Section 3.5 Derivatives of Trig Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\sin f(x)) = \cos(f(x)) \cdot f'(x)$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\cos f(x)) = -\sin(f(x)) \cdot f'(x)$$

ex: $\frac{d}{dx}\left(\overset{c \cdot f(x)}{2\sin(3x)}\right) = \overset{c \cdot f'(x)}{2\cos(3x) \cdot 3} = 6\cos(3x)$

ex: $\frac{d}{dx}\left(\cos^2 x\right) = \frac{d}{dx}\left((\cos x)^2\right) = 2(\cos x)^1 \cdot (-\sin x)$

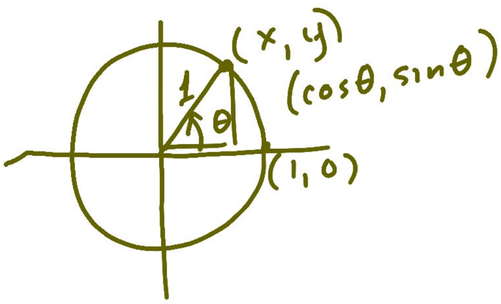
$$= -2\cos x \sin x$$

$$= -2\sin(2x)$$

ex: $\frac{d}{dx}\left(\cos(x^2)\right) = -\sin(x^2) \cdot 2x = -2x\sin(x^2)$

ex: $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$$= \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{(\cos x)^2}$$



$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

ex: $g(\theta) = \sin(\tan\theta)$

$g'(\theta) = \cos(\tan\theta) \cdot \sec^2\theta$

p131
3-31 odd

ex: $z = \theta e^{\cos\theta}$

$$z' = \underset{\text{1st}}{\theta}\,\underbrace{e^{\cos\theta} \cdot (-\sin\theta)}_{\text{deriv 2nd}} + \underbrace{e^{\cos\theta}}_{\text{2nd}} \cdot \underbrace{1}_{\text{deriv of 1st}}$$

$$z = e^{\cos\theta}\left(-\theta\sin\theta + 1\right)$$

44p126 $\quad f(t) = 2e^{\overbrace{-2e^{2t}}} = \underset{c}{2} \cdot \underbrace{e^{-2e^{2t}}}_{f(x)}$

$$f'(t) = 2e^{-2e^{2t}} \cdot -2e^{2t} \cdot 2 = -8e^{-2e^{2t}}e^{2t}$$

$=$

53p126

$F(2) = 1 \qquad G(4) = 2$

$F'(2) = 5 \qquad G'(4) = 6$

$F(4) = 3 \qquad G(3) = 4$

$F'(4) = 7 \qquad G'(3) = 8$

c) $H(4) = G(F(4))$
$= G(3)$
$= 4$

a) $H(4)$ if $H(x) = F(G(x)) = F(\underbrace{G(4)}) = F(2) = 1$

b) $H'(4) \quad H(x) = F(G(x))$

$H'(4) = F'(G(4)) \cdot G'(4)$
$= F'(2) \cdot G'(4)$
$= 5 \cdot 6$
$= 30$

d) $H'(4) = G'(F(4))F'(4)$
$H'(4) = G'(3)F'(4)$
$H'(4) = 8 \cdot 7$
$= 56$

e) $H'(4) = \dfrac{G \cdot F' - F \cdot G'}{G^2}$

$= \dfrac{2 \cdot 7 - 3 \cdot 6}{2^2}$

$= -1$

54) $f(1)=4 \quad f'(1)=3$

a) $g'(1)$ if $g(x)=\sqrt{f(x)}$

$$g'(x)=\frac{1}{2}(f(x))^{-1/2}\cdot f'(x)$$

$$g'(1)=\frac{1}{2}\cdot(f(1))^{-1/2}\cdot f'(1)$$

$$g'(1)=\frac{1}{2}\cdot 4^{-1/2}\cdot 3=\frac{1}{2}\cdot\frac{1}{\sqrt{4}}\cdot 3$$

$$g'(1)=\frac{3}{4}$$

b) $h'(1)$ if $h(x)=f(\sqrt{x})$

$$h'(x)=f'(\sqrt{x})\cdot\frac{1}{2}x^{-1/2}$$

$$h'(1)=f'(\sqrt{1})\cdot\frac{1}{2\sqrt{1}}$$

$$=3\cdot\frac{1}{2}$$

$$=\frac{3}{2}$$

56) $y=k(x)$

$k'(1)=2$

a) $k(2x)$

$$k'(2x)\cdot 2=k'\left(2\cdot\frac{1}{2}\right)\cdot 2$$

$$=k'(1)\cdot 2$$

$$=2\cdot 2=4$$

b) $k(x+1)$

$$k'(x+1)=k'(0+1)=k'(1)=2$$

c) $k\left(\frac{1}{4}x\right)$

$$k'\left(\frac{1}{4}x\right)\cdot\frac{1}{4}=k'(1)\cdot\frac{1}{4}=2\cdot\frac{1}{4}=\frac{1}{2}$$

58) $x^5e^{(x^6)}$

$$\frac{1}{6}e^{(x^6)}$$