)(x - 5y)**55.** (ab + 1)(ab - 1)**56.** (ab + 2)(ab - 2)**57.**  $(x + y^2)(x - y^2)$ **58.**  $(x^2 + y)(x^2 - y)$ **59.**  $(3a^2b + a)(3a^2b - a)$ **60.**  $(5a^2b + a)(5a^2b - a)$ **61.**  $(3xy^2 - 4y)(3xy^2 + 4y)$ **62.**  $(7xy^2 - 10y)(7xy^2 + 10y)$ **63.**  $(a + b)(a^2 - b^2)$ **64.**  $(a - b)(a^2 + b^2)$ **65.**  $(x + y)(x^2 + 3xy + y^2)$ **66.**  $(x + y)(x^2 + 5xy + y^2)$ **67.**  $(x - y)(x^2 - 3xy + y^2)$ **68.**  $(x - y)(x^2 - 4xy + y^2)$ **69.** (xy + ab)(xy - ab)**70.**  $(xy + ab^2)(xy - ab^2)$ **71.**  $(x^2 + 1)(x^4y + x^2 + 1)$ **72.**  $(x^2 + 1)(xy^4 + y^2 + 1)$ **73.**  $(x^2y^2 - 3)^2$ **74.**  $(x^2y^2 - 5)^2$ **75.** (x + y + 1)(x + y - 1)

**76.** (x + y + 1)(x - y + 1)

In Exercises 77–80, write a polynomial in two variables that describes the total area of each shaded region. Express each polynomial as the sum or difference of terms.



#### **Practice PLUS**

In Exercises 81–86, find each product. As we said in the Section 5.3 opener, cut to the chase in each part of the polynomial multiplication: Use only the special-product formula for the sum and difference of two terms or the formulas for the square of a binomial.

- **81.**  $[(x^3y^3 + 1)(x^3y^3 1)]^2$
- **82.**  $[(1 a^3b^3)(1 + a^3b^3)]^2$
- **83.**  $(xy 3)^2(xy + 3)^2$  (Do not begin by squaring a binomial.)
- **84.**  $(ab 4)^2(ab + 4)^2$  (Do not begin by squaring a binomial.)
- **85.** [x + y + z][x (y + z)]**86.** (a - b - c)(a + b + c)

#### **Application Exercises**

**87**. The number of board feet, *N*, that can be manufactured from a tree with a diameter of *x* inches and a length of *y* feet is modeled by the formula

$$N = \frac{1}{4}x^2y - 2xy + 4y$$

A building contractor estimates that 3000 board feet of lumber is needed for a job. The lumber company has just milled a fresh load of timber from 20 trees that averaged 10 inches in diameter and 16 feet in length. Is this enough to complete the job? If not, how many additional board feet of lumber is needed? **88**. The storage shed shown in the figure has a volume given by the polynomial

$$2x^2y + \frac{1}{2}\pi x^2y.$$



- **a.** A small business is considering having a shed installed like the one shown in the figure. The shed's height, 2x, is 26 feet and its length, y, is 27 feet. Using x = 13 and y = 27, find the volume of the storage shed.
- **b.** The business requires at least 18,000 cubic feet of storage space. Should they construct the storage shed described in part (a)?

An object that is falling or vertically projected into the air has its height, in feet, above the ground given by

$$s = -16t^2 + v_0t + s_0,$$

where s is the height, in feet,  $v_0$  is the original velocity of the object, in feet per second, t is the time the object is in motion, in seconds, and  $s_0$  is the height, in feet, from which the object is dropped or projected. The figure shows that a ball is thrown straight up from a rooftop at an original velocity of 80 feet per second from a height of 96 feet. The ball misses the rooftop on its way down and eventually strikes the ground. Use the formula and this information to solve Exercises 89–91.



- **89**. How high above the ground will the ball be 2 seconds after being thrown?
- **90.** How high above the ground will the ball be 4 seconds after being thrown?
- **91.** How high above the ground will the ball be 6 seconds after being thrown? Describe what this means in practical terms.

The graph visually displays the information about the thrown ball described in Exercises 89–91. The horizontal axis represents the ball's time in motion, in seconds. The vertical axis represents the ball's height above the ground, in feet. Use the graph to solve Exercises 92–97.



- **92**. During which time period is the ball rising?
- **93**. During which time period is the ball falling?
- **94**. Identify your answer from Exercise 90 as a point on the graph.
- **95.** Identify your answer from Exercise 89 as a point on the graph.
- 96. After how many seconds does the ball strike the ground?
- **97.** After how many seconds does the ball reach its maximum height above the ground? What is a reasonable estimate of this maximum height?

#### Writing in Mathematics

- **98**. What is a polynomial in two variables? Provide an example with your description.
- **99.** Explain how to find the degree of a polynomial in two variables.
- **100.** Suppose that you take up sky diving. Explain how to use the formula for Exercises 89–91 to determine your height above the ground at every instant of your fall.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 101–104, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

- **101.** I use the same procedures for operations with polynomials in two variables as I did when performing these operations with polynomials in one variable.
- **102.** Adding polynomials in several variables is the same as adding like terms.
- **103.** I used FOIL to find the product of x + y and  $x^2 xy + y^2$ .
- **104.** I used FOIL to multiply 5xy and 3xy + 4.

In Exercises 105–108, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

- **105.** The degree of  $5x^{24} 3x^{16}y^9 7xy^2 + 6$  is 24.
- **106.** In the polynomial  $4x^2y + x^3y^2 + 3x^2y^3 + 7y$ , the term  $x^3y^2$  has degree 5 and no numerical coefficient.

**107.**  $(2x + 3 - 5y)(2x + 3 + 5y) = 4x^2 + 12x + 9 - 25y^2$ 

**108.** 
$$(6x^2y - 7xy - 4) - (6x^2y + 7xy - 4) = 0$$

In Exercises 109–110, find a polynomial in two variables that describes the area of the shaded region of each figure. Write the polynomial as the sum or difference of terms.



#### **MID-CHAPTER CHECK POINT**

# **111.** Use the formulas for the volume of a rectangular solid and a cylinder to derive the polynomial in Exercise 88 that describes the volume of the storage building.

#### **Review Exercises**

**112.** Solve for W: 
$$R = \frac{L + 3W}{2}$$
. (Section 2.4, Example 4)

- **113.** Subtract: -6.4 (-10.2). (Section 1.6, Example 2)
- **114.** Write the point-slope form and slope-intercept form of the equation of a line passing through the point (-2, 5) and parallel to the line whose equation is 3x y = 9. (Section 3.5, Example 3)

#### **Preview Exercises**

*Exercises* 115–117 *will help you prepare for the material covered in the next section.* 

**115.** Find the missing exponent, designated by the question mark, in the final step.

$$\frac{x^7}{x^3} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^?$$

**116.** Simplify: 
$$\frac{(x^2)^3}{5^3}$$
.  
**117.** Simplify:  $\frac{(2a^3)^5}{(b^4)^5}$ .

#### Section 5.1–Section 5.4

What You Know: We learned to add, subtract, and multiply polynomials. We used a number of fast methods for finding products of polynomials, including the FOIL method for multiplying binomials, a special-product formula for the product of the sum and difference of two terms  $[(A + B)(A - B) = A^2 - B^2]$ , and special-product formulas for squaring binomials  $[(A + B)^2 = A^2 + 2AB + B^2; (A - B)^2 = A^2 - 2AB + B^2]$ . Finally, we applied all of these operations to polynomials in several variables.

In Exercises 1–21, perform the indicated operations.

1.  $(11x^2y^3)(-5x^2y^3)$ 2.  $11x^2y^3 - 5x^2y^3$ 3. (3x + 5)(4x - 7)4. (3x + 5) - (4x - 7)5.  $(2x - 5)(x^2 - 3x + 1)$ 6.  $(2x - 5) + (x^2 - 3x + 1)$ 7.  $(8x - 3)^2$ 8.  $(-10x^4)(-7x^5)$ 

- 9.  $(x^{2} + 2)(x^{2} 2)$ 10.  $(x^{2} + 2)^{2}$ 11. (9a - 10b)(2a + b)12.  $7x^{2}(10x^{3} - 2x + 3)$ 13.  $(3a^{2}b^{3} - ab + 4b^{2}) - (-2a^{2}b^{3} - 3ab + 5b^{2})$ 14. 2(3y - 5)(3y + 5)15.  $(-9x^{3} + 5x^{2} - 2x + 7) + (11x^{3} - 6x^{2} + 3x - 7)$ 16.  $10x^{2} - 8xy - 3(y^{2} - xy)$ 17.  $(-2x^{5} + x^{4} - 3x + 10) - (2x^{5} - 6x^{4} + 7x - 13)$ 18.  $(x + 3y)(x^{2} - 3xy + 9y^{2})$ 19.  $(5x^{4} + 4)(2x^{3} - 1)$
- **20.**  $(y 6z)^2$  **21.** (2x + 3)(2x - 3) - (5x + 4)(5x - 4)**22.** Graph:  $y = 1 - x^2$ .



## **Dividing Polynomials**

## Objectives

- 1 Use the quotient rule for exponents.
- 2 Use the zero-exponent rule.
- 3 Use the quotientsto-powers rule.
- 4 Divide monomials.
- 5 Check polynomial division.
- 6 Divide a polynomial by a monomial.

Use the quotient rule for

exponents.

NOT AVAILABLE FOR ELECTRONIC VIEWING

In the dramatic arts, ours is the era of the movies. As individuals and as a nation, we've grown up with them. Our images of love, war, family, country—even of things that terrify us—owe much to what we've seen on screen. In this section's exercise set, we'll model our love for movies with polynomials and polynomial division. Before discussing polynomial division, we must develop some additional rules for working with exponents.

#### The Quotient Rule for Exponents

Consider the quotient of two exponential expressions, such as the quotient of  $2^7$  and  $2^3$ . We are dividing 7 factors of 2 by 3 factors of 2. We are left with 4 factors of 2:

$$\frac{2^{7}}{2^{3}} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 2 \cdot 2 \cdot 2 \cdot 2$$

$$3 \text{ factors of } 2 \qquad \text{Divide out pairs of factors: } \frac{2}{2} = 1. \qquad 4 \text{ factors of } 2$$

Thus,

$$\frac{2^7}{2^3} = 2^4$$
.

We can quickly find the exponent, 4, on the quotient by subtracting the original exponents:

$$\frac{2^7}{2^3} = 2^{7-3}.$$

This suggests the following rule:

The Quotient Rule

$$\frac{b^m}{b^n} = b^{m-n}, \quad b \neq 0$$

When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.

ISBN 0-558-95172-4
#### EXAMPLE 1

#### Using the Quotient Rule

Divide each expression using the quotient rule:

a. 
$$\frac{2^8}{2^4}$$
 b.  $\frac{x^{13}}{x^3}$  c.  $\frac{y^{15}}{y}$ .

#### Solution

a. 
$$\frac{2^8}{2^4} = 2^{8-4} = 2^4 \text{ or } 16$$
  
b.  $\frac{x^{13}}{x^3} = x^{13-3} = x^{10}$   
c.  $\frac{y^{15}}{y} = \frac{y^{15}}{y^1} = y^{15-1} = y^{14}$ 

**CHECK POINT 1** Divide each expression using the quotient rule:

$5^{12}$	$x^9$	$y^{20}$
a. $5^4$	b. $\overline{x^2}$	c. $\frac{y}{y}$ .

2 Use the zero-exponent rule.

#### Zero as an Exponent

A nonzero base can be raised to the 0 power. The quotient rule can be used to help determine what zero as an exponent should mean. Consider the quotient of  $b^4$  and  $b^4$ , where b is not zero. We can determine this quotient in two ways.



This means that  $b^0$  must equal 1.

#### The Zero-Exponent Rule

If *b* is any real number other than 0,

$$b^0 = 1.$$

# EXAMPLE 2 Using the Zero-Exponent Rule

Use the zero-exponent rule to simplify each expression:

a. 
$$7^0$$
 b.  $(-5)^0$  c.  $-5^0$  d.  $10x^0$  e.  $(10x)^0$ .  
Solution  
a.  $7^0 = 1$  Any nonzero number raised to the 0 power is 1.  
b.  $(-5)^0 = 1$  Any nonzero number raised to the 0 power is 1.

c.  $-5^0 = -1$ Only 5 is raised to the 0 power. d.  $10x^0 = 10 \cdot 1 = 10$ Only x is raised to the 0 power. e.  $(10x)^0 = 1$ The entire expression, 10x, is raised to the 0 power.

**CHECK POINT 2** Use the zero-exponent rule to simplify each expression:

**a.**  $14^0$  **b.**  $(-10)^0$  **c.**  $-10^0$  **d.**  $20x^0$  **e.**  $(20x)^0$ .

Use the quotients-topowers rule.

## The Quotients-to-Powers Rule for Exponents

We have seen that when a product is raised to a power, we raise every factor in the product to the power:

$$(ab)^n = a^n b^n.$$

There is a similar property for raising a quotient to a power.

#### **Quotients to Powers**

If a and b are real numbers and b is nonzero, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

When a quotient is raised to a power, raise the numerator to the power and divide by the denominator raised to the power.

## EXAMPLE 3 Using the Quotients-to-Powers Rule

Simplify each expression using the quotients-to-powers rule:

a. 
$$\left(\frac{x}{4}\right)^2$$
 b.  $\left(\frac{x^2}{5}\right)^3$  c.  $\left(\frac{2a^3}{b^4}\right)^5$ .

Solution

a.  $\left(\frac{x}{4}\right)^2 = \frac{x^2}{4^2} = \frac{x^2}{16}$ b.  $\left(\frac{x^2}{5}\right)^3 = \frac{(x^2)^3}{5^3} = \frac{x^{2\cdot3}}{5\cdot5\cdot5} = \frac{x^6}{125}$ c.  $\left(\frac{2a^3}{b^4}\right)^5 = \frac{(2a^3)^5}{(b^4)^5}$   $= \frac{2^5(a^3)^5}{(b^4)^5}$   $= \frac{2^5(a^3)^5}{(b^4)^5}$   $= \frac{2^5a^{3\cdot5}}{b^{4\cdot5}}$   $= \frac{32a^{15}}{b^{20}}$ Square the numerator and the denominator. Raise the numerator and the denominator to the fifth power. Raise each factor in the numerator to the fifth power. To raise exponential expressions to powers, multiply exponents:  $(b^m)^n = b^{mn}$ . **CHECK POINT 3** Simplify each expression using the quotients-to-powers rule:

**a.** 
$$\left(\frac{x}{5}\right)^2$$
 **b.**  $\left(\frac{x^4}{2}\right)^3$  **c.**  $\left(\frac{2a^{10}}{b^3}\right)^4$ .

#### Study Tip

Try to avoid the following common errors that can occur when simplifying exponential expressions.

Correct	Incorrect	Description of Error
$\frac{2^{20}}{2^4} = 2^{20-4} = 2^{16}$	$\frac{2^{20}}{2^4} = 2^5$	Exponents should be subtracted, not divided.
$-8^0 = -1$	$-8^{0} = 1$	Only 8 is raised to the 0 power.
$\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2} = \frac{x^2}{25}$	$\left(\frac{x}{5}\right)^2 = \frac{x^2}{5}$	The numerator and denominator must both be squared.

Divide monomials.

#### **Dividing Monomials**

Now that we have developed three additional properties of exponents, we are ready to turn to polynomial division. We begin with the quotient of two monomials, such as  $16x^{14}$  and  $8x^2$ . This quotient is obtained by dividing the coefficients, 16 and 8, and then dividing the variables using the quotient rule for exponents.

$$\frac{16x^{14}}{8x^2} = \frac{16}{8}x^{14-2} = 2x^{12}$$
Divide coefficients and subtract exponents.

#### **Dividing Monomials**

To divide monomials, divide the coefficients and then divide the variables. Use the quotient rule for exponents to divide the variables: Keep the variable and subtract the exponents.



#### Study Tip

Look at the solution to Example 4(b). Rather than subtracting exponents for division that results in a 0 exponent, you might prefer to divide out  $x^3$ .

2.23	2	1
8.23	$=\frac{1}{8}$	4

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5 Check polynomial division.

#### **Checking Division of Polynomial Problems**

The answer to a division problem can be checked. For example, consider the following problem:



The quotient is correct if the product of the divisor and the quotient is the dividend. Is the quotient shown in the preceding equation correct?



Because the product of the divisor and the quotient is the dividend, the answer to the division problem is correct.

#### **Checking Division of Polynomials**

To check a quotient in a division problem, multiply the divisor and the quotient. If this product is the dividend, the quotient is correct.

6 Divide a polynomial by a monomial.

#### Study Tip

Try to avoid this common error:



#### Dividing a Polynomial That Is Not a Monomial by a Monomial

To divide a polynomial by a monomial, we divide each term of the polynomial by the monomial. For example,

$$\frac{10x^8 + 15x^6}{5x^3} = \frac{10x^8}{5x^3} + \frac{15x^6}{5x^3} = \frac{10}{5}x^{8-3} + \frac{15}{5}x^{6-3} = 2x^5 + 3x^3.$$
monomial divisor
Divide the first term by 5x^3.

Is the quotient correct? Multiply the divisor and the quotient.

$$5x^{3}(2x^{5} + 3x^{3}) = 5x^{3} \cdot 2x^{5} + 5x^{3} \cdot 3x^{3}$$
$$= 5 \cdot 2x^{3+5} + 5 \cdot 3x^{3+3} = 10x^{8} + 15x^{6}$$

Because this product gives the dividend, the quotient is correct.

#### Dividing a Polynomial That Is Not a Monomial by a Monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

### EXAMPLE 5 Dividing a Polynomial by a Monomial

Find the quotient:  $(-12x^8 + 4x^6 - 8x^3) \div 4x^2$ .

#### Solution

$$\frac{-12x^8 + 4x^6 - 8x^3}{4x^2}$$
Rewrite the division in a vertical format.  

$$= \frac{-12x^8}{4x^2} + \frac{4x^6}{4x^2} - \frac{8x^3}{4x^2}$$
Divide each term of the polynomial by the monomial  

$$= \frac{-12}{4}x^{8-2} + \frac{4}{4}x^{6-2} - \frac{8}{4}x^{3-2}$$
Divide coefficients and subtract exponents.  

$$= -3x^6 + x^4 - 2x$$
Simplify.

To check the answer, multiply the divisor and the quotient.

$$4x^{2}(-3x^{6} + x^{4} - 2x) = 4x^{2}(-3x^{6}) + 4x^{2} \cdot x^{4} - 4x^{2}(2x)$$
  

$$= 4(-3)x^{2+6} + 4x^{2+4} - 4 \cdot 2x^{2+1}$$
  

$$= -12x^{8} + 4x^{6} - 8x^{3}$$
  
This is the dividend.

Because the product of the divisor and the quotient is the dividend, the answer—that is, the quotient—is correct.

**CHECK POINT 5** Find the quotient:  $(-15x^9 + 6x^5 - 9x^3) \div 3x^2$ .

# EXAMPLE 6 Dividing a Polynomial by a Monomial

Divide:  $\frac{16x^5 - 9x^4 + 8x^3}{2x^3}$ .

#### Solution

$$\frac{16x^5 - 9x^4 + 8x^3}{2x^3}$$
This is the given polynomial division.  

$$= \frac{16x^5}{2x^3} - \frac{9x^4}{2x^3} + \frac{8x^3}{2x^3}$$
Divide each term by  $2x^3$ .  

$$= \frac{16}{2}x^{5-3} - \frac{9}{2}x^{4-3} + \frac{8}{2}x^{3-3}$$
Divide coefficients and subtract exponents  
Did you immediately write the last term as  

$$= 8x^2 - \frac{9}{2}x + 4x^0$$
Simplify.  

$$= 8x^2 - \frac{9}{2}x + 4$$

$$x^0 = 1, \text{ so } 4x^0 = 4 \cdot 1 = 4.$$

4?

Check the answer by showing that the product of the divisor and the quotient is the dividend.

CHECK POINT 6 Divide:  $\frac{25x^9 - 7x^4 + 10x^3}{5x^3}$ .

# EXAMPLE 7 Dividing Polynomials in Two Variables

Divide:  $(15x^5y^4 - 3x^3y^2 + 9x^2y) \div 3x^2y$ .

#### Solution

**MyMathLab** 

$$\frac{15x^5y^4 - 3x^3y^2 + 9x^2y}{3x^2y}$$
Rewrite the division in a vertical format.  

$$= \frac{15x^5y^4}{3x^2y} - \frac{3x^3y^2}{3x^2y} + \frac{9x^2y}{3x^2y}$$
Divide each term of the polynomial by the monomial.  

$$= \frac{15}{3}x^{5-2}y^{4-1} - \frac{3}{3}x^{3-2}y^{2-1} + \frac{9}{3}x^{2-2}y^{1-1}$$
Divide coefficients and subtract exponents.  

$$= 5x^3y^3 - xy + 3$$
Simplify.

Check the answer by showing that the product of the divisor and the quotient is the dividend.

CHECK POINT 7 Divide: 
$$(18x^7y^6 - 6x^2y^3 + 60xy^2) \div 6xy^2$$
.

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5.5 EXERCISE SET

#### **Practice Exercises**

In Exercises 1–10, divide each expression using the quotient rule. Express any numerical answers in exponential form.

1. 
$$\frac{3^{20}}{3^5}$$
  
2.  $\frac{3^{30}}{3^{10}}$   
3.  $\frac{x^6}{x^2}$   
4.  $\frac{x^8}{x^4}$   
5.  $\frac{y^{13}}{y^5}$   
6.  $\frac{y^{19}}{y^6}$   
7.  $\frac{5^6 \cdot 2^8}{5^3 \cdot 2^4}$   
8.  $\frac{3^6 \cdot 2^8}{3^3 \cdot 2^4}$   
9.  $\frac{x^{100}y^{50}}{x^{25}y^{10}}$ 

*In Exercises 11–24, use the zero-exponent rule to simplify each expression.* 

<b>11.</b> 2 <sup>0</sup>	<b>12.</b> 4 <sup>0</sup>	<b>13</b> . $(-2)^0$
<b>14.</b> $(-4)^0$	<b>15.</b> $-2^0$	<b>16.</b> $-4^0$
<b>17.</b> $100y^0$	<b>18</b> . 200 <i>y</i> <sup>0</sup>	<b>19</b> . $(100y)^0$
<b>20.</b> $(200y)^0$	<b>21.</b> $-5^0 + (-5)^0$	<b>22.</b> $-6^0 + (-6)^0$
<b>23</b> . $-\pi^0 - (-\pi)^0$	<b>24.</b> $-\sqrt{3^{\circ}}$	$\overline{0} - \left(-\sqrt{3}\right)^0$

*In Exercises* 25–36, *simplify each expression using the quotientsto-powers rule. If possible, evaluate exponential expressions.* 

**25.** 
$$\left(\frac{x}{3}\right)^2$$
**26.**  $\left(\frac{x}{5}\right)^2$ **27.**  $\left(\frac{x^2}{4}\right)^3$ **28.**  $\left(\frac{x^2}{3}\right)^3$ **29.**  $\left(\frac{2x^3}{5}\right)^2$ **30.**  $\left(\frac{3x^4}{7}\right)^2$ **31.**  $\left(\frac{-4}{3a^3}\right)^3$ **32.**  $\left(\frac{-5}{2a^3}\right)^3$ 



DOWNLOA

In Exercises 37–52, divide the monomials. Check each answer by showing that the product of the divisor and the quotient is the dividend.

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**37.** 
$$\frac{30x^{10}}{10x^5}$$
**38.**  $\frac{45x^{12}}{15x^4}$ **39.**  $\frac{-8x^{22}}{4x^2}$ **40.**  $\frac{-15x^{40}}{3x^4}$ **41.**  $\frac{-9y^8}{18y^5}$ **42.**  $\frac{-15y^{13}}{45y^9}$ **43.**  $\frac{7y^{17}}{5y^5}$ **44.**  $\frac{9y^{19}}{7y^{11}}$ **45.**  $\frac{30x^7y^5}{5x^2y}$ **46.**  $\frac{40x^9y^5}{2x^2y}$ **47.**  $\frac{-18x^{14}y^2}{36x^2y^2}$ **48.**  $\frac{-15x^{16}y^2}{45x^2y^2}$ **49.**  $\frac{9x^{20}y^{20}}{7x^{20}y^{20}}$ **50.**  $\frac{7x^{30}y^{30}}{15x^{30}y^{30}}$ **51.**  $\frac{-5x^{10}y^{12}z^6}{50x^2y^3z^2}$ **52.**  $\frac{-8x^{12}y^{10}z^4}{40x^2y^3z^2}$ 

In Exercises 53–78, divide the polynomial by the monomial. Check each answer by showing that the product of the divisor and the quotient is the dividend.

**53.** 
$$\frac{10x^4 + 2x^3}{2}$$
 **54.**  $\frac{20x^4 + 5x^3}{5}$ 

55. 
$$\frac{14x^4 - 7x^3}{7x}$$
  
56.  $\frac{24x^4 - 8x^3}{8x}$   
57.  $\frac{y^7 - 9y^2 + y}{y}$   
58.  $\frac{y^8 - 11y^3 + y}{y}$   
59.  $\frac{24x^3 - 15x^2}{-3x}$   
60.  $\frac{10x^3 - 20x^2}{-5x}$   
61.  $\frac{18x^5 + 6x^4 + 9x^3}{3x^2}$   
62.  $\frac{18x^5 + 24x^4 + 12x^3}{6x^2}$   
63.  $\frac{12x^4 - 8x^3 + 40x^2}{4x}$   
64.  $\frac{49x^4 - 14x^3 + 70x^2}{-7x}$   
65.  $(4x^2 - 6x) \div x$   
66.  $(16y^2 - 8y) \div y$   
67.  $\frac{30z^3 + 10z^2}{-5z}$   
68.  $\frac{12y^4 - 42y^2}{-4y}$   
69.  $\frac{8x^3 + 6x^2 - 2x}{2x}$   
70.  $\frac{9x^3 + 12x^2 - 3x}{3x}$   
71.  $\frac{25x^7 - 15x^5 - 5x^4}{5x^3}$   
72.  $\frac{49x^7 - 28x^5 - 7x^4}{7x^3}$   
73.  $\frac{18x^7 - 9x^6 + 20x^5 - 10x^4}{-2x^4}$   
74.  $\frac{25x^8 - 50x^7 + 3x^6 - 40x^5}{-5x^5}$ 

75. 
$$\frac{12x^2y^2 + 6x^2y - 15xy^2}{3xy}$$
76. 
$$\frac{18a^3b^2 - 9a^2b - 27ab^2}{9ab}$$
77. 
$$\frac{20x^7y^4 - 15x^3y^2 - 10x^2y}{-5x^2y}$$
78. 
$$\frac{8x^6y^3 - 12x^8y^2 - 4x^{14}y^6}{-4x^6y^2}$$

#### **Practice PLUS**

In Exercises 79–82, simplify each expression.

79. 
$$\frac{2x^{3}(4x+2) - 3x^{2}(2x-4)}{2x^{2}}$$
  
80. 
$$\frac{6x^{3}(3x-1) + 5x^{2}(6x-3)}{3x^{2}}$$
  
81. 
$$\left(\frac{18x^{2}y^{4}}{9xy^{2}}\right) - \left(\frac{15x^{5}y^{6}}{5x^{4}y^{4}}\right)$$
  
82. 
$$\left(\frac{9x^{3} + 6x^{2}}{3x}\right) - \left(\frac{12x^{2}y^{2} - 4xy^{2}}{2xy^{2}}\right)$$
  
83. Divide the sum of  $(y + 5)^{2}$  and  $(y + 5)(y - 5)$  by 2y.

**84.** Divide the sum of  $(y + 4)^2$  and (y + 4)(y - 4) by 2y.

In Exercises 85–86, the variable n in each exponent represents a natural number. Divide the polynomial by the monomial. Then use polynomial multiplication to check the quotient.

**85.** 
$$\frac{12x^{15n} - 24x^{12n} + 8x^{3n}}{4x^{3n}}$$
**86.** 
$$\frac{35x^{10n} - 15x^{8n} + 25x^{2n}}{5x^{2n}}$$

#### **Application Exercises**

The bar graphs show U.S. film box-office receipts, in millions of dollars, and box-office admissions, in millions of tickets sold, for five selected years.



#### United States Film Box-Office Receipts and Admissions

Sources: U.S. Department of Commerce, Motion Picture Association of America, National Association of Theatre Owners

Rec

The following polynomial models of degree 2 can be used to describe the data in the bar graphs on the previous page:

eipts, in millions of dollars	Admissions, in millions of tickets sold
$R = 3.6x^2 + 158x + 279$	0 $A = -0.2x^2 + 2^2$

$$A = -0.2x^2 + 21x + 1015$$

The variable x represents the number of years after 1980.

Use this information to solve Exercises 87-88.

- 87. a. Use the data displayed by the bar graphs on the previous page to find the average admission charge for a film ticket in 2000. Round to two decimal places, or to the nearest cent.
  - b. Use the models to write an algebraic expression that describes the average admission charge for a film ticket x years after 1980.
  - c. Use the model from part (b) to find the average admission charge for a film ticket in 2000. Round to the nearest cent. Does the model underestimate or overestimate the actual average charge that you found in part (a)? By how much?
  - d. Can the polynomial division for the model in part (b) be performed using the methods that you learned in this section? Explain your answer.
- **88.** a. Use the data displayed by the bar graphs on the previous page to find the average admission charge for a film ticket in 2005. Round to two decimal places, or to the nearest cent.
  - b. Use the models to write an algebraic expression that describes the average admission charge for a film ticket x years after 1980.
  - c. Use the model from part (b) to find the average admission charge for a film ticket in 2005. Round to the nearest cent. Does the model underestimate or overestimate the actual average charge that you found in part (a)? By how much?
  - d. Can the polynomial division for the model in part (b) be performed using the methods that you learned in this section? Explain your answer.

#### Writing in Mathematics

- **89.** Explain the quotient rule for exponents. Use  $\frac{3^6}{2^2}$  in your explanation.
- **90**. Explain how to find any nonzero number to the 0 power.
- **91**. Explain the difference between  $(-7)^0$  and  $-7^0$ .
- 92. Explain how to simplify an expression that involves a quotient raised to a power. Provide an example with your explanation.
- 93. Explain how to divide monomials. Give an example.
- 94. Explain how to divide a polynomial that is not a monomial by a monomial. Give an example.
- 95. Are the expressions

$$\frac{12x^2+6x}{3x} \quad \text{and} \quad 4x+2$$

equal for every value of x? Explain.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 96–99, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

- **96.** Because division by 0 is undefined, numbers to 0 powers should not be written in denominators.
- 97. The quotient rule is applied by dividing the exponent in the numerator by the exponent in the denominator.
- **98.** I divide monomials by dividing coefficients and subtracting exponents.
- **99.** I divide a polynomial by a monomial by dividing each term of the monomial by the polynomial.

In Exercises 100–103, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

**100.**  $x^{10} \div x^2 = x^5$  for all nonzero real numbers x.

6x

**101.** 
$$\frac{12x^3 - 6x}{2x} = 6x^2 -$$
  
**102.**  $\frac{x^2 + x}{x} = x$ 

- **103.** If a polynomial in *x* of degree 6 is divided by a monomial in x of degree 2, the degree of the quotient is 4.
- **104.** What polynomial, when divided by  $3x^2$ , yields the trinomial  $6x^6 - 9x^4 + 12x^2$  as a quotient?

In Exercises 105–106, find the missing coefficients and exponents designated by question marks.

**105.** 
$$\frac{?x^8 - ?x^6}{3x^2} = 3x^5 - 4x^3$$
  
**106.**  $\frac{3x^{14} - 6x^{12} - ?x^7}{2x^2} = -x^7 + 2x^5 + 3$ 

#### **Review Exercises**

**107.** Find the absolute value: |-20.3|. (Section 1.3, Example 8)

**108.** Express  $\frac{7}{8}$  as a decimal. (Section 1.3, Example 4)

**109.** Graph: 
$$y = \frac{1}{3}x + 2$$
. (Section 3.4, Example 3)

#### **Preview Exercises**

Exercises 110-112 will help you prepare for the material covered in the next section. In each exercise, perform the long division without using a calculator, and then state the quotient and the remainder. **110**. 19)494

111. 24)2958 **112**. 98)25.187



# Long Division of Polynomials; Synthetic Division

# **Objectives**



2 Divide polynomials using synthetic division.

NOT AVAILABLE FOR ELECTRONIC VIEWING "He was an arithmetician rather than a mathematician. None of the humor, the music, or the mysticism of higher mathematics ever entered his head."

-John Steinbeck

"Arithmetic has a very great and elevating effect. He who can properly divide is to be considered a god." — Plato

"You cannot ask us to take sides against arithmetic."

-Winston Churchill

So, what's the deal? Will performing the repetitive procedure of long division (don't reach for that calculator!) have an elevating effect? Or will it confine you to a computational box that allows neither humor nor music to enter? Forget the box: Mathematician Wilhelm Leibniz believed that music is nothing but unconscious arithmetic. But do think elevation, if not to the level of an ancient Greek god, then to new, algebraic highs. The bottom line: Understanding long division of whole numbers lays the foundation for performing the division of a polynomial by a binomial, such as

$$x+3\overline{)x^2+10x+21}.$$

Divisor has two terms and is a binomial. The polynomial dividend has three terms and is a trinomial.

In this section, you will learn how to perform such divisions.

# The Steps in Dividing a Polynomial by a Binomial

Dividing a polynomial by a binomial may remind you of long division. Let's review long division of whole numbers by dividing 3983 by 26.





The quotient is 153 and the remainder is 5. This can be written as

Quatiant	1 - 2 5	Remainder
Quotient	$153 \overline{26}$ .	Divisor

This answer can be checked. Multiply the divisor and the quotient. Then add the remainder. If the result is the dividend, the answer is correct. In this case, we have



Because we obtained the dividend, the answer to the division problem,  $153\frac{5}{26}$ , is correct.

When a divisor is a binomial, the four steps used to divide whole numbersdivide, multiply, subtract, bring down the next term – form the repetitive procedure for dividing a polynomial by a binomial.

#### EXAMPLE 1 Dividing a Polynomial by a Binomial

 $x(x + 3) = x^{2} + 3x$   $x + 3)x^{2} + 10x + 21$  $x^{2} + 3x$ 

Change signs of the polynomial being subtracted.

Divide  $x^2 + 10x + 21$  by x + 3.

**Solution** The following steps illustrate how polynomial division is very similar to numerical division.

$$(x + 3)x^2 + 10x + 21$$

Arrange the terms of the dividend  $(x^2 + 10x + 21)$  and the divisor (x + 3) in descending powers of x.

 $\frac{x}{x+3)x^2+10x+21}$ DIVIDE x<sup>2</sup> (the first term in the dividend) by x (the first term in the divisor):  $\frac{x^2}{x} = x$ . Align like terms.

MULTIPLY each term in the divisor (x + 3) by x, aligning terms of the product under like terms in the dividend.

 $x + 3)\overline{x^2 + 10x + 21}$   $x + 3)\overline{x^2 + 10x + 21}$   $x^2 + 3x$   $x^2 + 3x$  x

 $x + 3)\overline{x^{2} + 10x + 21}$   $x^{2} + 3x \qquad \downarrow$   $x^{2} + 3x \qquad \downarrow$  7x + 21BRING DOWN 21 from the original dividend and add algebraically to form a new dividend.

Use long division to divide by a polynomial containing more than one term.



The quotient is x + 7 and the remainder is 0. We will not list a remainder of 0 in the answer. Thus,

$$\frac{x^2 + 10x + 21}{x + 3} = x + 7.$$

When dividing polynomials by binomials, the answer can be checked. Find the product of the divisor and the quotient and add the remainder. If the result is the dividend, the answer to the division problem is correct. For example, let's check our work in Example 1.



Multiply the divisor and the quotient and add the remainder, 0:

$$(x + 3)(x + 7) + 0 = x^{2} + 7x + 3x + 21 + 0 = x^{2} + 10x + 21.$$
  
Divisor Quotient Remainder This is the dividend.

Because we obtained the dividend, the quotient is correct.

**CHECK POINT 1** Divide  $x^2 + 14x + 45$  by x + 9.

Before considering additional examples, let's summarize the general procedure for dividing a polynomial by a binomial.

#### Dividing A Polynomial by a Binomial

- 1. Arrange the terms of both the dividend and the divisor in descending powers of the variable.
- **2**. **Divide** the first term in the dividend by the first term in the divisor. The result is the first term of the quotient.
- **3**. **Multiply** every term in the divisor by the first term in the quotient. Write the resulting product beneath the dividend with like terms lined up.
- 4. Subtract the product from the dividend.
- **5**. **Bring down** the next term in the original dividend and write it next to the remainder to form a new dividend.
- **6.** Use this new expression as the dividend and repeat this process until the remainder can no longer be divided. This will occur when the degree of the remainder (the highest exponent on a variable in the remainder) is less than the degree of the divisor.

 $2x^2$ 

In our next division, we will obtain a nonzero remainder.

EXAMPLE 2 Dividing a Polynomial by a Binomial Divide:  $\frac{7x - 9 - 4x^2 + 4x^3}{2x - 1}$ .

**Solution** We begin by writing the dividend in descending powers of *x*.

$$7x - 9 - 4x^2 + 4x^3 = 4x^3 - 4x^2 + 7x - 9$$

Think of 9 as  $9x^0$ . The powers descend from 3 to 0.

 $2x - 1)\overline{4x^3 - 4x^2 + 7x - 9}$  This is the problem with the dividend

in descending powers of x.

$$2x^{2}$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$2x^{2}$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2}$$

$$2x^{2}$$

$$2x^{2}$$

$$2x^{2}$$

$$2x^{2}$$

$$2x^{2}$$

$$2x^{2}$$

$$2x^{2}$$

$$2x - 1\overline{\smash{\big)}4x^3 - 4x^2 + 7x - 9} \qquad \text{subtract: } 4x^3 - 4x^2 - (4x^3 - 2x^2) \\ \stackrel{\ominus}{=} \frac{4x^3 \stackrel{\oplus}{=} 2x^2}{-2x^2} \qquad = 4x^3 - 4x^2 - 4x^3 + 2x^2 \\ = -2x^2.$$

Change signs of the polynomial being subtracted.

$$2x^{2}$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} \downarrow \qquad -2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$2x - 1)\overline{4x^{3} - 4x^{2} + 7x - 9}$$

$$4x^{3} - 2x^{2} + 7x$$

$$-2x^{2} + 7x$$

$$4x^{3} - 2x^{2} + 7x$$

$$-2x^{2} + 7x$$

$$= -2x^{2} + 7x - (-2x^{2} + x)$$

$$= -2x^{2} + 7x + 2x^{2} - 2x^{2} + 7x + 2x^{2} + 7x + 2x^{2} - 2x^{2} + 7x + 7x$$

x

$$3|2x-1| = 6x-3$$

$$2x - 1\overline{)4x^{3} - 4x^{2} + 7x - 9} = \frac{2x^{2} - x}{6x - 9}$$

$$3|2x-1| = 6x-3$$

$$2x - 1\overline{)4x^{3} - 4x^{2} + 7x - 9} = \frac{2x^{2} - x + 3}{6x - 9}$$

$$2x - 1\overline{)4x^{3} - 4x^{2} + 7x - 9} = \frac{2x^{2} - x + 3}{6x - 9}$$

$$2x - 1\overline{)4x^{3} - 4x^{2} + 7x - 9} = \frac{2x^{2} - x + 3}{6x - 9}$$

$$3|2x-1| = 6x - 3$$

$$2x - 1\overline{)4x^{3} - 4x^{2} + 7x - 9} = \frac{4x^{3} - 2x^{2}}{-2x^{2} + 7x} = \frac{2x^{2} + x}{6x - 9}$$

$$3|2x - 1| = 6x - 3$$

$$2x - 1\overline{)4x^{3} - 4x^{2} + 7x - 9} = \frac{6x - 3}{6x - 9}$$

$$4x^{3} - 2x^{2} = -2x^{2} + 7x = \frac{6x - 9}{-2x^{2} + 7x} = \frac{6x - 9 - (6x - 3)}{-2x^{2} + 7x} = -6.$$

$$3|2x - 1| = 6x - 3$$

$$4x^{3} - 2x^{2} = -6.$$

$$4x^{3} - 2x^{2} = -6.$$

$$3|2x - 1| = 6x - 3$$

$$4x^{3} - 2x^{2} = -6.$$

$$3|2x - 1| = 6x - 3$$

$$4x^{3} - 2x^{2} = -6.$$

$$4x^{3} - 2x^{2} = -6.$$
Remainder

The quotient is  $2x^2 - x + 3$  and the remainder is -6. When there is a nonzero remainder, as in this example, list the quotient, plus the remainder above the divisor. Thus,

$$\frac{7x - 9 - 4x^2 + 4x^3}{2x - 1} = 2x^2 - x + 3 + \frac{-6}{2x - 1}$$
Remainder above divisor

or

$$\frac{7x - 9 - 4x^2 + 4x^3}{2x - 1} = 2x^2 - x + 3 - \frac{6}{2x - 1}$$

Check this result by showing that the product of the divisor and the quotient,

$$(2x-1)(2x^2-x+3)$$

plus the remainder, -6, is the dividend,  $7x - 9 - 4x^2 + 4x^3$ .

CHECK POINT 2 Divide:  $\frac{6x + 8x^2 - 12}{2x + 3}$ .

If a power of the variable is missing in a dividend, add that power of the variable with a coefficient of 0 and then divide. In this way, like terms will be aligned as you carry out the division.

EXAMPLE 3 **Dividing a Polynomial with Missing Terms** Divide:  $\frac{8x^3 - 1}{2x - 1}$ .

**Solution** We write the dividend,  $8x^3 - 1$ , as

$$8x^3 + 0x^2 + 0x - 1.$$
  
Use a coefficient of 0 with missing terms.

By doing this, we will keep all like terms aligned.

$$4x^{2}(2x-1) = 8x^{3} - 4x^{2}$$

$$2x - 1) = 8x^{3} - 4x^{2}$$

$$2x - 1) = 8x^{3} + 0x^{2} + 0x - 1$$

$$9 = 8x^{3} - 4x^{2}$$

$$9 = 8x^{3} - 4x^{2}$$

$$4x^{2} + 0x$$

Divide 
$$\left(\frac{8x^3}{2x} = 4x^2\right)$$
, multiply  
[ $4x^2(2x - 1) = 8x^3 - 4x^2$ ],  
subtract, and bring down the next  
term.  
The new dividend is  $4x^2 + 0x$ .

The new dividend is 
$$4x^2 + 0x$$
.  
 $4x^2 + 2x$   
 $2x - 1)8x^3 + 0x^2 + 0x - 1$   
 $8x^3 - 4x^2$   
 $4x^2 + 0x$   
 $9x^2 - 2x$   
 $2x - 1$   
Divide  $\left(\frac{4x^2}{2x} = 2x\right)$ , multiply  
 $[2x(2x - 1) = 4x^2 - 2x]$ ,  
subtract, and bring down the next  
term.  
The new dividend is  $2x - 1$ .  
 $4x^2 + 2x + 1$   
 $4x^2 + 2x + 1$   
 $(2x - 1)$   
 $(2x - 1)$   
 $(2x - 1)$ 

 $\frac{4x^{2} + 2x + 1}{2x - 1)8x^{3} + 0x^{2} + 0x - 1}$   $\frac{8x^{3} - 4x^{2}}{4x^{2} + 0x}$   $\frac{1(2x - 1)}{5(2x - 1)} = 2x - 1$   $\frac{4x^{2} - 2x}{2x - 1}$   $\frac{4x^{2} - 2x}{2x - 1}$ The remainder is 0.

$$\frac{3x^3 - 1}{2x - 1} = 4x^2 + 2x + 1.$$

Check this result by showing that the product of the divisor and the quotient

$$(2x-1)(4x^2+2x+1)$$

plus the remainder, 0, is the dividend,  $8x^3 - 1$ .

CHECK POINT 3 Divide: 
$$\frac{x^3-1}{x-1}$$
.

 $\longrightarrow \ominus 2x \stackrel{\oplus}{=} 1$ Thus,

1(2x-1) = 2x - 1

 $2x(2x-1) = 4x^2 - 2x$ 

# The graphs of $y_1 = \frac{8x^3 - 1}{2x - 1}$ and $y_2 = 4x^2 + 2x + 1$ are shown below.

Using Technology **Graphic Connections** 



The graphs coincide. Thus,

$$\frac{8x^3 - 1}{2x - 1} = 4x^2 + 2x + 1.$$

8

#### **EXAMPLE 4** Long Division of Polynomials

Divide  $6x^4 + 5x^3 + 3x - 5$  by  $3x^2 - 2x$ .

**Solution** We write the dividend,  $6x^4 + 5x^3 + 3x - 5$ , as  $6x^4 + 5x^3 + 0x^2 + 3x - 5$  to keep all like terms aligned.



The division process is finished because the degree of 7x - 5, which is 1, is less than the degree of the divisor  $3x^2 - 2x$ , which is 2. The answer is

$$\frac{6x^4 + 5x^3 + 3x - 5}{3x^2 - 2x} = 2x^2 + 3x + 2 + \frac{7x - 5}{3x^2 - 2x}.$$

**CHECK POINT 4** Divide  $2x^4 + 3x^3 - 7x - 10$  by  $x^2 - 2x$ .

#### **Dividing Polynomials Using Synthetic Division**

We can use **synthetic division** to divide polynomials if the divisor is of the form x - c. This method provides a quotient more quickly than long division. Let's compare the two methods showing  $x^3 + 4x^2 - 5x + 5$  divided by x - 3.



On the next page, we observe the relationship between the polynomials in the long division process and the numbers that appear in synthetic division.

2 Divide polynomials using synthetic division.



**Synthetic Division** To divide a polynomial by x - c:

- **1**. Arrange polynomials in descending powers, with a 0 coefficient for any missing term.
- **2.** Write *c* for the divisor, x c. To the right, write the coefficients of the dividend.
- **3.** Write the leading coefficient of the dividend on the bottom row.
- **4.** Multiply *c* (in this case, 3) times the value just written on the bottom row. Write the product in the next column in the second row.
- **5.** Add the values in this new column, writing the sum in the bottom row.
- **6**. Repeat this series of multiplications and additions until all columns are filled in.

7. Use the numbers in the last row to write the quotient, plus the remainder above the divisor. The degree of the first term of the quotient is one less than the degree of the first term of the dividend. The final value in this row is the remainder.

Example  $(x-3)x^3 + 4x^2 - 5x + 5$ 3 1 4 -5 5 3 1 4 -5 5  $\downarrow$  Bring down 1. 3 1 4 -5 5 Multiply by 3:  $3 \cdot 1 = 3$ . 3 1 4 -5 5  $3 \quad \text{Add.} \\ 7 \quad \checkmark$ 1 4 -5 5 3 21 Add. 16 Multiply by 3:  $3 \cdot 7 = 21$ . 1 4 -5 5 21 48 Add. 3 7 16 53 Multiply by 3:  $3 \cdot 16 = 48$ .



#### EXAMPLE 5 **Using Synthetic Division**

Use synthetic division to divide  $5x^3 + 6x + 8$  by x + 2.

**Solution** The divisor must be in the form x - c. Thus, we write x + 2 as x - (-2). This means that c = -2. Writing a 0 coefficient for the missing  $x^2$ -term in the dividend, we can express the division as follows:

$$x - (-2)\overline{)5x^3 + 0x^2 + 6x + 8}.$$

Now we are ready to set up the problem so that we can use synthetic division.

Use the coefficients of the dividend  

$$5x^3 + 0x^2 + 6x + 8$$
 in descending powers of x.  
his is c in  
 $x - (-2)$ . -2 5 0 6 8

We begin the synthetic division process by bringing down 5. This is followed by a series of multiplications and additions.

1. Bring down 5.  
2. Multiply: 
$$-2(5) = -10$$
.  
3. Add:  $0 + (-10) = -10$ .  
3. Add:  $0 + (-10) = -10$ .  
3. Add:  $0 + (-10) = -10$ .  
4. Multiply:  $-2(-10) = 20$ .  
5. Add:  $6 + 20 = 26$ .  
5. Add:  $8 + (-52) = -44$ .  
5. Add. 5.  
5. Add:  $8 + (-52) = -44$ .  
5. Add. 5.  
5. Add:  $8 + (-52) = -44$ .  
5. Add. 5.  
5. Add:  $8 + (-52) = -44$ .  
5. Add. 5.  
5. Add:  $8 + (-52) = -44$ .  
5. Add. 5.  
5. Add.

The numbers in the last row represent the coefficients of the quotient and the remainder. The degree of the first term of the quotient is one less than that of the dividend. Because the degree of the dividend,  $5x^3 + 6x + 8$ , is 3, the degree of the quotient is 2. This means that the 5 in the last row represents  $5x^2$ .



Thus,

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In Exercises 1–40, divide as indicated. Check each answer by showing that the product of the divisor and the quotient, plus the remainder, is the dividend.

1.  $\frac{x^2 + 6x + 8}{x + 2}$ **2.**  $\frac{x^2 + 7x + 10}{x + 5}$ **3.**  $\frac{2x^2 + x - 10}{x - 2}$ 4.  $\frac{2x^2 + 13x + 15}{x + 5}$ 5.  $\frac{x^2 - 5x + 6}{x - 3}$ 6.  $\frac{x^2 - 2x - 24}{x + 4}$ **8.**  $\frac{2y^2 - 13y + 21}{y - 3}$ 7.  $\frac{2y^2 + 5y + 2}{y + 2}$ 9.  $\frac{x^2 - 3x + 4}{x + 2}$ 10.  $\frac{x^2 - 7x + 5}{x + 3}$ **11.**  $\frac{5y+10+y^2}{y+2}$ 12.  $\frac{-8y + y^2 - 9}{y - 3}$ **13.**  $\frac{x^3 - 6x^2 + 7x - 2}{x - 1}$ 14.  $\frac{x^3 + 3x^2 + 5x + 3}{x + 1}$ **15.**  $\frac{12y^2 - 20y + 3}{2y - 3}$  **16.**  $\frac{4y^2 - 8y - 5}{2y + 1}$ **17.**  $\frac{4a^2 + 4a - 3}{2a - 1}$  **18.**  $\frac{2b^2 - 9b - 5}{2b + 1}$ **19.**  $\frac{3y - y^2 + 2y^3 + 2}{2y + 1}$ **20.**  $\frac{9y + 18 - 11y^2 + 12y^3}{4y + 3}$ **21.**  $\frac{6x^2 - 5x - 30}{2x - 5}$ **22.**  $\frac{4y^2 + 8y + 3}{2y - 1}$ **23.**  $\frac{x^3 + 4x - 3}{x - 2}$ **24.**  $\frac{x^3 + 2x^2 - 3}{x - 2}$ 

25. 
$$\frac{4y^3 + 8y^2 + 5y + 9}{2y + 3}$$
  
26. 
$$\frac{2y^3 - y^2 + 3y + 2}{2y + 1}$$
  
27. 
$$\frac{6y^3 - 5y^2 + 5}{3y + 2}$$
  
28. 
$$\frac{4y^3 + 3y + 5}{2y - 3}$$
  
29. 
$$\frac{27x^3 - 1}{3x - 1}$$
  
30. 
$$\frac{8x^3 + 27}{2x + 3}$$
  
31. 
$$\frac{81 - 12y^3 + 54y^2 + y^4 - 108y}{y - 3}$$
  
32. 
$$\frac{8y^3 + y^4 + 16 + 32y + 24y^2}{y + 2}$$
  
33. 
$$\frac{4y^2 + 6y}{2y - 1}$$
  
34. 
$$\frac{10x^2 - 3x}{x + 3}$$
  
35. 
$$\frac{y^4 - 2y^2 + 5}{y - 1}$$
  
36. 
$$\frac{y^4 - 6y^2 + 3}{y - 1}$$
  
37. 
$$(4x^4 + 3x^3 + 4x^2 + 9x - 6) \div (x^2 + 3)$$

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**38.**  $(3x^5 - x^3 + 4x^2 - 12x - 8) \div (x^2 - 2)$ 

**39.**  $(15x^4 + 3x^3 + 4x^2 + 4) \div (3x^2 - 1)$ 

**40.** 
$$(18x^4 + 9x^3 + 3x^2) \div (3x^2 + 1)$$

In Exercises 41–58, divide using synthetic division. In the first two exercises, begin the process as shown.

**41.** 
$$(2x^2 + x - 10) \div (x - 2)$$
 2 2 1 -10  
**42.**  $(x^2 + x - 2) \div (x - 1)$  1 1 1 -2  
**43.**  $(3x^2 + 7x - 20) \div (x + 5)$ 

44. 
$$(5x^2 - 12x - 8) \div (x + 3)$$
  
45.  $(4x^3 - 3x^2 + 3x - 1) \div (x - 1)$   
46.  $(5x^3 - 6x^2 + 3x + 11) \div (x - 2)$   
47.  $(6x^5 - 2x^3 + 4x^2 - 3x + 1) \div (x - 2)$   
48.  $(x^5 + 4x^4 - 3x^2 + 2x + 3) \div (x - 3)$   
49.  $(x^2 - 5x - 5x^3 + x^4) \div (5 + x)$   
50.  $(x^2 - 6x - 6x^3 + x^4) \div (6 + x)$   
51.  $(3x^3 + 2x^2 - 4x + 1) \div (x - \frac{1}{3})$   
52.  $(2x^4 - x^3 + 2x^2 - 3x + 1) \div (x - \frac{1}{2})$ 

**53.** 
$$\frac{x^5 + x^3 - 2}{x - 1}$$
**54.** 
$$\frac{x^7 + x^5 - 10x^3 + 12}{x + 2}$$

**55.** 
$$\frac{x^4 - 256}{x - 4}$$
  
**56.**  $\frac{x^7 - 128}{x - 2}$   
**57.**  $\frac{2x^5 - 3x^4 + x^3 - x^2 + 2x - 1}{x + 2}$ 

**58.** 
$$\frac{x^5 - 2x^4 - x^3 + 3x^2 - x + 1}{x - 2}$$

#### **Practice PLUS**

In Exercises 59-68, divide as indicated.

**59.**  $\frac{x^4 + y^4}{x + y}$ **60.**  $\frac{x^5 + y^5}{x + y}$ 

61. 
$$\frac{3x^{4} + 5x^{3} + 7x^{2} + 3x - 2}{x^{2} + x + 2}$$
62. 
$$\frac{x^{4} - x^{3} - 7x^{2} - 7x - 2}{x^{2} - 3x - 2}$$
63. 
$$\frac{4x^{3} - 3x^{2} + x + 1}{x^{2} + x + 1}$$
64. 
$$\frac{x^{4} - x^{2} + 1}{x^{2} + x + 1}$$
65. 
$$\frac{x^{5} - 1}{x^{2} - x + 2}$$
66. 
$$\frac{5x^{5} - 7x^{4} + 3x^{3} - 20x^{2} + 28x - 12}{x^{3} - 4}$$
67. 
$$\frac{4x^{3} - 7x^{2}y - 16xy^{2} + 3y^{3}}{x - 3y}$$
68. 
$$\frac{12x^{3} - 19x^{2}y + 13xy^{2} - 10y^{3}}{4x - 5y}$$

- **69.** Divide the difference between  $4x^3 + x^2 2x + 7$  and  $3x^3 2x^2 7x + 4$  by x + 1.
- **70.** Divide the difference between  $4x^3 + 2x^2 x 1$  and  $2x^3 x^2 + 2x 5$  by x + 2.

#### **Application Exercises**

**71**. Write a simplified polynomial that represents the length of the rectangle.



**72**. Write a simplified polynomial that represents the measure of the base of the parallelogram.



You just signed a contract for a new job. The salary for the first year is \$30,000 and there is to be a percent increase in your salary each year. The algebraic expression

$$\frac{30,000x^n - 30,000}{x - 1}$$

describes your total salary over n years, where x is the sum of 1 and the yearly percent increase, expressed as a decimal. Use this information to solve Exercises 73–74.

- **73. a.** Use the given expression and write a quotient of polynomials that describes your total salary over three years.
  - **b.** Simplify the expression in part (a) by performing the division.
  - **c.** Suppose you are to receive an increase of 5% per year. Thus, *x* is the sum of 1 and 0.05, or 1.05. Substitute 1.05 for *x* in the expression in part (a) as well as in the simplified form of the expression in part (b). Evaluate each expression. What is your total salary over the three-year period?
- **74. a.** Use the given expression and write a quotient of polynomials that describes your total salary over four years.
  - **b.** Simplify the expression in part (a) by performing the division.
  - **c.** Suppose you are to receive an increase of 8% per year. Thus, *x* is the sum of 1 and 0.08, or 1.08. Substitute 1.08 for *x* in the expression in part (a) as well as in the simplified form of the expression in part (b). Evaluate each expression. What is your total salary over the four-year period?

#### Writing in Mathematics

- **75.** In your own words, explain how to divide a polynomial by a binomial. Use  $\frac{x^2 + 4}{x + 2}$  in your explanation.
- **76.** When dividing a polynomial by a binomial, explain when to stop dividing.
- **77.** After dividing a polynomial by a binomial, explain how to check the answer.
- **78.** When dividing a binomial into a polynomial with missing terms, explain the advantage of writing the missing terms with zero coefficients.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 79–82, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning. Each statement applies to the division problem

$$\frac{x^3+1}{x+1}$$

**79.** The purpose of writing  $x^3 + 1$  as  $x^3 + 0x^2 + 0x + 1$  is to keep all like terms aligned.

- **80.** Rewriting  $x^3 + 1$  as  $x^3 + 0x^2 + 0x + 1$  can change the value of the variable expression for certain values of x.
- **81.** There's no need to apply the long-division process to this problem because I can work the problem in my head and see that the quotient must be  $x^2 + 1$ .
- **82**. The degree of the quotient must be 3 1.

In Exercises 83–86, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

**83.** If  $4x^2 + 25x - 3$  is divided by 4x + 1, the remainder is 9.

- **84.** If polynomial division results in a remainder of zero, then the product of the divisor and the quotient is the dividend.
- **85.** A nonzero remainder indicates that the answer to a polynomial long-division problem is not a polynomial.
- **86**. When a polynomial is divided by a binomial, the division process stops when the last term of the dividend is brought down.
- **87.** When a certain polynomial is divided by 2x + 4, the quotient is

$$x - 3 + \frac{17}{2x + 4}.$$

What is the polynomial?

- **88.** Find the number k such that when  $16x^2 2x + k$  is divided by 2x 1, the remainder is 0.
- **89**. Describe the pattern that you observe in the following quotients and remainders.

$$\frac{x^3 - 1}{x + 1} = x^2 - x + 1 - \frac{2}{x + 1}$$
$$\frac{x^5 - 1}{x + 1} = x^4 - x^3 + x^2 - x + 1 - \frac{2}{x + 1}$$

Use this pattern to find  $\frac{x^7 - 1}{x + 1}$ . Verify your result by dividing.

#### **Technology Exercises**

In Exercises 90–94, use a graphing utility to determine whether the divisions have been performed correctly. Graph each side of the given equation in the same viewing rectangle. The graphs should coincide. If they do not, correct the expression on the right side by using polynomial division. Then use your graphing utility to show that the division has been performed correctly.

**90.** 
$$\frac{x^2 - 4}{x - 2} = x + 2$$
  
**91.**  $\frac{x^2 - 25}{x - 5} = x - 5$ 

2

**92.** 
$$\frac{2x^2 + 13x + 15}{x - 5} = 2x + 3$$

**93.** 
$$\frac{6x^2 + 16x + 8}{3x + 2} = 2x - 4$$
  
**94.**  $\frac{x^3 + 3x^2 + 5x + 3}{x + 1} = x^2 - 2x + 3$ 

#### **Review Exercises**

**95.** Solve the system:

$$7x - 6y = 17$$
$$3x + y = 18.$$

(Section 4.3, Example 2)

**96**. What is 6% of 20? (Section 2.4, Example 7)

**97.** Solve: 
$$\frac{x}{3} + \frac{2}{5} = \frac{x}{5} - \frac{2}{5}$$
 (Section 2.3, Example 4)

#### **Preview Exercises**

*Exercises* 98–100 will help you prepare for the material covered in the next section.

**98. a.** Find the missing exponent, designated by the question mark, in each final step.

$$\frac{7^3}{7^5} = \frac{7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7} = \frac{1}{7^7}$$
$$\frac{7^3}{7^5} = 7^{3-5} = 7^7$$

**b.** Based on your two results for  $\frac{7^3}{7^5}$ , what can you conclude?

**99.** Simplify: 
$$\frac{(2x^3)^4}{x^{10}}$$
.  
**100.** Simplify:  $\left(\frac{x^5}{x^2}\right)^3$ .



# **Objectives**

- 1 Use the negative exponent rule.
- 2 Simplify exponential expressions.
- 3 Convert from scientific notation to decimal notation.
- 4 Convert from decimal notation to scientific notation.
- 5 Compute with scientific notation.
- 6 Solve applied problems using scientific notation.

# Negative Exponents and Scientific Notation



We frequently encounter very large and very small numbers. Governments throughout the world are concerned about the billions of tons of carbon dioxide that the global population of 6.5 billion people release into the atmosphere each year. In the photo shown above, the national debt of the United States was about \$8.2 trillion. A typical atom has a diameter of about one-ten-billionth of a meter. Exponents provide a way of putting these large and small numbers in perspective.

1 Use the negative exponent rule.

#### **Negative Integers as Exponents**

A nonzero base can be raised to a negative power. The quotient rule can be used to help determine what a negative integer as an exponent should mean. Consider the quotient of  $b^3$  and  $b^5$ , where b is not zero. We can determine this quotient in two ways.



Notice that  $\frac{b^3}{b^5}$  equals both  $b^{-2}$  and  $\frac{1}{b^2}$ . This means that  $b^{-2}$  must equal  $\frac{1}{b^2}$ . This example is a special case of the **negative exponent rule**.

#### The Negative Exponent Rule

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n}.$$

#### Study Tip

A negative exponent does not make the value of an expression negative. For example,

$$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

is positive. Avoid these common errors:

**Incorrect!** 



#### EXAMPLE 1 Using the Negative Exponent Rule

Use the negative exponent rule to write each expression with a positive exponent. Then simplify the expression.

a.  $7^{-2}$  b.  $4^{-3}$  c.  $(-2)^{-4}$  d.  $-2^{-4}$  e.  $5^{-1}$ 

#### Solution



e.  $5^{-1} = \frac{1}{5^1} = \frac{1}{5}$ 

CHECK POINT 1 Use the negative exponent rule to write each expression with a positive exponent. Then simplify the expression.

a.	6 <sup>-2</sup>	b.	5 <sup>-3</sup>	c. $(-3)^{-4}$
d.	-3 <sup>-4</sup>	e.	8 <sup>-1</sup>	

Negative exponents can also appear in denominators. For example,

$$\frac{1}{2^{-10}} = \frac{1}{\frac{1}{2^{10}}} = 1 \div \frac{1}{2^{10}} = 1 \cdot \frac{2^{10}}{1} = 2^{10}.$$

In general, if a negative exponent appears in a denominator, an expression can be written with a positive exponent using

$$\frac{1}{b^{-n}} = b^n$$

For example,

$$\frac{1}{2^{-3}} = 2^3 = 8 \quad \text{and} \quad \frac{1}{(-6)^{-2}} = (-6)^2 = 36.$$
Change only the sign of the exponent and not the sign of the base, -6.

#### Negative Exponents in Numerators and Denominators

If b is any real number other than 0 and n is a natural number, then

$$b^{-n} = \frac{1}{b^n}$$
 and  $\frac{1}{b^{-n}} = b^n$ .

When a negative number appears as an exponent, switch the position of the base (from numerator to denominator or from denominator to numerator) and make the exponent positive. The sign of the base does not change.

#### EXAMPLE 2 Using Negative Exponents

Write each expression with positive exponents only. Then simplify, if possible.

a. 
$$\frac{4^{-3}}{5^{-2}}$$
 b.  $\left(\frac{3}{4}\right)^{-2}$  c.  $\frac{1}{4x^{-3}}$  d.  $\frac{x^{-5}}{y^{-1}}$ 

Solution

**a.** 
$$\frac{4^{-3}}{5^{-2}} = \underbrace{5 \cdot 5}_{4^3} = \frac{5 \cdot 5}{4 \cdot 4 \cdot 4} = \frac{25}{64}$$

Switch the position of the bases and make the exponents positive.

**b.** 
$$\left(\frac{3}{4}\right)^{-2} = \frac{3^{-2}}{4^{-2}} = \frac{4^2}{3^2} = \frac{4 \cdot 4}{3 \cdot 3} = \frac{16}{9}$$

Switch the position of the bases and make the exponents positive.

c. 
$$\frac{1}{4x^{-3}} = \frac{x^3}{4}$$

Switch the position of the base and make the exponent positive. Note that only x is raised to the -3 power.

d. 
$$\frac{x^{-5}}{v^{-1}} = \frac{y^1}{x^5} = \frac{y}{x^5}$$

simplify, if possible.

**CHECK POINT 2** Write each expression with positive exponents only. Then

a. 
$$\frac{2^{-3}}{7^{-2}}$$
 b.  $\left(\frac{4}{5}\right)^{-2}$  c.  $\frac{1}{7y^{-2}}$  d.  $\frac{x^{-1}}{y^{-8}}$ 

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2 Simplify exponential expressions.

#### Simplifying Exponential Expressions

Properties of exponents are used to simplify exponential expressions. An exponential expression is **simplified** when

- Each base occurs only once.
- No parentheses appear.
- No powers are raised to powers.
- No negative or zero exponents appear.

#### Simplifying Exponential Expressions

**1.** If necessary, be sure that each base appears only once, using

$$b^m \cdot b^n = b^{m+n}$$
 or  $\frac{b^m}{b^n} = b^{m-n}$ .  $x^4 \cdot x^3 = x^{4+3} = x^7$ 

2. If necessary, remove parentheses using

$$(ab)^n = a^n b^n$$
 or  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ .  $(xy)^3 = x^3 y^3$ 

**3**. If necessary, simplify powers to powers using

$$(b^m)^n = b^{mn}.$$
  $(x^4)^3 = x^{4\cdot 3} = x^{12}$ 

**Example** 

**4**. If necessary, rewrite exponential expressions with zero powers as  $1 (b^0 = 1)$ . Furthermore, write the answer with positive exponents using

$$b^{-n} = \frac{1}{b^n}$$
 or  $\frac{1}{b^{-n}} = b^n$ .  $\frac{x^5}{x^8} = x^{5-8} = x^{-3} = \frac{1}{x^3}$ 

The following examples show how to simplify exponential expressions. In each example, assume that any variable in a denominator is not equal to zero.

#### EXAMPLE 3 Simplifying an Exponential Expression

Simplify:  $x^{-9} \cdot x^4$ .

Solution



**CHECK POINT 3** Simplify:  $x^{-12} \cdot x^2$ .



### Study Tip

There is often more than one way to simplify an exponential expression. For example, you may prefer to simplify Example 3 as follows:

$$x^{-9} \cdot x^4 = \frac{x^4}{x^9} = x^{4-9} = x^{-5} = \frac{1}{x^5}.$$

#### Solution

a. 
$$\frac{x^4}{x^{20}} = x^{4-20} = x^{-16} = \frac{1}{x^{16}}$$
  
b.  $\frac{25x^6}{5x^8} = \frac{25}{5} \cdot \frac{x^6}{x^8} = 5x^{6-8} = 5x^{-2} = \frac{5}{x^2}$   
c.  $\frac{10y^7}{-2y^{10}} = \frac{10}{-2} \cdot \frac{y^7}{y^{10}} = -5y^{7-10} = -5y^{-3} = -\frac{5}{y^3}$ 

#### **CHECK POINT 4** Simplify:

**a.** 
$$\frac{x^2}{x^{10}}$$
 **b.**  $\frac{75x^3}{5x^9}$  **c.**  $\frac{50y^8}{-25y^{14}}$ .

EXAMPLE 5  
Simplify: 
$$\frac{(5x^3)^2}{x^{10}}$$
.

# Simplifying an Exponential Expression

#### Solution

$$\frac{(5x^3)^2}{x^{10}} = \frac{5^2(x^3)^2}{x^{10}}$$
 Raise each factor in the product to the second power  
using  $(ab)^n = a^n b^n$ .  
$$= \frac{5^2 x^{3/2}}{x^{10}}$$
 Raise powers to powers using  $(b^m)^n = b^{mn}$ .  
$$= \frac{25x^6}{x^{10}}$$
 Simplify.  
$$= 25x^{6-10}$$
 When dividing with the same base, subtract exponents:  
 $\frac{b^m}{b^n} = b^{m-n}$ .  
$$= 25x^{-4}$$
 Simplify. The base, x, now appears only once.  
$$= \frac{25}{x^4}$$
 Rewrite with a positive exponent using  $b^{-n} = \frac{1}{b^n}$ .

#### EXAMPLE 6

# Simplifying an Exponential Expression

Simplify: 
$$\left(\frac{x^5}{x^2}\right)^{-3}$$
.

# Solution

Method 1. First perform the division within the parentheses.

$$\frac{x^5}{x^2} \bigg)^{-3} = (x^{5-2})^{-3}$$
 Within parentheses, divide by subtracting exponents:  $\frac{b^m}{b^n} = b^{m-n}$ .  

$$= (x^3)^{-3}$$
 Simplify. The base, x, now appears only once.  

$$= x^{3(-3)}$$
 Raise powers to powers:  $(b^m)^n = b^{mn}$ .  

$$= x^{-9}$$
 Simplify.  

$$= \frac{1}{x^9}$$
 Rewrite with a positive exponent using  $b^{-n} = \frac{1}{b^n}$ .

**Method 2.** Remove parentheses first by raising the numerator and the denominator to the -3 power.

Which method do you prefer?

CHECK POINT 6 Simplify:

plify:	$\left(\frac{x^8}{x^4}\right)^{-5}.$	

Table 5.1Names of La Numbers	
$10^{2}$	hundred
10 <sup>3</sup>	thousand
10 <sup>6</sup>	million
10 <sup>9</sup>	billion
10 <sup>12</sup>	trillion
10 <sup>15</sup>	quadrillion
$10^{18}$	quintillion
$10^{21}$	sextillion
10 <sup>24</sup>	septillion
10 <sup>27</sup>	octillion
$10^{30}$	nonillion
$10^{100}$	googol
10 <sup>googol</sup>	googolplex

3 Convert from scientific notation to decimal notation.

### Scientific Notation

As of December 2006, the national debt of the United States was about \$8.6 trillion. This is the amount of money the government has had to borrow over the years, mostly by selling bonds, because it has spent more than it has collected in taxes. A stack of \$1 bills equaling the national debt would rise to twice the distance from Earth to the moon. Because a trillion is  $10^{12}$  (see **Table 5.1**), the national debt can be expressed as

 $8.6 \times 10^{12}$ .

The number 8.6  $\times$  10<sup>12</sup> is written in a form called *scientific notation*.

#### **Scientific Notation**

A positive number is written in scientific notation when it is expressed in the form

 $a \times 10^n$ ,

where *a* is a number greater than or equal to 1 and less than  $10 (1 \le a < 10)$  and *n* is an integer.

It is customary to use the multiplication symbol,  $\times$ , rather than a dot, when writing a number in scientific notation.

Here are two examples of numbers in scientific notation:

- Each day,  $2.6 \times 10^7$  pounds of dust from the atmosphere settle on Earth.
- The length of the AIDS virus is  $1.1 \times 10^{-4}$  millimeter.

We can use *n*, the exponent on the 10 in  $a \times 10^n$ , to change a number in scientific notation to decimal notation. If *n* is **positive**, move the decimal point in *a* to the **right** *n* places. If *n* is **negative**, move the decimal point in *a* to the **left** |n| places.

#### EXAMPLE 7 **Converting from Scientific to Decimal Notation**

Write each number in decimal notation:

a.  $2.6 \times 10^7$ **b.**  $1.1 \times 10^{-4}$ .

**Solution** In each case, we use the exponent on the 10 to move the decimal point. In part (a), the exponent is positive, so we move the decimal point to the right. In part (b), the exponent is negative, so we move the decimal point to the left.



To convert a positive number from decimal notation to scientific notation, we reverse the procedure of Example 7.

#### Converting from Decimal to Scientific Notation

Write the number in the form  $a \times 10^{n}$ .

- Determine a, the numerical factor. Move the decimal point in the given number to obtain a number greater than or equal to 1 and less than 10.
- Determine n, the exponent on  $10^n$ . The absolute value of n is the number of places the decimal point was moved. The exponent n is positive if the given number is greater than 10 and negative if the given number is between 0 and 1.

#### Using Technology

Convert from decimal

notation to scientific

notation.

You can use your calculator's EE (enter exponent) or EXP key to convert from decimal to scientific notation. Here is how it's done for 0.000023:

#### Many Scientific Calculators

Keystrol	kes	Display
.000023	EE =	2.3 - 05
Many Graphing Calculators		
Use the mode setting for scientific notation.		
Keystrokes Display		
.000023	ENTER	2.3E-5

#### **Converting from Decimal Notation** to Scientific Notation

Write each number in scientific notation:

a. 4.600.000 b. 0.000023.

#### Solution



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**CHECK POINT 8** Write each number in scientific notation:

a. 7,410,000,000

b. 0.00000092.

Compute with scientific notation.

#### Computations with Scientific Notation

Properties of exponents are used to perform computations with numbers that are expressed in scientific notation.

#### Computations with Numbers in Scientific Notation

#### **Multiplication**

$$a \times 10^n$$
 ×  $(b \times 10^m) = (a \times b) \times 10^{n+1}$ 

Add the exponents on 10 and multiply the other parts of the numbers separately.

- m

Division

$$\frac{a \times 10^n}{b \times 10^m} = \left(\frac{a}{b}\right) \times 10^{n-m}$$

Subtract the exponents on 10 and divide the other parts of the numbers separately.

#### **Exponentiation**

 $(a \times 10^n)^m = a^n \times 10^{nm}$ Multiply exponents on 10 and raise the other part of the number to the power.

After the computation is completed, the answer may require an adjustment before it is expressed in scientific notation.

#### **Computations with Scientific Notation** EXAMPLE 9

Perform the indicated computations, writing the answers in scientific notation:

b.  $\frac{1.2 \times 10^6}{4.8 \times 10^{-3}}$ 

a.  $(4 \times 10^5)(2 \times 10^9)$ 

$$(2 \times 10)$$

c. 
$$(5 \times 10^{-4})^3$$
.

#### Solution

a. 
$$(4 \times 10^5)(2 \times 10^9) = (4 \times 2) \times (10^5 \times 10^9)$$
  
=  $8 \times 10^{5+9}$ 

 $= 8 \times 10^{14}$ 

Add the exponents on 10 and multiply the other parts.

Simplify.

b. 
$$\frac{1.2 \times 10^{6}}{4.8 \times 10^{-3}} = \left(\frac{1.2}{4.8}\right) \times \left(\frac{10^{6}}{10^{-3}}\right)$$
 Regroup.  
=  $0.25 \times 10^{6-(-3)}$  Subtract the exponents on 10 and divide the other parts.  
=  $0.25 \times 10^{9}$  Simplify. Because 0.25 is not between 1 and 10, it must be written in scientific notation.  
=  $2.5 \times 10^{-1} \times 10^{9}$  0.25 =  $2.5 \times 10^{-1}$   
=  $2.5 \times 10^{-1+9}$  Add the exponents on 10.  
=  $2.5 \times 10^{8}$  Simplify.

Using Technology

 $(4 \times 10^5)(2 \times 10^9)$  On а Calculator:

Many Scientific Calculators

Display: 8. 14

#### Many Graphing Calculators



Display: 8E14

SBN 0-558-95172-4

c. 
$$(5 \times 10^{-4})^3 = 5^3 \times (10^{-4})^3$$
 (ab)<sup>n</sup> = a<sup>n</sup>b<sup>n</sup>. Cube each factor in parentheses.  
=  $125 \times 10^{-4\cdot3}$  Multiply the exponents and cube the other part of the number.  
=  $125 \times 10^{-12}$  Simplify. 125 must be written in scientific notation.  
=  $1.25 \times 10^2 \times 10^{-12}$  125 =  $1.25 \times 10^2$   
=  $1.25 \times 10^{2+(-12)}$  Add the exponents on 10.  
=  $1.25 \times 10^{-10}$  Simplify.

× 107

a. 
$$(3 \times 10^8)(2 \times 10^2)$$
  
b.  $\frac{8.4 \times 10^4}{4 \times 10^{-4}}$   
c.  $(4 \times 10^{-2})^3$ .

### **Applications: Putting Numbers in Perspective**

Due to tax cuts and spending increases, the United States began accumulating large deficits in the 1980s. To finance the deficit, the government had borrowed \$8.6 trillion as of December 2006. The graph in **Figure 5.5** shows the national debt increasing over time.



Example 10 shows how we can use scientific notation to comprehend the meaning of a number such as 8.6 trillion.

#### EXAMPLE 10 The National Debt

As of December 2006, the national debt was \$8.6 trillion, or  $8.6 \times 10^{12}$  dollars. At that time, the U.S. population was approximately 300,000,000 (300 million), or  $3 \times 10^8$ . If the national debt was evenly divided among every individual in the United States, how much would each citizen have to pay?

**Solution** The amount each citizen must pay is the total debt,  $8.6 \times 10^{12}$  dollars, divided by the number of citizens,  $3 \times 10^8$ .

$$\frac{8.6 \times 10^{12}}{3 \times 10^8} = \left(\frac{8.6}{3}\right) \times \left(\frac{10^{12}}{10^8}\right)$$
$$\approx 2.87 \times 10^{12-8}$$
$$= 2.87 \times 10^4$$
$$= 28.700$$

Every U.S. citizen would have to pay approximately \$28,700 to the federal government to pay off the national debt.

6 Solve applied problems using scientific notation.

FIGURE 5.5

Source: Office of Management and Budget

#### Using Technology

Here is the keystroke sequence for solving Example 10,  $\frac{8.6 \times 10^{12}}{3 \times 10^8}$ , using a calculator:



The quotient is displayed by pressing | = | on a scientific calculator or | ENTER | on a graphing calculator. The answer can be displayed in scientific or decimal notation. Consult your manual.

**CHECK POINT 10** Pell Grants help low-income undergraduate students pay for college. In 2006, the federal cost of this program was \$13 billion ( $$13 \times 10^9$ ) and there were 5.1 million  $(5.1 \times 10^6)$  grant recipients. How much, to the nearest hundred dollars, was the average grant?

#### **Blitzer Bonus**

The earthquake that ripped through northern California on October 17, 1989, measured 7.1 on the Richter scale, killed more than 60 people, and injured more than 2400. Shown here is San Francisco's Marina district, where shock waves tossed houses off their foundations and into the street.



#### Earthquakes and Exponents

The Richter scale is misleading because it is not actually a 1 to 8, but rather a 1 to 10 million scale. Each level indicates a tenfold increase in magnitude from the previous level, making a 7.0 earthquake a million times greater than a 1.0 quake.

The following is a translation of the Richter scale:

Magnitude (10 <sup>R–1</sup> )
$10^{1-1} = 10^0 = 1$
$10^{2-1} = 10^1 = 10$
$10^{3-1} = 10^2 = 100$
$10^{4-1} = 10^3 = 1000$
$10^{5-1} = 10^4 = 10,000$
$10^{6-1} = 10^5 = 100,000$
$10^{7-1} = 10^6 = 1,000,000$
$10^{8-1} = 10^7 = 10,000,000$





In Exercises 29–78, simplify each exponential expression. Assume that variables represent nonzero real numbers.

<b>29.</b> $x^{-8} \cdot x^3$	<b>30.</b> $x^{-11} \cdot x^5$
<b>31</b> . $(4x^{-5})(2x^2)$	<b>32.</b> $(5x^{-7})(3x^3)$
<b>33</b> . $\frac{x^3}{x^9}$	<b>34.</b> $\frac{x^5}{x^{12}}$
<b>35.</b> $\frac{y}{y^{100}}$	<b>36.</b> $\frac{y}{y^{50}}$
<b>37.</b> $\frac{30z^5}{10z^{10}}$	<b>38.</b> $\frac{45z^4}{15z^{12}}$
<b>39.</b> $\frac{-8x^3}{2x^7}$	<b>40.</b> $\frac{-15x^4}{3x^9}$
<b>41.</b> $\frac{-9a^5}{27a^8}$	<b>42.</b> $\frac{-15a^8}{45a^{13}}$
<b>43</b> . $\frac{7w^5}{5w^{13}}$	<b>44.</b> $\frac{7w^8}{9w^{14}}$
<b>45.</b> $\frac{x^3}{(x^4)^2}$	<b>46.</b> $\frac{x^5}{(x^3)^2}$
<b>47.</b> $\frac{y^{-3}}{(y^4)^2}$	<b>48.</b> $\frac{y^{-5}}{(y^3)^2}$
<b>49.</b> $\frac{(4x^3)^2}{x^8}$	<b>50.</b> $\frac{(5x^3)^2}{x^7}$
<b>51.</b> $\frac{(6y^4)^3}{y^{-5}}$	<b>52.</b> $\frac{(4y^5)^3}{y^{-4}}$
<b>53.</b> $\left(\frac{x^4}{x^2}\right)^{-3}$	<b>54.</b> $\left(\frac{x^6}{x^2}\right)^{-3}$
<b>55.</b> $\left(\frac{4x^5}{2x^2}\right)^{-4}$	<b>56.</b> $\left(\frac{6x^7}{2x^2}\right)^{-4}$
<b>57.</b> $(3x^{-1})^{-2}$	<b>58.</b> $(4x^{-1})^{-2}$
<b>59.</b> $(-2y^{-1})^{-3}$	<b>60.</b> $(-3y^{-1})^{-3}$
<b>61.</b> $\frac{2x^5 \cdot 3x^7}{15x^6}$	<b>62.</b> $\frac{3x^3 \cdot 5x^{14}}{20x^{14}}$
<b>63.</b> $(x^3)^5 \cdot x^{-7}$	<b>64.</b> $(x^4)^3 \cdot x^{-5}$
<b>65.</b> $(2y^3)^4y^{-6}$	<b>66.</b> $(3y^4)^3y^{-7}$



In Exercises 79–90, write each number in decimal notation without the use of exponents.

<b>79.</b> $8.7 \times 10^2$	<b>80.</b> 2.75 × 10 <sup>3</sup>
<b>81.</b> $9.23 \times 10^5$	<b>82.</b> $7.24 \times 10^4$
<b>83.</b> $3.4 \times 10^0$	<b>84.</b> $9.115 \times 10^{0}$
<b>85.</b> 7.9 × 10 <sup>-1</sup>	<b>86.</b> $8.6 \times 10^{-1}$
<b>87.</b> 2.15 × 10 <sup>-2</sup>	<b>88.</b> $3.14 \times 10^{-2}$
<b>89.</b> $7.86 \times 10^{-4}$	<b>90.</b> $4.63 \times 10^{-5}$

In Exercises 91–106, write each number in scientific notation.

<b>91</b> . 32,400	<b>92</b> . 327,000
<b>93</b> . 220,000,000	<b>94</b> . 370,000,000,000
<b>95.</b> 713	<b>96.</b> 623
<b>97</b> . 6751	<b>98</b> . 9832
<b>99.</b> 0.0027	<b>100.</b> 0.00083
<b>101</b> . 0.0000202	<b>102.</b> 0.00000103
<b>103</b> . 0.005	<b>104.</b> 0.006
<b>105.</b> 3.14159	<b>106.</b> 2.71828

In Exercises 107–126, perform the indicated computations. Write the answers in scientific notation.

<b>107.</b> $(2 \times 10^3)(3 \times 10^2)$	
<b>108.</b> $(3 \times 10^4)(3 \times 10^2)$	
<b>109.</b> $(2 \times 10^5)(8 \times 10^3)$	
<b>110.</b> $(4 \times 10^3)(5 \times 10^4)$	
<b>111.</b> $\frac{12 \times 10^6}{4 \times 10^2}$	112. $rac{20 imes 10^{20}}{10 imes 10^{10}}$
112 $\frac{15 \times 10^4}{15}$	<b>114.</b> $\frac{18 \times 10^2}{9 \times 10^{-3}}$
$\frac{113}{5 \times 10^{-2}}$	
<b>115</b> $\frac{15 \times 10^{-4}}{15}$	$18 \times 10^{-2}$
5 × $10^2$	$9 \times 10^{3}$
<b>117</b> $\frac{180 \times 10^6}{100}$	118. $\frac{180 \times 10^8}{2 \times 10^4}$
$2 \times 10^{3}$	
119 $3 \times 10^4$	120 $5 \times 10^2$
$12 \times 10^{-3}$	$20 \times 10^{-3}$
<b>121.</b> $(5 \times 10^2)^3$	<b>122.</b> $(4 \times 10^3)^2$
<b>123.</b> $(3 \times 10^{-2})^4$	
<b>124.</b> $(2 \times 10^{-3})^5$	
<b>125.</b> $(4 \times 10^6)^{-1}$	<b>126.</b> $(5 \times 10^4)^{-1}$

ISBN 0-558-95172-4

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#### **Practice PLUS**

In Exercises 127–134, simplify each exponential expression. Assume that variables represent nonzero real numbers.

127. 
$$\frac{(x^{-2}y)^{-3}}{(x^{2}y^{-1})^{3}}$$
128. 
$$\frac{(xy^{-2})^{-2}}{(x^{-2}y)^{-3}}$$
129. 
$$(2x^{-3}yz^{-6})(2x)^{-5}$$
130. 
$$(3x^{-4}yz^{-7})(3x)^{-3}$$
131. 
$$\left(\frac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2}$$
132. 
$$\left(\frac{x^{4}y^{5}z^{6}}{x^{-4}y^{-5}z^{-6}}\right)^{-4}$$
133. 
$$\frac{(2^{-1}x^{-2}y^{-1})^{-2}(2x^{-4}y^{3})^{-2}(16x^{-3}y^{3})^{0}}{(2x^{-3}y^{-5})^{2}}$$
134. 
$$\frac{(2^{-1}x^{-3}y^{-1})^{-2}(2x^{-6}y^{4})^{-2}(9x^{3}y^{-3})^{0}}{(2x^{-4}y^{-6})^{2}}$$

In Exercises 135–138, perform the indicated computations. *Express answers in scientific notation.* 

**135.** 
$$(5 \times 10^3)(1.2 \times 10^{-4}) \div (2.4 \times 10^2)$$
  
**136.**  $(2 \times 10^2)(2.6 \times 10^{-3}) \div (4 \times 10^3)$   
**137.**  $\frac{(1.6 \times 10^4)(7.2 \times 10^{-3})}{(3.6 \times 10^8)(4 \times 10^{-3})}$   
**138.**  $\frac{(1.2 \times 10^6)(8.7 \times 10^{-2})}{(2.9 \times 10^6)(3 \times 10^{-3})}$ 

#### **Application Exercises**

*In Exercises 139–142, rewrite the number in each statement in scientific notation.* 

- **139.** King Mongkut of Siam (the king in the musical *The King and I*) had 9200 wives.
- **140.** The top-selling music album of all time is "Their Greatest Hits" by the Eagles, selling 28,000,000 copies.
- **141.** The volume of a bacterium is 0.000000000000025 cubic meter.
- **142.** Home computers can perform a multiplication in 0.00000000036 second.

In Exercises 143–146, use  $10^6$  for one million and  $10^9$  for one billion to rewrite the number in each statement in scientific notation.

Pet Ownership in the United States



Source: American Pet Product Manufacturers Association

- 143. Americans are caregivers to 90.5 million cats.
- 144. Americans are caregivers to 73.9 million dogs.



Source: The International Institute for Strategic Studies, "The Military Balance 2005–2006"

- 145. In 2005, the United States spent \$465 billion on defense.
- **146.** In 2005, the United States spent more on defense than the four next top-spending nations combined. These nations spent \$228 billion on defense.

The bar graph shows the total amount Americans paid in federal taxes, in trillions of dollars, and the U.S. population, in millions, from 2002 through 2005. Exercises 147–148 are based on the numbers displayed by the graph.



Sources: Internal Revenue Service and U.S. Census Bureau

- **147. a.** In 2005, the United States government collected \$2.27 trillion in taxes. Express this number in scientific notation.
  - **b.** In 2005, the population of the United States was approximately 298 million. Express this number in scientific notation.
  - c. Use your scientific notation answers from parts (a) and (b) to answer this question: If the total 2005 tax collections were evenly divided among all Americans, how much would each citizen pay? Express the answer in decimal notation, rounded to the nearest dollar.

- **148. a.** In 2004, the United States government collected \$2.02 trillion in taxes. Express this number in scientific notation.
  - **b.** In 2004, the population of the United States was approximately 295 million. Express this number in scientific notation.
  - **c.** Use your scientific notation answers from parts (a) and (b) to answer this question: If the total 2004 tax collections were evenly divided among all Americans, how much would each citizen pay? Express the answer in decimal notation, rounded to the nearest dollar.
- **149.** In 2007, the population of the United States was approximately  $3.1 \times 10^8$  and each person spent about \$120 per year on ice cream. Express the total annual spending on ice cream for that year in scientific notation.
- **150.** A human brain contains  $3 \times 10^{10}$  neurons and a gorilla brain contains  $7.5 \times 10^9$  neurons. How many times as many neurons are in the brain of a human as in the brain of a gorilla?

Use the motion formula d = rt, distance equals rate times time, and the fact that light travels at the rate of  $1.86 \times 10^5$  miles per second, to solve Exercises 151–152.

- **151.** If the moon is approximately  $2.325 \times 10^5$  miles from Earth, how many seconds does it take moonlight to reach Earth?
- **152.** If the sun is approximately  $9.14 \times 10^7$  miles from Earth, how many seconds, to the nearest tenth of a second, does it take sunlight to reach Earth?

#### Writing in Mathematics

- **153.** Explain the negative exponent rule and give an example.
- 154. How do you know if an exponential expression is simplified?
- **155.** How do you know if a number is written in scientific notation?
- **156.** Explain how to convert from scientific to decimal notation and give an example.
- **157.** Explain how to convert from decimal to scientific notation and give an example.
- **158**. Describe one advantage of expressing a number in scientific notation over decimal notation.

#### **Critical Thinking Exercises**

**Make Sense?** In Exercises 159–162, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning.

- **159.** There are many exponential expressions that are equal to  $36x^{12}$ , such as  $(6x^6)^2$ ,  $(6x^3)(6x^9)$ ,  $36(x^3)^9$ , and  $6^2(x^2)^6$ .
- **160.** If  $5^{-2}$  is raised to the third power, the result is a number between 0 and 1.
- **161.** The population of Colorado is approximately  $4.6 \times 10^{12}$ .
- **162.** I wrote a number where there is no advantage to using scientific notation instead of decimal notation.

In Exercises 163–170, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

**163.** 
$$4^{-2} < 4^{-3}$$
  
**164.**  $5^{-2} > 2^{-5}$   
**165.**  $(-2)^4 = 2^{-4}$   
**166.**  $5^2 \cdot 5^{-2} > 2^5 \cdot 2^{-5}$   
**167.**  $534.7 = 5.347 \times 10^3$   
**168.**  $\frac{8 \times 10^{30}}{4 \times 10^{-5}} = 2 \times 10^{25}$ 

**169.** 
$$(7 \times 10^5) + (2 \times 10^{-3}) = 9 \times 10^2$$

**170.** 
$$(4 \times 10^3) + (3 \times 10^2) = 4.3 \times 10^3$$

**171.** The mad Dr. Frankenstein has gathered enough bits and pieces (so to speak) for  $2^{-1} + 2^{-2}$  of his creature-to-be. Write a fraction that represents the amount of his creature that must still be obtained.

#### **Technology Exercises**

- **172.** Use a calculator in a fraction mode to check any five of your answers in Exercises 1–22.
- **173.** Use a calculator to check any three of your answers in Exercises 79–90.
- **174.** Use a calculator to check any three of your answers in Exercises 91–106.
- **175.** Use a calculator with an <u>EE</u> or <u>EXP</u> key to check any four of your computations in Exercises 107–126. Display the result of the computation in scientific notation.

#### **Review Exercises**

- **176.** Solve: 8 6x > 4x 12. (Section 2.7, Example 7)
- **177.** Simplify:  $24 \div 8 \cdot 3 + 28 \div (-7)$ . (Section 1.8, Example 8)
- **178.** List the whole numbers in this set:

$$\bigg\{-4, -\frac{1}{5}, 0, \pi, \sqrt{16}, \sqrt{17}\bigg\}.$$

(Section 1.3, Example 5)

#### **Preview Exercises**

*Exercises* 179–181 will help you prepare for the material covered in the first section of the next chapter. In each exercise, find the product.

**179.** 
$$4x^{3}(4x^{2} - 3x + 1)$$
  
**180.**  $9xy(3xy^{2} - y + 9)$   
**181.**  $(x + 3)(x^{2} + 5)$ 

# **GROUP PROJECT**



#### ■ PUTTING NUMBERS INTO PERSPECTIVE

A large number can be put into perspective by comparing it with another number. For example, we put the \$8.6 trillion national debt into perspective by comparing it to the number of U.S. citizens. We did the same thing with total tax collections in Exercises 147–148 of Exercise Set 5.7.

For this project, each group member should consult an almanac, a newspaper, or the World Wide Web to find a number greater than one million. Explain to other members of the group the context in which the large number is used. Express the number in scientific notation. Then put the number into perspective by comparing it with another number.

# **Chapter 5 Summary**

#### **Definitions and Concepts**

## Examples

#### Section 5.1 Adding and Subtracting Polynomials

Polynomials

A polynomial is a single term or the sum of two or more terms containing variables with whole number exponents. A monomial is a polynomial with exactly one term; a binomial has exactly two terms; a trinomial has exactly three terms. The degree of a polynomial is the highest power of all the terms. The standard form of a polynomial is written in descending powers of the variable.

•
Monomial: $2x^5$
Degree is 5.
Binomial: $6x^3 + 5x$
Degree is 3.
Trinomial: $7x + 4x^2 - 5$
Degree is 2.

To add polynomials, add like terms.

The opposite, or additive inverse, of a polynomial is that polynomial with the sign of every coefficient changed. To subtract two polynomials, add the first polynomial and the opposite of the polynomial being subtracted.

The graphs of equations defined by polynomials of degree 2, shaped like bowls or inverted bowls, can be obtained using the point-plotting method.

$$(6x^3 + 5x^2 - 7x) + (-9x^3 + x^2 + 6x)$$
  
=  $(6x^3 - 9x^3) + (5x^2 + x^2) + (-7x + 6x)$   
=  $-3x^3 + 6x^2 - x$ 

 $(5y^3 - 9y^2 - 4) - (3y^3 - 12y^2 - 5)$ =  $(5y^3 - 9y^2 - 4) + (-3y^3 + 12y^2 + 5)$ =  $(5y^3 - 3y^3) + (-9y^2 + 12y^2) + (-4 + 5)$ =  $2y^3 + 3y^2 + 1$ 





# Summary **393**

Definitions and Concepts	Examples	
Section 5.2 Multiplying Polynomials		
Properties of Exponents Product Rule: $b^m \cdot b^n = b^{m+n}$ Power Rule: $(b^m)^n = b^{mn}$ Products to Powers: $(ab)^n = a^n b^n$	$x^{3} \cdot x^{8} = x^{3+8} = x^{11}$ $(x^{3})^{8} = x^{3\cdot 8} = x^{24}$ $(-5x^{2})^{3} = (-5)^{3}(x^{2})^{3} = -125x^{6}$	
To multiply monomials, multiply coefficients and add exponents.	$(-6x^4)(3x^{10}) = -6 \cdot 3x^{4+10} = -18x^{14}$	
To multiply a monomial and a polynomial, multiply each term of the polynomial by the monomial.	$2x^{4}(3x^{2} - 6x + 5)$ = $2x^{4} \cdot 3x^{2} - 2x^{4} \cdot 6x + 2x^{4} \cdot 5$ = $6x^{6} - 12x^{5} + 10x^{4}$	
To multiply polynomials when neither is a monomial, multiply each term of one polynomial by each term of the other polynomial. Then combine like terms.	(2x + 3)(5x2 - 4x + 2) $= 2x(5x2 - 4x + 2) + 3(5x2 - 4x + 2)$ $= 10x3 - 8x2 + 4x + 15x2 - 12x + 6$ $= 10x3 + 7x2 - 8x + 6$	
Section 5.3 Special Products		
The FOIL method may be used when multiplying two binomials: First terms multiplied. Outside terms multiplied. Inside terms multiplied. Last terms multiplied.	$(3x + 7)(2x - 5) = 3x \cdot 2x + 3x(-5) + 7 \cdot 2x + 7(-5)$ $= 6x^{2} - 15x + 14x - 35$ $= 6x^{2} - x - 35$	
The Product of the Sum and Difference of Two Terms $(A + B)(A - B) = A^2 - B^2$	$(4x + 7)(4x - 7) = (4x)^2 - 7^2$ = 16x <sup>2</sup> - 49	
The Square of a Binomial Sum $(A + B)^2 = A^2 + 2AB + B^2$	$(x^{2} + 6)^{2} = (x^{2})^{2} + 2 \cdot x^{2} \cdot 6 + 6^{2}$ $= x^{4} + 12x^{2} + 36$	
The Square of a Binomial Difference $(A - B)^2 = A^2 - 2AB + B^2$	$(9x - 3)^2 = (9x)^2 - 2 \cdot 9x \cdot 3 + 3^2$ = 81x <sup>2</sup> - 54x + 9	
Section 5.4 Polynomials in Several Variables		
To evaluate a polynomial in several variables, substitute the given value for each variable and perform the resulting computation.	Evaluate $4x^2y + 3xy - 2x$ for $x = -1$ and $y = -3$ . $4x^2y + 3xy - 2x$	

 $= 4(-1)^2(-3) + 3(-1)(-3) - 2(-1)$ 

= 4(1)(-3) + 3(-1)(-3) - 2(-1)

= -12 + 9 + 2 = -1



To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.  $\frac{8x^6 - 4x^3 + 10x}{2x}$ =  $\frac{8x^6}{2x} - \frac{4x^3}{2x} + \frac{10x}{2x}$ =  $4x^{6-1} - 2x^{3-1} + 5x^{1-1} = 4x^5 - 2x^2 + 5$ 

#### Section 5.6 Long Division of Polynomials; Synthetic Division

To divide a polynomial by a binomial, begin by arranging the polynomial in descending powers of the variable. If a power of a variable is missing, add that power with a coefficient of 0. Repeat the four steps—divide, multiply, subtract, bring down the next term—until the degree of the remainder is less than the degree of the divisor.

Divide: 
$$(2x^3 - x^2 - 7) \div (x - 2)$$
.  

$$\frac{2x^2 + 3x + 6}{x - 2)2x^3 - x^2 + 0x - 7}$$

$$\frac{2x^3 - 4x^2}{3x^2 + 0x}$$

$$\frac{3x^2 - 6x}{6x - 7}$$

$$\frac{6x - 12}{5}$$
The answer is  $2x^2 + 3x + 6 + \frac{5}{x - 2}$ .
## **Definitions and Concepts**

# **Examples**

#### Section 5.6 Long Division of Polynomials; Synthetic Division (continued)

A shortcut to long division, called synthetic division, can be used to divide a polynomial by a binomial of the form x - c. Here is the division problem shown on the previous page using synthetic division.



The answer is  $2x^2 + 3x + 6 + \frac{5}{x-2}$ .



Negative Exponents in Numerators and Denominators If  $b \neq 0$ ,  $b^{-n} = \frac{1}{b^n}$  and  $\frac{1}{b^{-n}} = b^n$ .

$$6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$
$$\frac{1}{(-2)^{-4}} = (-2)^4 = 16$$
$$\left(\frac{2}{3}\right)^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{3^3}{2^3} = \frac{27}{8}$$

An exponential expression is simplified when

- Each base occurs only once.
- No parentheses appear.
- No powers are raised to powers.
- No negative or zero exponents appear.

A positive number in scientific notation is expressed as  $a \times 10^n$ , where  $1 \le a < 10$  and *n* is an integer.

Simplify: 
$$\frac{(2x^4)^3}{x^{18}}$$
.  
 $\frac{(2x^4)^3}{x^{18}} = \frac{2^3(x^4)^3}{x^{18}} = \frac{8x^{4\cdot 3}}{x^{18}} = \frac{8x^{12}}{x^{18}} = 8x^{12-18} = 8x^{-6} = \frac{8}{x^6}$ 

Write  $2.9 \times 10^{-3}$  in decimal notation.

$$2.9 \times 10^{-3} = 0.0029 = 0.0029$$

Write 16,000 in scientific notation.

$$16,000 = 1.6 \times 10^4$$

Use properties of exponents with base 10

 $10^{m} \cdot 10^{n} = 10^{m+n}, \quad \frac{10^{m}}{10^{n}} = 10^{m-n}, \text{ and } (10^{m})^{n} = 10^{mn}$  to perform computations with scientific notation.

$$(5 \times 10^{3})(4 \times 10^{-8})$$
  
= 5 \cdot 4 \times 10^{3-8}  
= 20 \times 10^{-5}  
= 2 \times 10^{1} \times 10^{-5} = 2 \times 10^{-4}

## **CHAPTER 5 REVIEW EXERCISES**

**5.1** In Exercises 1–3, identify each polynomial as a monomial, binomial, or trinomial. Give the degree of the polynomial.

- **1.**  $7x^4 + 9x$ **2.**  $3x + 5x^2 - 2$
- **3**. 16*x*

In Exercises 4–8, add or subtract as indicated.

- **4.**  $(-6x^3 + 7x^2 9x + 3) + (14x^3 + 3x^2 11x 7)$
- **5.**  $(9y^3 7y^2 + 5) + (4y^3 y^2 + 7y 10)$
- **6.**  $(5y^2 y 8) (-6y^2 + 3y 4)$ **7.**  $(13x^4 - 8x^3 + 2x^2) - (5x^4 - 3x^3 + 2x^2 - 6)$
- **8**. Subtract  $x^4 + 7x^2 11x$  from  $-13x^4 6x^2 + 5x$ .
- In Exercises 9–11, add or subtract as indicated.

**9.** Add.  $7y^4 - 6y^3 + 4y^2 - 4y$  $y^3 - y^2 + 3y - 4$ 

**10.** Subtract.  $7x^2 - 9x + 2$   $-(4x^2 - 2x - 7)$  **11.** Subtract.  $5x^3 - 6x^2 - 9x + 14$  $-(-5x^3 + 3x^2 - 11x + 3)$ 

*In Exercises 12–13, graph each equation.* **12.**  $y = x^2 + 3$ **13.**  $y = 1 - x^2$ 

**5.2** In Exercises 14–18, simplify each expression. **14.**  $x^{20} \cdot x^3$  **15.**  $y \cdot y^5 \cdot y^8$  **16.**  $(x^{20})^5$  **17.**  $(10y)^2$  **18.**  $(-4x^{10})^3$ 

In Exercises 19–27, find each product.

**19.**  $(5x)(10x^3)$  **20.**  $(-12y^7)(3y^4)$  **21.**  $(-2x^5)(-3x^4)(5x^3)$  **22.**  $7x(3x^2 + 9)$  **23.**  $5x^3(4x^2 - 11x)$  **24.**  $3y^2(-7y^2 + 3y - 6)$  **25.**  $2y^5(8y^3 - 10y^2 + 1)$  **26.**  $(x + 3)(x^2 - 5x + 2)$ **27.**  $(3y - 2)(4y^2 + 3y - 5)$ 

In Exercises 28–29, use a vertical format to find each product.

**28.**  $y^2 - 4y + 7$ <u>3y - 5</u> **29.**  $4x^3 - 2x^2 - 6x - 1$ 2x + 3 **30.** (x + 6)(x + 2) **31.** (3y - 5)(2y + 1) **32.**  $(4x^2 - 2)(x^2 - 3)$  **33.** (5x + 4)(5x - 4) **34.** (7 - 2y)(7 + 2y) **35.**  $(y^2 + 1)(y^2 - 1)$  **36.**  $(x + 3)^2$  **37.**  $(3y + 4)^2$  **38.**  $(y - 1)^2$  **39.**  $(5y - 2)^2$  **40.**  $(x^2 + 4)^2$  **41.**  $(x^2 + 4)(x^2 - 4)$ **42.**  $(x^2 + 4)(x^2 - 5)$ 

**5.3** In Exercises 30–42, find each product.

**43**. Write a polynomial in descending powers of *x* that represents the area of the shaded region.



**44**. The parking garage shown in the figure measures 30 yards by 20 yards. The length and the width are each increased by a fixed amount, *x* yards. Write a trinomial that describes the area of the expanded garage.



# 5.4

**45.** Evaluate  $2x^3y - 4xy^2 + 5y + 6$  for x = -1 and y = 2.

**46**. Determine the coefficient of each term, the degree of each term, and the degree of the polynomial:

$$4x^2y + 9x^3y^2 - 17x^4 - 12.$$

In Exercises 47–56, perform the indicated operations. **47.**  $(7x^2 - 8xy + y^2) + (-8x^2 - 9xy + 4y^2)$ **48.**  $(13x^3y^2 - 5x^2y - 9x^2) - (11x^3y^2 - 6x^2y - 3x^2 + 4)$ **49.**  $(-7x^2y^3)(5x^4y^6)$ **50.**  $5ab^2(3a^2b^3 - 4ab)$ **51.** (x + 7y)(3x - 5y)**52.** (4xy - 3)(9xy - 1)**53.**  $(3x + 5y)^2$ **54.**  $(xy - 7)^2$ **55.** (7x + 4y)(7x - 4y)**56.**  $(a - b)(a^2 + ab + b^2)$ 

<b>5.5</b> In E.	xercises 57–63, simplify each expression.
<b>57.</b> $\frac{6^{40}}{6^{10}}$	<b>58</b> . $\frac{x^{18}}{x^3}$
<b>59</b> . (-10) <sup>0</sup>	<b>60.</b> $-10^{0}$
<b>61</b> . 400 <i>x</i> <sup>0</sup>	<b>62.</b> $\left(\frac{x^4}{2}\right)^3$
$\textbf{63.} \left(\frac{-3}{2y^6}\right)^4$	
In Exercises	64–68, divide and check each answer.

64. 
$$\frac{-15y^8}{3y^2}$$
  
65.  $\frac{40x^4}{5x^2}$   
66.  $\frac{18x^4 - 12x^2 + 36x}{6x}$   
67.  $\frac{30x^8 - 25x^7 - 40x^5}{-5x^3}$   
68.  $\frac{27x^3y^2 - 9x^2y - 18xy^2}{3xy}$ 

<b>5.6</b> In Exercises 69–73, divide and check each answer.
<b>69.</b> $\frac{2x^2 + 3x - 14}{x - 2}$
<b>70.</b> $\frac{2x^3 - 5x^2 + 7x + 5}{2x + 1}$
<b>71.</b> $\frac{x^3 - 2x^2 - 33x - 7}{x - 7}$
<b>72.</b> $\frac{y^3 - 27}{y - 3}$
<b>73.</b> $(4x^4 + 6x^3 + 3x - 1) \div (2x^2 + 1)$
In Exercises 74–76, divide using synthetic division.

**74.** 
$$(4x^3 - 3x^2 - 2x + 1) \div (x + 1)$$
  
**75.**  $(3x^4 - 2x^2 - 10x - 20) \div (x - 2)$   
**76.**  $(x^4 + 16) \div (x + 4)$ 

**5.7** In Exercises 77–81, write each expression with positive exponents only and then simplify.

**77.** 
$$7^{-2}$$
 **78.**  $(-4)^{-3}$ 

**79.** 
$$2^{-1} + 4^{-1}$$

**80.** 
$$\frac{1}{5^{-2}}$$
 **81.**  $\left(\frac{2}{5}\right)^{-3}$ 

In Exercises 82–90, simplify each exponential expression. Assume that variables in denominators do not equal zero.

82. 
$$\frac{x^3}{x^9}$$
 83.  $\frac{30y^6}{5y^8}$ 

 84.  $(5x^{-7})(6x^2)$ 
 85.  $\frac{x^4 \cdot x^{-2}}{x^{-6}}$ 

 86.  $\frac{(3y^3)^4}{y^{10}}$ 
 87.  $\frac{y^{-7}}{(y^4)^3}$ 

 88.  $(2x^{-1})^{-3}$ 
 89.  $\left(\frac{x^7}{x^4}\right)^{-2}$ 

 90.  $\frac{(y^3)^4}{(y^{-2})^4}$ 

In Exercises 91-93, write each number in decimal notation without the use of exponents.

91.	$2.3 \times 10^{4}$	92.	1.76 $\times$	$10^{-3}$
93.	$9 \times 10^{-1}$			

In Exercises 94–97, write each number in scientific notation. 94. 73,900,000 **95.** 0.00062

96	0.38	97	38
30.	0.50	37.	5.0

In Exercises 98–100, perform the indicated computation. Write the answers in scientific notation.

**98.** 
$$(6 \times 10^{-3})(1.5 \times 10^{6})$$

**99.** 
$$\frac{2 \times 10^2}{4 \times 10^{-3}}$$

**100.**  $(4 \times 10^{-2})^2$ 

In Exercises 101–102, use 10<sup>6</sup> for one million and 10<sup>9</sup> for one billion to rewrite the number in each statement in scientific notation.

- 101. According to the Tax Foundation, in 2005, individuals and companies in the United States spent approximately \$257 billion on tax preparation costs.
- 102. According to the Internal Revenue Service, in 2005, approximately 175 million tax returns were filed.
- 103. Use your scientific notation answers from Exercises 101 and 102 to answer this question: If the total 2005 tax preparation costs were evenly divided among all tax returns, how much would it cost to prepare each return? Express the answer in decimal notation, rounded to the nearest dollar.

#### CHAPTER 5 TEST

Remember to use your Chapter Test Prep Video CD to see the worked-out solutions to the test questions you want to review.

**1.** Identify  $9x + 6x^2 - 4$  as a monomial, binomial, or trinomial. Give the degree of the polynomial.

In Exercises 2–3, add or subtract as indicated.

- **2.**  $(7x^3 + 3x^2 5x 11) + (6x^3 2x^2 + 4x 13)$
- **3.**  $(9x^3 6x^2 11x 4) (4x^3 8x^2 13x + 5)$
- 4. Graph the equation:  $y = x^2 3$ . Select integers for x, starting with -3 and ending with 3.

In Exercises 5–11, find each product.

**5.**  $(-7x^3)(5x^8)$ **6.**  $6x^2(8x^3 - 5x - 2)$ 7.  $(3x + 2)(x^2 - 4x - 3)$ **8.** (3y + 7)(2y - 9)**9.** (7x + 5)(7x - 5)**10.**  $(x^2 + 3)^2$ **11.**  $(5x - 3)^2$ **12.** Evaluate  $4x^2y + 5xy - 6x$  for x = -2 and y = 3. In Exercises 13–15, perform the indicated operations. **13.**  $(8x^2y^3 - xy + 2y^2) - (6x^2y^3 - 4xy - 10y^2)$ **14.** (3a - 7b)(4a + 5b)**15.**  $(2x + 3y)^2$ In Exercises 16–18, divide and check each answer. **16.**  $\frac{-25x^{16}}{5x^{16}}$ **17.**  $\frac{15x^4 - 10x^3 + 25x^2}{5x}$ **18.**  $\frac{2x^3 - 3x^2 + 4x + 4}{2x + 1}$ 

**19.** Divide using synthetic division:

$$(3x^4 + 11x^3 - 20x^2 + 7x + 35) \div (x + 5).$$

In Exercises 20–21, write each expression with positive exponents only and then simplify.

**20.** 
$$10^{-2}$$
 **21.**  $\frac{1}{4^{-1}}$ 

In Exercises 22–27, simplify each expression.

**22.** 
$$(-3x^2)^3$$
  
**23.**  $\frac{20x}{5x^8}$   
**24.**  $(-7x^{-8})(3x^2)$   
**25.**  $\frac{(2y^3)^4}{y^8}$   
**26.**  $(5x^{-4})^{-2}$   
**27.**  $\left(\frac{x^{10}}{x^5}\right)^{-3}$ 

- **28.** Write  $3.7 \times 10^{-4}$  in decimal notation.
- **29**. Write 7,600,000 in scientific notation.

*In Exercises 30–31, perform the indicated computation. Write the answers in scientific notation.* 

**30.** 
$$(4.1 \times 10^2)(3 \times 10^{-5})$$

**31.** 
$$\frac{8.4 \times 10^6}{4 \times 10^{-2}}$$

**32.** Write a polynomial in descending powers of x that represents the area of the figure.



# CUMULATIVE REVIEW EXERCISES (CHAPTERS 1-5)

In Exercises 1–2, perform the indicated operation or operations.

**1.**  $(-7)(-5) \div (12 - 3)$ 

- **2.**  $(3-7)^2(9-11)^3$
- **3.** What is the difference in elevation between a plane flying 14,300 feet above sea level and a submarine traveling 750 feet below sea level?

#### In Exercises 4–5, solve each equation.

**4.** 2(x + 3) + 2x = x + 4

- **5.**  $\frac{x}{5} \frac{1}{3} = \frac{x}{10} \frac{1}{2}$
- **6.** The length of a rectangular sign is 2 feet less than three times its width. If the perimeter of the sign is 28 feet, what are its dimensions?
- 7. Solve  $7 8x \le -6x 5$  and graph the solution set on a number line.

- **8.** You invested \$6000 in two accounts paying 12% and 14% annual interest. At the end of the year, the total interest from these investments was \$772. How much was invested at each rate?
- **9.** You need to mix a solution that is 70% antifreeze with one that is 30% antifreeze to obtain 20 liters of a mixture that is 60% antifreeze. How many liters of each of the solutions must be used?
- **10.** Graph  $y = -\frac{2}{5}x + 2$  using the slope and y-intercept.
- **11.** Graph x 2y = 4 using intercepts.
- **12.** Find the slope of the line passing through the points (-3, 2) and (2, -4). Is the line rising, falling, horizontal, or vertical?
- **13.** The slope of a line is -2 and the line passes through the point (3, -1). Write the line's equation in point-slope form and slope-intercept form.

In Exercises 14–15, solve each system by the method of your choice.

**14.** 
$$3x + 2y = 10$$
  
 $4x - 3y = -15$   
**15.**  $2x + 3y = -6$ 

- **15.** 2x + 3y = -6y = 3x - 13
- **16.** You are choosing between two long-distance telephone plans. One has a monthly fee of \$15 with a charge of \$0.05 per minute for all long-distance calls. The other plan has a monthly fee of \$5 with a charge of \$0.07 per minute for all long-distance calls. For how many minutes of long-distance calls will the costs for the two plans be the same? What will be the cost for each plan?
- **17**. Write in scientific notation: 0.0024.

**18.** Subtract: 
$$(9x^5 - 3x^3 + 2x - 7) - (6x^5 + 3x^3 - 7x - 9).$$

**19.** Divide: 
$$\frac{x^3 + 3x^2 + 5x + 3}{x + 1}$$
.  
**20.** Simplify: 
$$\frac{(3x^2)^4}{x^{10}}$$
.